QCD Sum Rules Based on Commutation Relations

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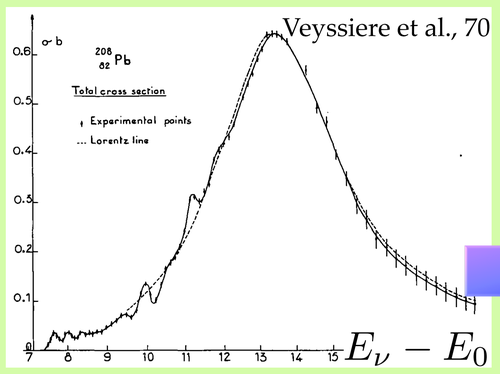
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1. Introduction

- Dipole sum rule (giant resonance)

$$\sigma_{\rm tot} = \frac{4\pi^2 e^2}{\hbar c}\sum_\nu (E_\nu - E_0)|\langle\nu|D|0\rangle|^2$$
$$= \frac{4\pi^2 e^2}{\hbar c}\langle 0|[D,[H,D]]|0\rangle$$
$$= \frac{2\pi^2 e^2\hbar}{mc}\frac{NZ}{A}(1+K)$$



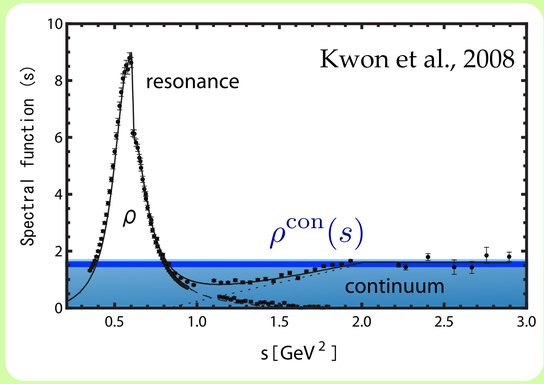
What about sum rules in QFT such as QCD ?

- QCD sum rules based on OPE [Shifman, Vainshtein & Zhakharov, 78]

$$\int \mathrm{d}s\; s^n(\rho(s) - \rho^{\rm con}(s)) = \text{condensates}$$

Resonance ⇔ $\langle 0|\bar{q}q|0\rangle$, $\langle 0|G^{a\mu\nu}G^a_{\mu\nu}|0\rangle$ etc.

Derive/ Generalize QCD sum rules from CRs without OPE



2. Sum rules based on CRs

- Spectral function

$$\rho(q^2) = -\frac{1}{3q^2}\sum_p (2\pi)^4\delta^{(4)}(q-p)\langle 0|j_\mu(0)|p\rangle\langle p|j^\mu(0)|0\rangle$$

- Energy weighted sum rules at zero 3MOM $\boxed{s\to\omega^2}$

$$\int_0^\infty \frac{\mathrm{d}s}{2\pi}\, s^n(\rho(s)-\rho^{\rm con}(s)) = -\frac{1}{3}\int \mathrm{d}^3x\,\langle 0|[[j_\mu(0,\vec{x}),\mathrm{H}]_{2n-1},j^\mu(0)]|0\rangle_{\rm NP}$$

$$\int_0^\infty \frac{\mathrm{d}s}{2\pi}\, s^{n-\frac{1}{2}}(\rho(s)-\rho^{\rm con}(s)) = \frac{2}{3V}\langle 0|([Q_\mu,\mathrm{H}]_{n-1})^2|0\rangle_{\rm NP}$$

- Perturbative UV divergence is subtracted

3. Hamiltonian formalism of QCD

- QCD Hamiltonian

$$\mathcal{H}_{\rm eff} = -gA^a_0\bar{q}_f\gamma^0 t^a q_f + \bar{q}_f(-i\gamma^k D_k + m_f)q_f + \frac{1}{2}\left((\vec{E}^a)^2 + (\vec{H}^a)^2\right)$$
$$+\vec{E}^a\cdot(\nabla A^a_0 - gf_{abc}\vec{A}^b A^c_0) + \partial_k B^a A^{ak} - \frac{\alpha}{2}(B^a)^2$$
$$+i\Pi^a_c\Pi^a_{\bar{c}} + gf_{abc}\Pi^a_c A^b_0 c^c - i\partial^k\bar{c}^a D^{\rm ad}_k c^a$$

