Indistinguishability of particles and thermodynamics

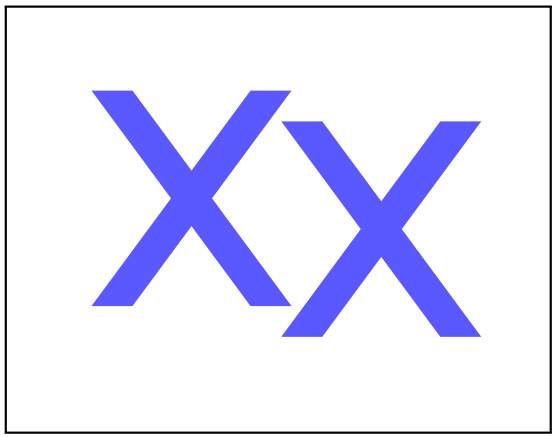
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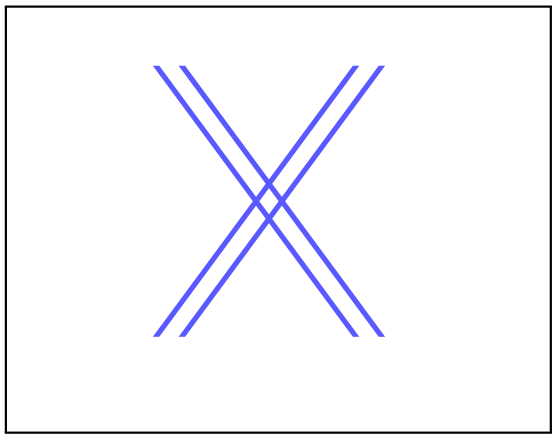
SEWM 2012, 10 July – 13 July 2012
Swansea University, Swansea, United Kingdom

For more details cf. arXiv:1206.0865

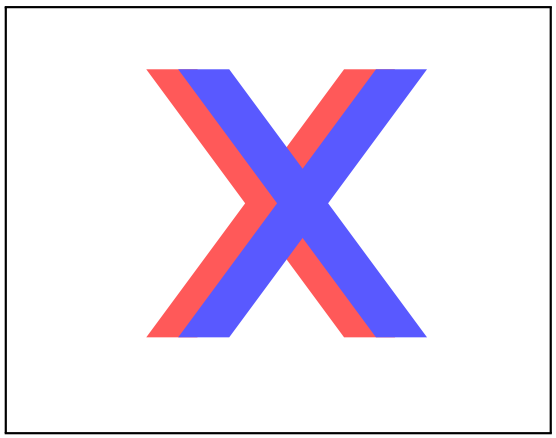
Some picture riddles: how many X do you see?



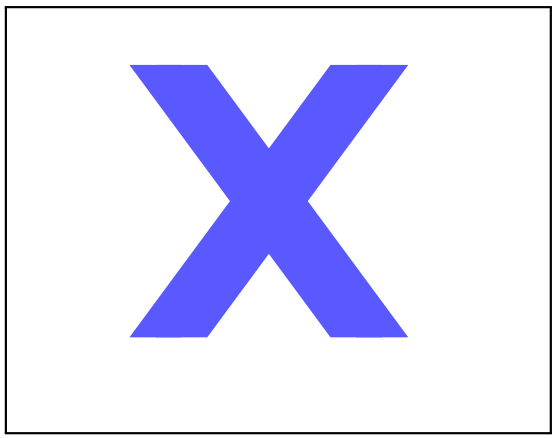
a.)



b.)



c.)



d.)

a.) there are two clean Xs

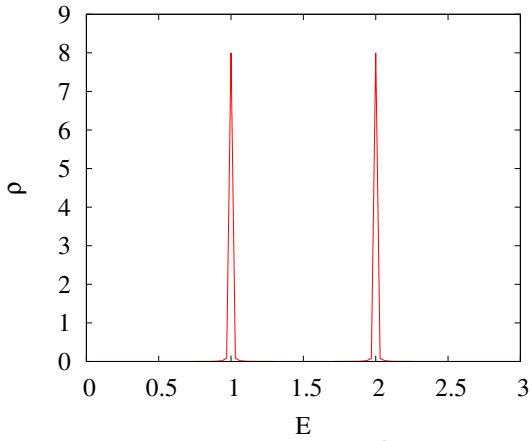
b.) the two Xs are distinguishable because they are thin

