Confining gauge theories with adjoint Higgs on $R^3 \times S^1$

Hiromichi Nishimura and Michael Ogilvie
Department of Physics, Washington University, St. Louis, MO 63130, USA

Washington University in St.Louis

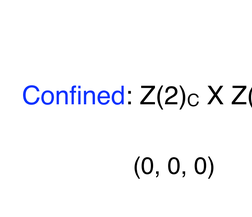
Background / Motivation

- Confining gauge theories[1] on $R^3 \times S^1$

- At small circumference L of S^1, the effective potential for the Polyakov loop P is reliable.

Confined: $\langle \mathrm{Tr} P(\vec{x}) \rangle = 0 \rightarrow F_q = \infty$

Deconfined: $\langle \mathrm{Tr} P(\vec{x}) \rangle \neq 0 \rightarrow F_q = \mathrm{Finite}$

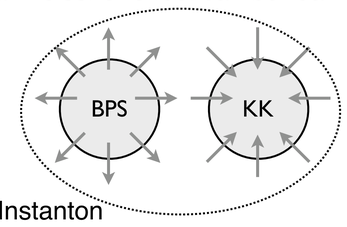


- A nontrivial Polyakov loop gives rise to BPS and KK monopoles, which are constituents of instanton[2]. The dual Meissner effect can be realized.

For SU(2)

SSB: SU(2)→U(1)

$S_{BPS} + S_{KK} = 8\pi^2/g^2 = S_{Instanton}$



Instanton

- Introduce scalar fields in the theory to study interrelationship between the confined and Higgs phases[3].

Determining phases

- One-loop effective potential

$$L = \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{1}{2}\left(D_\mu\phi\right)^T \cdot D_\mu\phi + V(\phi) + V_d$$

Background fields: $\bar{A}_4 = \begin{pmatrix} 2\theta/gL & 0 \\ 0 & -2\theta/gL \end{pmatrix} \quad \bar{\phi} = \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix}$

$$U = \underbrace{\frac{1}{2}m^2v^2 + \frac{1}{4}\lambda v^4}_{\text{0-loop}} + \underbrace{\frac{2}{\pi^2L^4}\left[\left(\theta - \frac{\pi}{2}\right)^4 + \frac{\pi^2 a}{2}\left(\theta - \frac{\pi}{2}\right)^2\right]}_{\text{1-loop}} + O(L^{-2})$$

$(a = 4N_f N_4^2/\pi - 1)$

- Symmetry and order parameters

Global Symmetry: $Z(2)_C \times Z(2)_H$

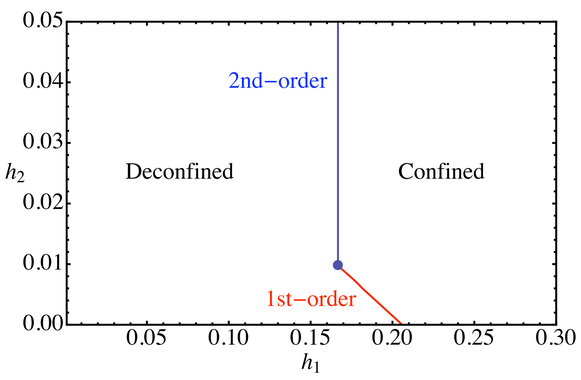
- Center Symmetry $Z(2)_C$: P → - P
- Reflection Symmetry $Z(2)_H$: ϕ → - ϕ

Gauge-Invariant Order Parameters

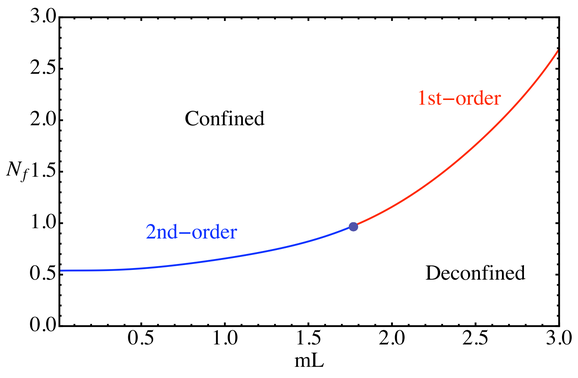
- <Tr P>: Transforms under $Z(2)_C$
- <Tr P^2ϕ>:Transforms under $Z(2)_H$
- <Tr Pϕ>:Transforms under $Z(2)_C$ & $Z(2)_H$

