New insights into dense two-color matter

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Abstract

We study SU(2) lattice gauge theory with two Wilson quark flavours at non-zero chemical potential and low temperature. As the baryon chemical potential is varied, we find further evidence of three distinct physical regimes: vacuum/hadronic, quarkyonic and deconfined phases. In this presentation we focus mainly on the following observables: quark number susceptibility, chiral condensate (using naive fermions) and eigenvalues of the Dirac operator.

Introduction

- Intensive efforts are underway to unveil the phase structure of strongly interacting matter at high density and low temperature.
- A wealth of information exists regarding possible phases and their properties in various models but no reliable, quantitative results are available yet.
- Many questions could in principle be answered by lattice QCD simulations, but unfortunately their practical feasibility is limited, due to the existence of the so called "sign problem".
- Lattice simulations may still be used to study QCD-like theories without a sign problem providing first-principles, nonperturbative results: this is the main aim of the present study.
- We focus on QCD with gauge group SU(2) (two-color QCD or QC$_2$D); it is of particular interest because it shares most of the salient features of real QCD (eg, confinement, dynamical chiral symmetry breaking and long-range interactions). It differs from QCD in that the baryons of the theory are bosons, and the lightest baryon is a pseudo-Goldstone boson, degenerate with the pion.
- We study QC$_2$D with conventional Wilson action for the gauge fields and two flavours of Wilson fermion plus a *diquark source* term (μ baryon chemical potential, $M(\mu)$ is the Wilson fermion matrix):

$$S_f = \bar\psi_1 M(\mu)\psi_1 + \bar\psi_2 M(\mu)\psi_2 + J\left\{\psi_2^{tr}(C\gamma_5)\tau_2\psi_1 - \bar\psi_1(C\gamma_5)\tau_2\bar\psi_2^{tr}\right\} . \quad (1)$$

- It is convenient to introduce the change of variables: $\bar\phi = -\psi_2^{tr}C\tau_2$, $\phi = C^{-1}\tau_2\bar\psi_2^{tr}$ and $\psi = \psi_1$, $\bar\psi = \bar\psi_1$. Using it, we can rewrite S_f as:

$$S_f = (\bar\psi\bar\phi)\begin{pmatrix} M(\mu) & J\gamma_5 \\ -\bar J\gamma_5 & M(-\mu)\end{pmatrix}\begin{pmatrix}\psi\\ \phi\end{pmatrix} \equiv \bar\Psi\mathcal{M}\Psi . \quad (2)$$

Expectations

At low temperature, increasing μ, we expect to see mainly three regimes:

- $\mu < \mu_0$: vacuum.
- $\mu \gtrsim \mu_0$: dilute tightly-bound quarks (weakly interacting baryons) BEC.
- $\mu \gg \mu_0$: weakly interacting quarks Cooper pairs BCS.

From chiral perturbation theory ($m_\pi \ll m_\rho$) we know that when $\mu \geq \mu_0 \equiv \frac{1}{2}m_\pi$:

- $n_q > 0$: non zero baryon density.
- $\langle qq\rangle \neq 0$: a gauge invariant superfluid order parameter (spontaneous breaking of U(1) baryon number).

Order Parameters and Observables

- Polyakov Loop: $L = \langle \mathrm{Tr}\prod_{n_4=1}^{N_\tau} U_4(n,n_4)\rangle$ (confinement).
- Diquark condensate: $\langle qq\rangle = \frac{1}{V}\frac{\partial \ln Z}{\partial j}$ (superfluidity).
  - BCS: $\langle qq\rangle \propto \mu^2\Delta$ (number of Cooper pairs condensed in the ground state).
  - BEC: $\langle qq\rangle^2 + \langle\bar qq\rangle^2 = 1, \quad \mu < m_\pi/2 \Rightarrow \langle qq\rangle = 0, \quad \mu > m_\pi/2 \Rightarrow \langle qq\rangle = \langle\bar qq\rangle_0\sqrt{1-\left(\frac{m_\pi}{2\mu}\right)^4}$
- Chiral condensate $\langle\bar qq\rangle$: the use of Wilson fermions precludes any direct study of chiral symmetry. Our strategy is to calculate $\langle\bar\psi\psi\rangle$ using a fermion formulation with manifest chiral symmetry (valence quarks), obtained by setting the Wilson parameter $r = 0$, and using for the gauge background ensembles those generated with Wilson quarks (sea quarks).
  We tuned the value of κ (for $r = 0$) to have the same value of m_π (for $r = 1$)
- Quark number density n_q
- Quark number susceptibility (it gives information about the deconfinement of *light quarks*)

$$\chi_q = \partial n_q/\partial\mu = \frac{T}{V_s}\left\{-\langle\left[-\bar\Psi\frac{\partial\mathcal{M}}{\partial\mu}\Psi\right]\rangle^2 + \langle\left[-\bar\Psi\frac{\partial\mathcal{M}}{\partial\mu}\Psi\right]^2\rangle + \langle\left[-\bar\Psi\frac{\partial^2\mathcal{M}}{\partial\mu^2}\Psi\right]\rangle\right\} \quad (3)$$

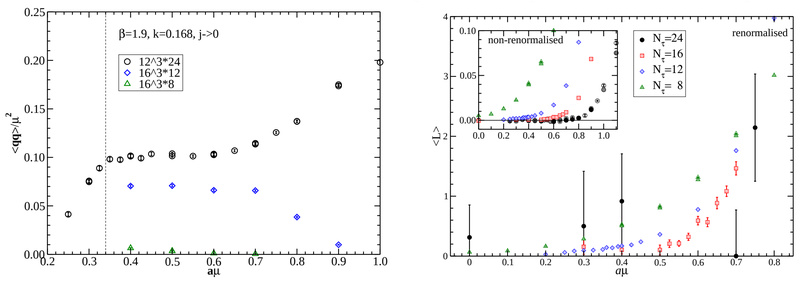
- Eigenvalues of the Dirac operator $\mathcal{M}$; they are determined using the ARPACK library (Implicitly Restarted Arnoldi Method)