c.) the two Xs are distinguishable because they have different color

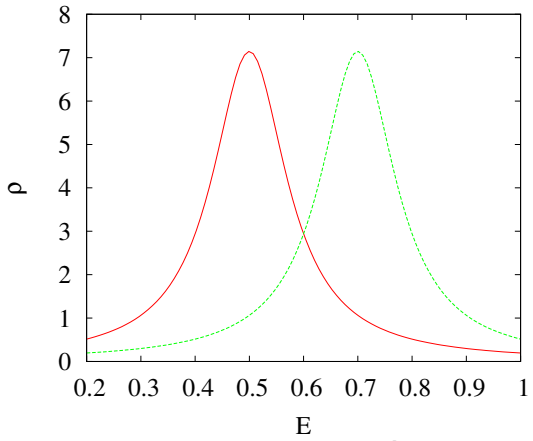
d.) there is only one thick X. . . or what else?

Some other picture riddles: how many peaks do you see?

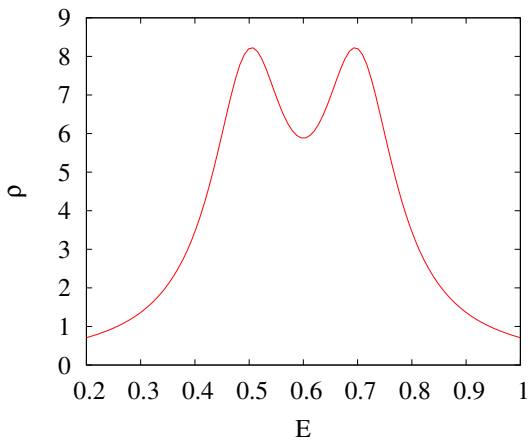
Peaks of ϱ are quasiparticles $\Rightarrow$ how many particles do you see?



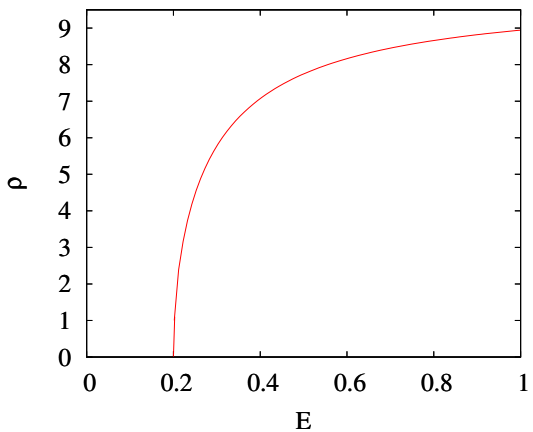
a.)



b.)



c.)



d.)

a.) there are two clean peaks

b.) the two peaks are distinguishable because they have different quantum numbers

c.) there is one peak with a dip, or two peaks . . .

d.) there is no peak, or there are infinitely many?

Most important question

how many peaks does a physical observable see?

How do we construct an effective model?

basic ideas

- respect symmetries of the fundamental model
- try to reproduce the spectrum most faithfully
- consistent field theory (unitary, causal, Lorentz-invariant)

Consequence: we must not have more fields than quantum numbers!

- particle number is a conserved quantity at tree level $\Rightarrow$ extra particles introduce extra quantum numbers, different physics
- physical reason: states with the same quantum numbers are indistinguishable
  $\Rightarrow$ we overcount the number of states

  if quasiparticles are well separated, or have different quantum numbers ✓

  problems in case of overlapping peaks or continuum ✗
- interactions can wash out the extra symmetry $\Rightarrow$ we need strong interactions, since we must change, in the overlapping peaks case, the tree level result by $\mathcal{O}(100\%)$
  $\Rightarrow$ non-perturbative theory is needed

Usual constructions:

- sigma models, Walecka model: one particle per quantum number ✓
  but they do not describe the excited hadron states ✗
- hadron resonance gas: describes excited hadron states ✓
  but it introduces many particles for each quantum channel ✗
- 2PI formalism: complete spectrum, correct symmetries ✓
  but perturbative ✗

New effective model construction is needed

It should reproduce the nonperturbative spectrum at fixed quantum numbers at tree level with one field

