Scaling of SU(2) gauge theory with mixed fundamental-adjoint action

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Abstract

We study the phase diagram of the SU(2) lattice gauge theory with fundamental-adjoint Wilson plaquette action. We confirm the presence of a first order bulk phase transition and we estimate the location of its end point in the bare parameter space. If this point is second order, the theory is one of the simplest realisations of a gauge theory with an infrared fixed point at finite bare couplings. All the relevant gauge observables are monitored in the vicinity of the fixed point with very good control over finite-size effects. The scaling properties of the low-lying glueball spectrum are studied while approaching the end point. We comment on possible implications of our results for the near-conformality of Minimal Walking Technicolor, the SU(2) gauge theory with two flavours of adjoint Dirac fermions.

The Model

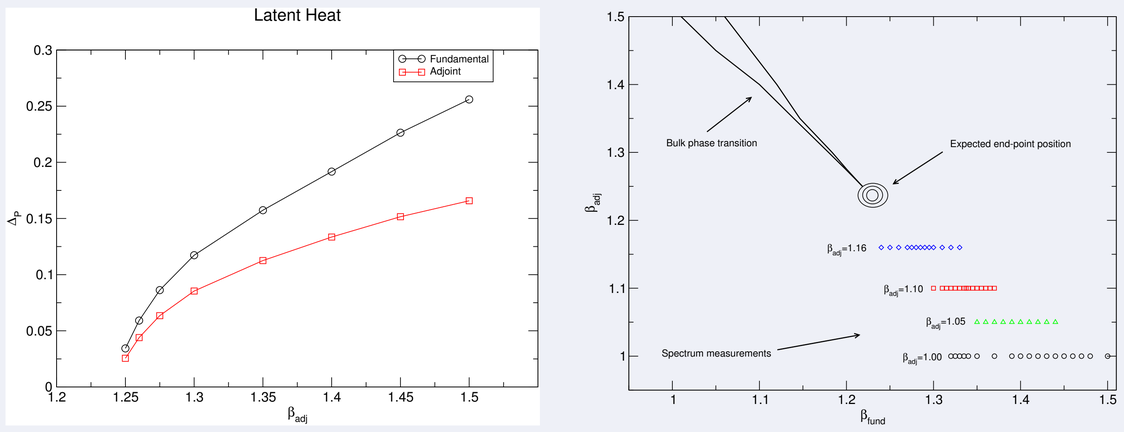
The Wilson plaquette action can be extended to include gauge representations other than the fundamental [1]. A simple realisation is

$$S = \beta_{\rm fund} \sum_{i,\mu>\nu} \left(1 - \frac{1}{N_c} \mathrm{Re\ Tr}_{\rm fund} U_{\mu\nu}(i)\right) + \beta_{\rm adj} \sum_{i,\mu>\nu} \left(1 - \frac{1}{N_c^2-1} \mathrm{Re\ Tr}_{\rm adj} U_{\mu\nu}(i)\right) ,$$

where the extra parameter $\beta_{\rm adj}$ is the coupling of the adjoint part. For $N_c = 2$, this pure gauge model can be seen as the leading contribution to the Minimal Walking Technicolor (MWT) action [2] in the heavy quark mass limit. The phase diagram of this lattice model has been studied extensively in the past (see e.g. [3] and references therein) and it features a bulk phase transition with an end-point. In principle, it is possible that the spectrum around the end-point mimics the features identified in [2] as a signature of conformality of MWT, hence invalidating the conclusion of that work. To check whether this is the case, we have analysed the scaling properties of the low-lying gluonic spectrum in the neighbourhood of the end-point. The nature of the end-point is still controversial, and simulations in his neighbourhood suffer from severe critical slowing down. In our work, we have used the algorithm described in [4], which has been shown to give significant improvements over traditional algorithms in the region of interest.

The Phase Diagram

In the two-dimensional parameter space at small $\beta_{\rm adj}$ we determine the location of a first order bulk transition characterised by a hysteresis cycle for the fundamental and adjoint plaquette expectation values. The transition turns into a crossover below the end-point, but to distinguish unambiguously the cross-over behaviour from the first order transition larger volumes are needed as we move towards the $\beta_{\rm adj} = 0$ axis.



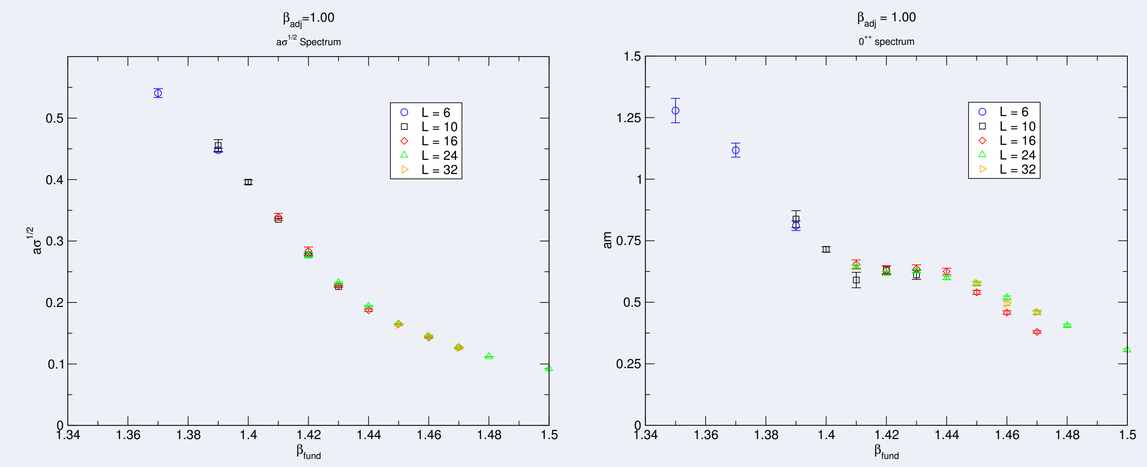
(a) (b)