- Heisenberg fields: $q_f,\ \bar{q}_f,\ A^a_\mu,\ B^a,\ E^a_k,\ c^a,\ \bar{c}^a,\ \Pi^a_c,\ \Pi^a_{\bar{c}}$ (quarks, gluons, ghosts)
- Canonical CRs: $\{q,\bar{q}\},\ [A^a,B^b],\ [A^a_i,E^{bj}],\ \{c^a,\Pi^b_c\}$
- Physical states are BRS invariant

4. Commutator anomaly

- Causality ➔ $[A(x),B(0)]_{\rm ET} = \sum C_i\hat{O}_i(0)\delta^{(3)}(x)$
- Commutator btw quark and gluon states

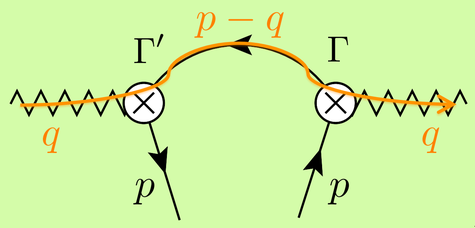
$$\int \mathrm{d}^3x\langle\alpha|[A(x),B(0)]_{\rm ET}|\beta\rangle = \lim_{q^0\to\infty}(-iq^0)\int \mathrm{d}^4x\; \mathrm{e}^{iq^0x^0}\langle\alpha|\mathrm{T}^*[A(x),B(0)]|\beta\rangle$$

- Free propagators ⇔ CCRs

$$\{q(x),\bar{q}(0)\}_{\rm ET} = \gamma^0\delta^{(3)}(x) \quad\Leftrightarrow\quad \lim_{q^0\to\infty}(-iq^0)\frac{i}{\not{q}-m} = \gamma^0$$
$$[A^a_k(x),E^{bk'}(0)]_{\rm ET} = i\delta^{ab}\delta_{kk'}\delta^{(3)}(x) \qquad \lim_{q^0\to\infty}(-iq^0)\delta^{ab}\delta_{kk'}\frac{-q_0}{q^2} = i\delta^{ab}\delta_{kk'}$$

- Commutator anomaly

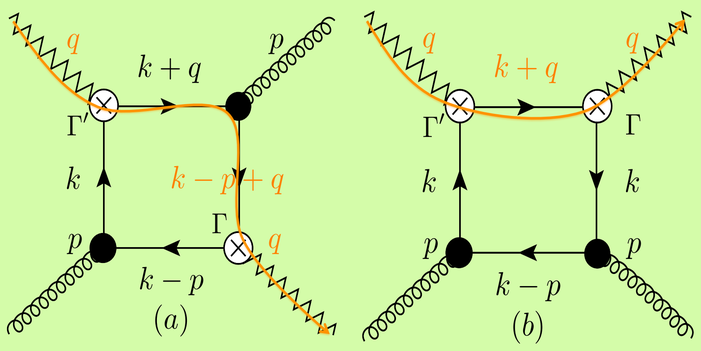
$\langle{\rm quark}|[\bar{q}\Gamma q,\bar{q}\Gamma' q]_{\rm ET}|{\rm quark}\rangle$



$$= (-iq^0)\Big(\langle{\rm quark}|\bar{q}_i\Gamma_{ij}\Gamma'_{kl}q_l|{\rm quark}\rangle S^{jk}_F(p-q) - \langle{\rm quark}|\bar{q}_k\Gamma'_{kl}\Gamma_{ij}q_j|{\rm quark}\rangle S^{li}_F(p+q)\Big)$$

$$= \langle{\rm quark}|\bar{q}_i\Gamma_{ij}\gamma^0_{jk}\Gamma'_{kl}q_l|{\rm quark}\rangle - \langle{\rm quark}|\bar{q}_k\Gamma'_{kl}\gamma^0_{li}\Gamma_{ij}q_j|{\rm quark}\rangle$$

$\langle{\rm gluon}|[\bar{q}\Gamma q,\bar{q}\Gamma' q]_{\rm ET}|{\rm gluon}\rangle$



(a) ➔ Deviation by gluon $\boxed{\alpha_s G^a_{\mu\nu}G^{a\mu\nu}}$

(b) ➔ Canonical commutator $\langle{\rm gluon}|\bar{q}\Gamma\gamma^0\Gamma' q - \bar{q}\Gamma'\gamma^0\Gamma q|{\rm gluon}\rangle$