Most generic quadratic model

$$\mathcal{L} = \frac{1}{2}\Phi(x)\mathcal{K}(i\partial)\Phi(x)$$

Relation of $\mathcal{K}$ kernel and ϱ spectral function:

$$G_R(k_0, \mathbf{k}) = \mathcal{K}^{-1}(k_0 + i\varepsilon, \mathbf{k}), \qquad \varrho = -2\,\mathrm{Im}\,G_R$$
$$G_R(k_0, \mathbf{k}) = \int \frac{d\omega}{2\pi} \frac{\varrho(\omega, \mathbf{k})}{k_0 - \omega + i\varepsilon}, \qquad \mathcal{K} = \mathrm{Re}\,G_R^{-1}$$

spectrum completely determines phyisics!

Consistency (similar to 2PI resummation case)

- unitarity fulfilled if $\varrho(\omega > 0) \geq 0$ ✓ (J. Polonyi, A. Siwek, Phys. Rev. D81, 085040 (2010).)
  physically: we take into account all possible states
- causality: x space-like vector $\langle [A(x), B(0)] \rangle = 0 \quad \Leftrightarrow \quad \varrho(x) = 0$
  Now ϱ is an input $\quad \Rightarrow \quad$ causality ✓
- energy and momentum conservation: space and time translation symmetry ✓
- Lorentz-invariance: if kernel is Lorentz-invariant ✓

Observables at finite temperature

- Note: no direct relation between effective potential and free energy!
- Energy density

$$\varepsilon = T_{00} = \int \frac{d^4p}{(2\pi)^4}\,\Theta(p_0)\left(p_0\frac{\partial\mathcal{K}}{\partial p_0} - \mathcal{K}\right) n(p_0)\varrho(p)$$

  - for construction: Noether current belonging to time translation symmetry, finite temperature averaging (KMS relation) and renormalization
  - nonlinear functional of ϱ!
- pressure, entropy, etc.: come from standard thermodynamics
- Number of degrees of freedom: intuitive characterization of the spectrum

$$N_{dof} = \int_0^\infty \frac{dp_0}{2\pi}\,\frac{1}{p_0}\left(p_0\frac{\partial\mathcal{K}}{\partial p_0} - \mathcal{K}\right)\,\varrho(p).$$

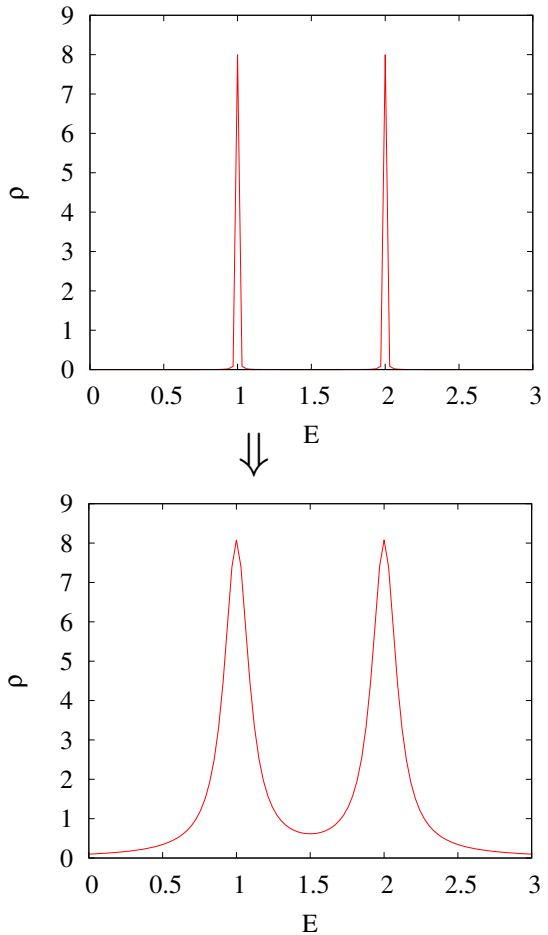
  - for construction: from ε with equivalent particle number à la Williams-Weizsacker
  - independent particles $\varrho(\omega) = \sum_{i=1}^{n} Z_i\delta(\omega - E_i) \quad \Rightarrow \quad N_{dof} = n!$

    counts the number of deltas, independently of the normalization
  - number of DoF is a dynamical quantity.