Results

Parameters of the simulations presented here: $\beta = 1.9$, $\kappa = 0.168$;
$m_\pi = 0.645(8)$, $m_\pi/m_\rho = 0.805(9)$, $a = 0.178(6)$ fm, $T_d(\mu = 0) = 217(23)$ MeV

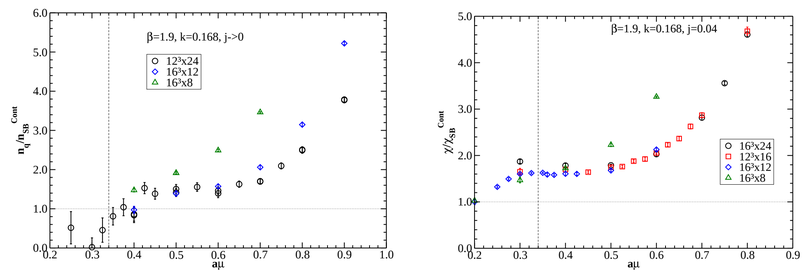
Correspondence lattice-temperature:
$N_\tau = 8 \to 132$MeV, $N_\tau = 12 \to 88$MeV, $N_\tau = 16 \to 66$MeV, $N_\tau = 24 \to 44$MeV

Diquark condensate and (renormalised) Polyakov loop at different temperature:



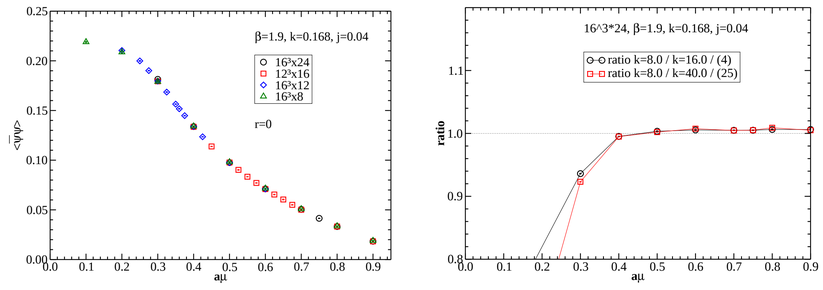
- $\langle qq\rangle$ estrapolated *linearly* to $j \to 0$.
- For the lowest temperatures $\langle qq\rangle \propto \mu^2$, *i.e.* BCS condensate develops, in a large range.
- For the highest T the diquark condensate is zero at all μ: the system is in the normal phase.
- No evidence of finite volume effects (we compared $12^3 \times 24$ with $16^3 \times 24$).
- The Polyakov Loop, for each T, increases rapidly from zero above a chemical potential $\mu_d(T)$.

Quark number density n_q and quark number susceptibility χ_q:



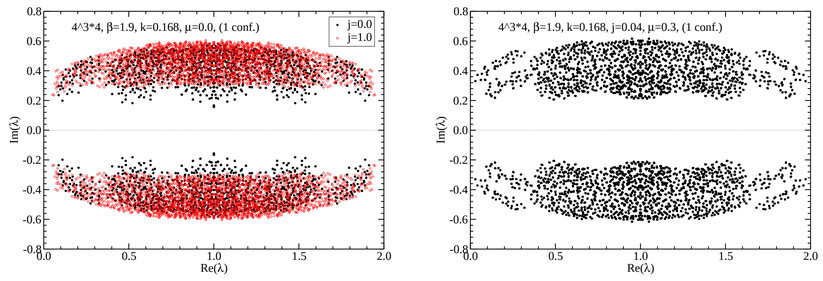
- There is no connection between the behaviour of the Polyakov Loop and χ_q.
- The different behaviour of the system at $N_\tau = 8$ suggests a different phase.

Chiral condensate and ratio $\frac{\kappa_1^2\langle\bar\psi\psi\rangle_{\kappa_1}}{\kappa_2^2\langle\bar\psi\psi\rangle_{\kappa_2}}$ ($r = 0$ and $\kappa = 8.0 \Rightarrow m_\pi = 0.66(2)$):



- The results suggest symmetry restoration for $a\mu \gtrsim 0.5$.
- In the symmetric phase $\langle\bar qq\rangle \propto m$ (using $\langle\bar qq\rangle = \kappa\langle\bar\psi\psi\rangle$).

Eigenvalue distribution (preliminary results, small volume):



- Note that there are no eigenvalues along the real axis.
- The introduction of j leaves the system without any degeneracy of the eigenvalues and they are pushed away from the real axis much further.

Conclusions

We find evidence of three regions at low T:

1. A vacuum/hadronic phase, with $\langle qq\rangle = 0, \langle L\rangle \approx 0, \langle\bar\psi\psi\rangle \neq 0, n_q \approx 0$, at low T and $\mu \lesssim \mu_0 = m_\pi/2$.
2. A quarkyonic phase, which is confined ($\langle L\rangle \approx 0$), chirally symmetric and characterised by SB scaling of bulk thermodynamic quantities and BCS scaling of the diquark condensate.
3. A deconfined quark–gluon plasma phase at high T (and/or large μ).

- We do not see a BEC phase (probably because the large ratio m_π/m_ρ).

