Quark number susceptibilities from resummed perturbative QCD

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I Introduction

- Recall the Lagrangian density of **QCD**:

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}{\rm Tr}\,(F_{\mu\nu}F^{\mu\nu}) + i\bar{\psi}\gamma^{\mu}D_{\mu}\psi + \mathcal{L}_{\rm gf} + \mathcal{L}_{\rm ghost} + \Delta\mathcal{L}_{\rm QCD}\,.$$

- Relation between **thermodynamic potential** Ω and **diagonal quark number susceptibility** χ, at vanishing chemical potential:

$$\chi\,(T) \;\doteq\; -\left.\frac{\partial^2\Omega\,(T,\mu)}{\partial\mu^2}\right|_{\mu=0}\,.$$

  N.B. For $N_f = 2+1$ HTLpt, the relation between χ and **baryon number susceptibility** χ_B is very simple (due to masslessness): $\chi_B \equiv \chi/3$.

- **Our motivation**:
  Studying the extent to which one can **understand recent high precision lattice data using weakly coupled quasiparticle picture**.

II Setup

- **Hard-Thermal-Loop perturbation theory** (**HTLpt**) setup [1]:
  - ⋆ Lagrangian density for HTLpt, **reorganizes the perturbative series** of thermal QCD:

$$\mathcal{L} = (\mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm HTL})\Big|_{g\to\sqrt{\delta}g} + \Delta\mathcal{L}_{\rm HTL}\,,$$

  - ⋆ HTL improvement term:

$$\mathcal{L}_{\rm HTL} = -\frac{1}{2}(1-\delta)m_D^2{\rm Tr}\left(F_{\mu\alpha}\left\langle\frac{y^\alpha y^\beta}{(y\cdot D)^2}\right\rangle_y F^{\mu}{}_{\beta}\right) + (1-\delta)\,im_q^2\bar{\psi}\gamma^{\mu}\left\langle\frac{y_\mu}{y\cdot D}\right\rangle_y\psi\,.$$

    N.B. δ: # of HTL dressed loops, formal expansion parameter; $\Delta\mathcal{L}_{\rm HTL}$: HTL counterterm(s); y^μ: lightlike 4-vector; $m_{D/q}$: **Debye/quark thermal mass parameters**.

  - ⋆ Adding $\mathcal{L}_{\rm HTL}$ **SHIFTS** the expansion to an ideal gas of **thermal quasiparticles**.
- **Dimensional Reduction** (**DR**) setup [2, 3]:
  - ⋆ Separation of the pressure:

$$p_{\rm QCD} \equiv p_{\rm hard} + p_{\rm soft}\,,$$

  - ⋆ $p_{\rm hard}$: Contributions from **hard modes** ($\propto 2\pi T$), computable using strict perturbation theory in the 4D theory,
  - ⋆ $p_{\rm soft}$: Contributions from **soft modes** ($\propto gT$), pressure of 3D Yang-Mills plus adjoint Higgs theory (EQCD):

$$\mathcal{L}_{\rm EQCD} = \frac{1}{2}{\rm Tr}\,F_{ij}^2 + {\rm Tr}\,[D_i, A_0]^2 + m_{\rm E}^2{\rm Tr}\,A_0^2 + \frac{ig^3}{3\pi^2}\sum_f \mu_f\,{\rm Tr}\,A_0^3 + \delta\mathcal{L}_{\rm E}\,.$$

III Computation

- HTLpt:
  - ⋆ Prescription for the **mass parameters**: **1-loop** m_D and m_q at finite T and μ, i.e.

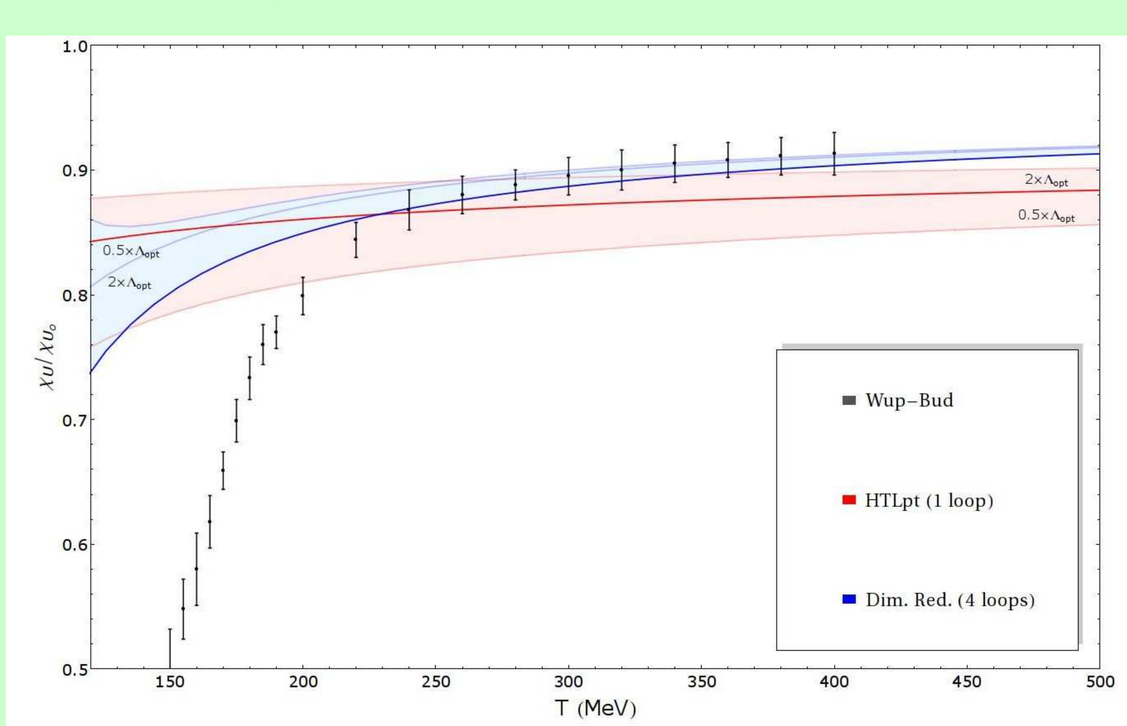
$$m_D^2 \equiv \alpha_S\,\pi\,\left(\frac{4N_c}{3}\,T^2 + 2N_f\,\frac{\mu^2}{\pi^2}\right)\,,$$
$$m_q^2 \equiv \alpha_S\,\pi\,\frac{N_c^2-1}{4N_c}\left(T^2 + \frac{\mu^2}{\pi^2}\right)\,,$$