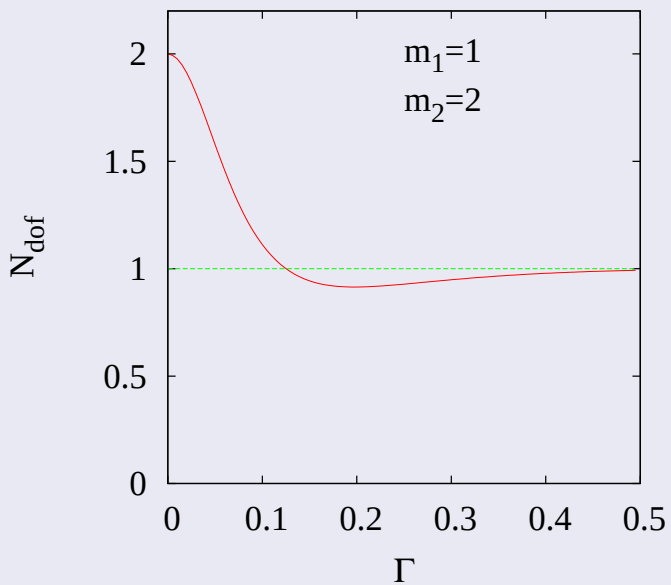
Two Breit-Wigner peaks

Change the width and compute the number of degrees of freedom!

for simplicity same width Γ, same height Z; masses $m_1 = 1$, $m_2 = 2$

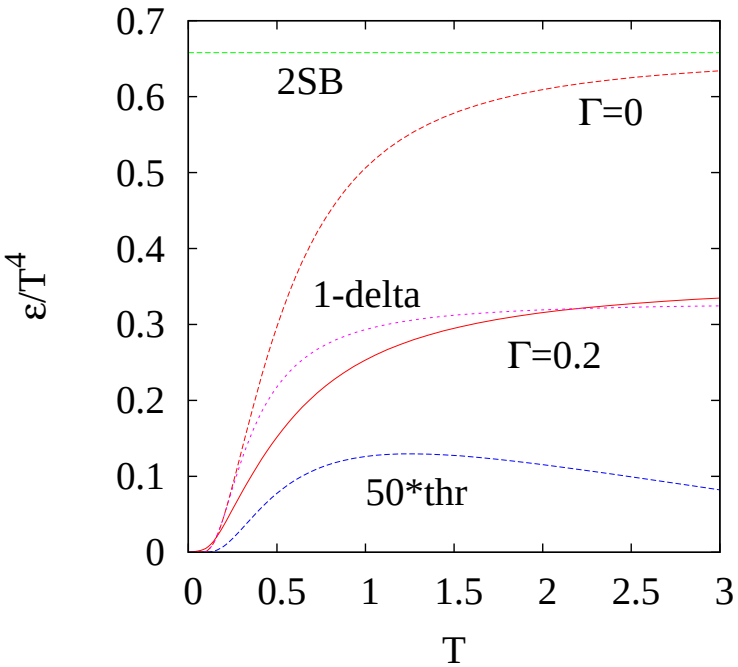


number of DoF



- already at $\Gamma/\Delta m \approx 0.1$ only one dof system!
- Gibbs paradox smoothed out!

Thermodynamics of the two Breit-Wigner system



$m_1 = 1,\ m_2 = 2$

- $\Gamma = 0$: 2 Dirac-delta ($\Gamma_1 = \Gamma_2 = 0$)
- $\Gamma = 0.2$: finite width peaks $\Gamma_1 = \Gamma_2 = 0.2$: if we had only one particle!

$\Rightarrow$ reduction of the number of degrees of freedom is observable in thermodynamics, too

Lowest curve: multiparticle threshold $\varrho(p) = \sqrt{1 - \frac{m^2}{p^2}}$

- negligible contribution to thermodynamics!
- overlapping Breit-Wigners $\Rightarrow$ destructive interference

Conclusions

We constructed an effective theory which is consistent, reproduces the complete spectrum and respect all symmetries

- gives nonlocal theory
- unitary, causal, E-conserving
- thermodynamics from Noether-currents, number of dof from Williams-Weizsacker ideas
- two Breit-Wigner system: already for $\Gamma/\Delta m \sim 0.1$ thermodynamics cannot resolve the peaks, effectively a one-dof system!

dynamical change of the number of dof is possible to describe