Figure 1: (a) The difference of the fundamental plaquette as a function of $\beta_{\rm adj}$ at the phase transition point; (b) The phase diagram of the system, with the parameters used in the simulations.

The difference between the fundamental plaquettes in the two phases can be used to identify the end-point of the transition. Fig. (1a) shows this difference as a function of $\beta_{\rm adj}$, together with the region where it is expected to vanish. This result seems to hint towards the presence of an end-point for the first order bulk transition in the region. Our results for the bulk transition are obtained using hypercubic lattices with size up to L=40. Below the end-point, down to the fundamental axis, we have checked that the transition becomes a crossover, signaled by the lack of volume scaling of the fundamental and adjoint plaquettes susceptibility. A summary of the phase diagram is plotted in Fig. (1b).

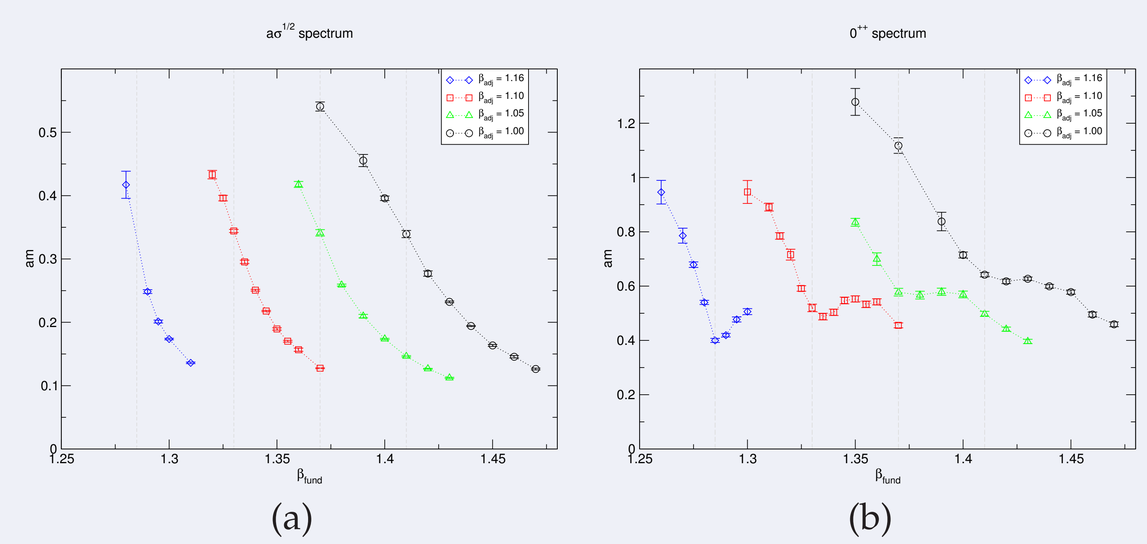
The Spectrum below the End-Point

On the points highlighted in Fig. (1b), we perform a full investigation of the gluonic spectrum, to investigate the scaling properties of masses while approaching the bulk transition end-point. For measuring the spectrum, we follow [5]. In Fig. 2 we show (preliminary) determinations of the string tension σ and of the 0^{++} glueball mass for $\beta_{\rm adj} = 1.00$ on various hypercubic lattices.



(a) (b)

Figure 2: (a) The string tension and (b) the 0^{++} glueball mass in the cross-over region at $\beta_{\rm adj} = 1.00$.



(a) (b)

Figure 3: (a) The string tension and (b) the 0^{++} glueball mass in the cross-over region at different values of $\beta_{\rm adj}$.

The behaviour of the string tension and the scalar glueball mass as a function of $\beta_{\rm fund}$ is summarised in Fig. 3 for four different values of the adjoint coupling. We note in particular that the latter is not monotonic in $\beta_{\rm fund}$ in the cross-over region.

Conclusions

The ratios of the measured masses do not seem to become constant as one approaches the bulk transition end-point. For example, by following a trajectory along the crossover line (highlighted by dashed lines in Fig. 3), the data show that the string tension remains constant while the scalar glueball mass decreases. This is very different from the behaviour observed in [2] for MWT. To confirm our preliminary findings, we are performing more extended simulations.

References

[1] G. Bhanot and M. Creutz, Phys. Rev. D **21** (1980) 2892.
[2] L. Del Debbio *et al.*, Phys. Rev. D **80** (2009) 074507.
[3] R. V. Gavai and M. Mathur, Phys. Rev. D **56** (1997) 32.
[4] A. Bazavov, B. A. Berg and U. M. Heller, Phys. Rev. D **72** (2005) 117501.
[5] B. Lucini, A. Rago and E. Rinaldi, JHEP **1008** (2010) 119.