5. Vector sum rules

- Vector current: $j^\mu_{\rm V} = (\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d)/2,\ j^\mu_{\rm A} = (\bar{u}\gamma^\mu\gamma^5 u - \bar{d}\gamma^\mu\gamma^5 d)/2$

$$\int_0^\infty \frac{\mathrm{d}s}{2\pi}\, s\Big(\rho_{\rm V}(s) - \rho^{\rm con}_{\rm V}(s)\Big) = -\frac{1}{3}\int \mathrm{d}^3x\;\langle 0|[[j^\mu_{\rm V}(0,x),\mathrm{H}],j_{{\rm V}\mu}(0,0)]|0\rangle_{\rm NP}$$
$$= \langle 0| - \frac{m_q}{2}\bar{q}q|0\rangle_{\rm NP}$$

Canonical commutator ➔ No gluon condensate

- Weinberg's sum rule holds

$$\int_0^\infty \frac{\mathrm{d}s}{2\pi}\, s(\rho_{\rm A}(s) - \rho_{\rm V}(s)) = \langle 0|\frac{4m_u}{3}\bar{u}u + \frac{4m_d}{3}\bar{d}d|0\rangle_{\rm NP}$$

- BJL modified commutator

$$[[\bar{u}\gamma^\mu u,\mathrm{H}],\bar{u}\gamma_\mu u] = -4i\bar{u}\gamma^k\overleftrightarrow{D}_k u + 12m_u\bar{u}u + \frac{\alpha_s}{2\pi}G^a_{kk'}G^a_{kk'}$$

Canonical commutator ➔ Chiral condensate

Commutator anomaly ➔ Gluon condensate

$$\int_0^\infty \frac{\mathrm{d}s}{2\pi}\, s\Big(\rho_{\rm V}(s) - \rho^{\rm con}_{\rm V}(s)\Big) = \langle 0| - \frac{m_q}{2}\bar{q}q - \frac{\alpha_s}{24\pi}G^a_{\mu\nu}G^{a\mu\nu}|0\rangle_{\rm NP}$$
$$\int_0^\infty \frac{\mathrm{d}s}{2\pi}\, s\Big(\rho_{\rm A}(s) - \rho^{\rm con}_{\rm A}(s)\Big) = \langle 0|\frac{5}{6}m_q\bar{q}q - \frac{\alpha_s}{24\pi}G^a_{\mu\nu}G^{a\mu\nu}|0\rangle_{\rm NP}$$

SVZ approach	Our approach
OPE as an operator identity	Commutators btw currents and the effective Hamiltonian
Perturbative calculation of the Wilson coefficients	Perturbative calculation of anomalies
Subtraction of contribution from the unit operator	Subtraction of perturbative vacuum graphs

6. Mass formulas by susceptibilities

- Sum rules by susceptibilities $\boxed{\chi_{\rm NP} = \langle 0|Q^2_0|0\rangle_{\rm NP}/V}$

$$\int_0^\infty \frac{\mathrm{d}s}{2\pi}\, s^{\frac{1}{2}}(\rho(s) - \rho^{\rm con}(s)) = \frac{8}{3}\chi_{\rm NP}$$

- Combining with dim 2 and 4 sum rules
  (Assuming $m^4_\# \gg |\langle m_q\bar{q}q\rangle_{\rm NP}|,\ |\langle\alpha_s G^2\rangle_{\rm NP}|$)

$$m_{\rho,\omega,\phi} = \left[\frac{32\pi^2}{3-2^{\frac{3}{2}}}(\chi_{\rho,\omega,\phi})_{\rm NP}\right]^{\frac{1}{3}}$$
$$m_{A_1} = \left[-128\pi^2(\chi_{A_1})_{\rm NP}\right]^{\frac{1}{3}} \quad \text{etc.}$$

- Fluctuation dominance ⇔ SSB dominance

c. f. Jaffe formulas $m_{\rm nucl} \sim \langle 0|\bar{q}q|0\rangle^{1/3}_{\rm NP}$

7. Discussions

In future investigation,

- Nucleon sum rules in vacuum/ in dense nuclear matter
- Medium QCD sum rules
- Other strongly interacting system