  - ⋆ Preliminary results correspond to a **high** T **truncation** (in powers of m_D/T and m_q/T) of the full LO result,
  - ⋆ Gluons contribute to the results through the **medium dependence** of m_D.
- DR:
  - ⋆ Existing 4-loop susceptibility result [2] improved using a novel resummation scheme, amounting to **keep all of the DR parameters unexpanded** [3], which **substantially decreases the renormalization scale dependence** through inclusion of a class of **higher order corrections**.
- $\Lambda_{\overline{\rm MS}}$ is chosen according to [3]: a phenomenological fit of the 4-loop DR pressure to **hadron resonance gas results**.
- Strange quark mass dependence included by rescaling of massless results by appropriate ratio of noninteracting result.

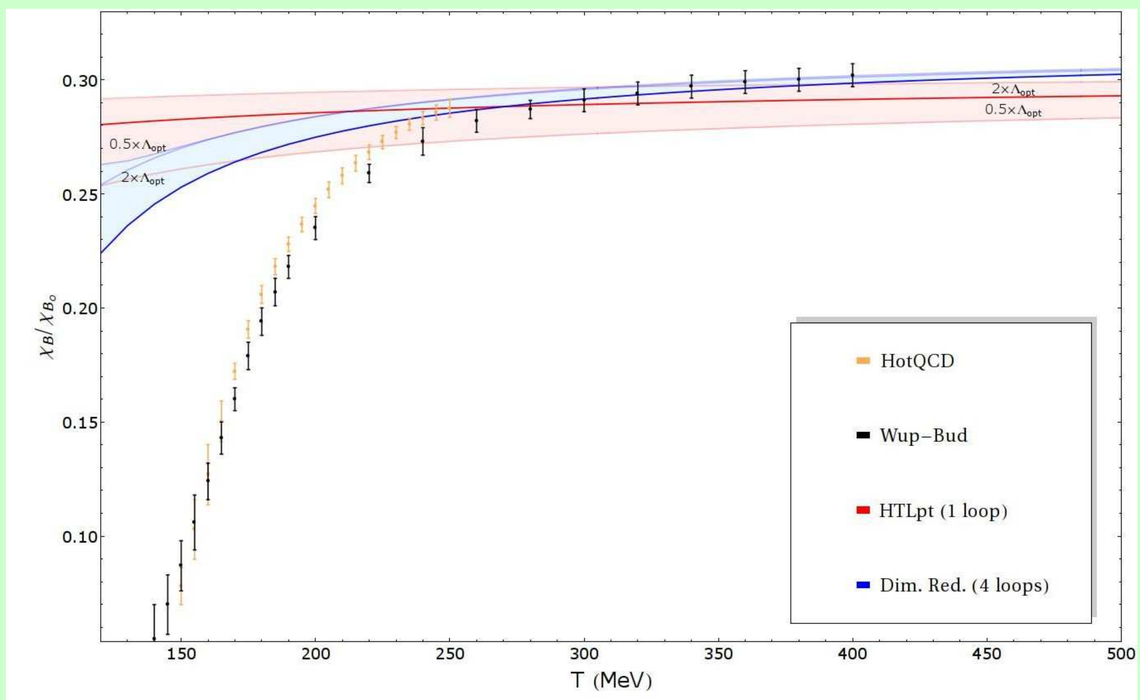
IV Preliminary results

Results for $N_f = 2+1$ (effect of the strange quark mass accounted for in DR) and $N_f = 2$. **2-loop perturbative running** used in both HTLpt and DR.

- $N_f = 2$ (lattice results from [5]):
  - ⋆ **Renormalization scale** fixed to the DR optimal scale $\bar{\Lambda} \approx 1.2910 \times 2\pi T$ [4], and varied by a factor of 2 (colored bands),
  - ⋆ $\Lambda_{\overline{\rm MS}} \equiv$ **180 MeV** [3].



- $N_f = 2+1$ (lattice results from [5, 6]):
  - ⋆ **Renormalization scale** fixed to the DR optimal scale $\bar{\Lambda} \approx 1.4455 \times 2\pi T$ [4], and varied by a factor of 2 (colored bands),
  - ⋆ $\Lambda_{\overline{\rm MS}} \equiv$ **200 MeV** [3].



V Conclusions

- Two perturbative resummation schemes are used to compute χ in QCD. The results are consistent with recent lattice data down to $T \approx 300$ MeV.
- HTLpt results are from the high T expansion of the free quasiparticle gas (1-loop); full computation underway [7].
- DR results amount to improvement of the existing 4-loop results from [2], using resummation scheme introduced in [3].

References

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