Center-stabilizing deformations

- $V_d = \left[h_1 (\mathrm{Tr}P)^2 + h_2 (\mathrm{Tr}P)^4\right]/L^4$

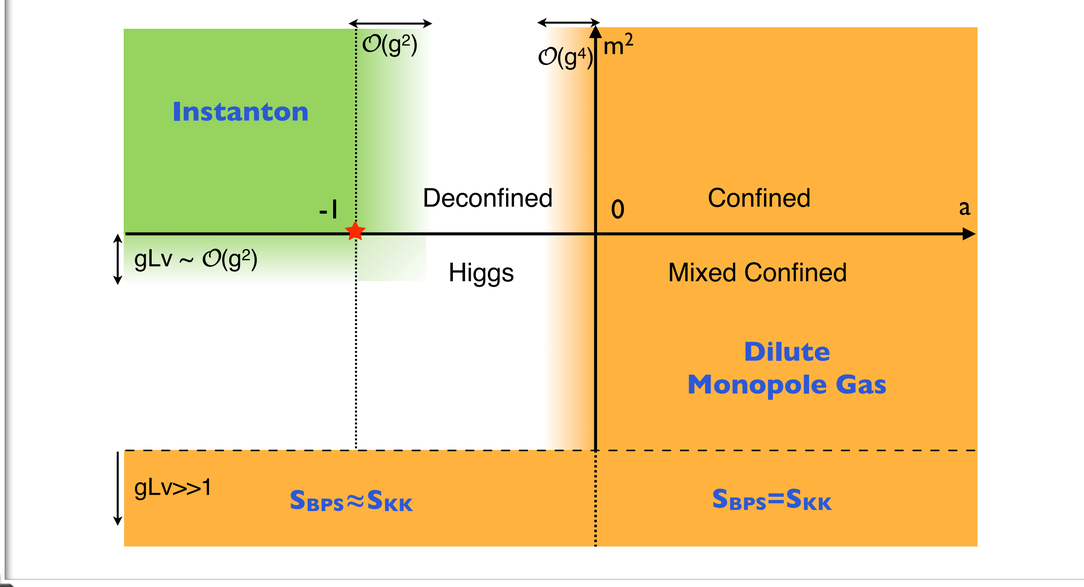


- N_f 2D-adjoint fermions of mass m with PBC



- Mixed confined phase is a new phase, where $Z(2)_C \times Z(2)_H \rightarrow Z(2)$, which acts nontrivially on P and ϕ.
- The "confined-Higgs phase," corresponding to (TrP, TrP^2ϕ, TrPϕ) = (0, ≠0, 0), is not possible.
- ★ marks where the deformation term is zero.

Phase diagrams



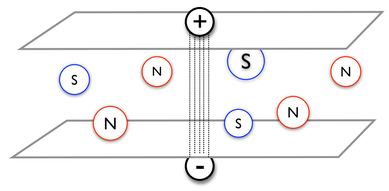
Topological effects

- Monopole actions depend on a linear combination of two Higgs:

$$L_E = \frac{1}{2}(B_i)^2 + \frac{1}{2}(D_i A_4)^2 + \frac{1}{2}(D_i\phi)^2$$

$$\rightarrow S_{BPS} = \frac{4\pi}{g^2}\sqrt{4\theta^2 + g^2L^2v^2}$$

$$\rightarrow S_{KK} = \frac{4\pi}{g^2}\sqrt{(2\pi - 2\theta)^2 + g^2L^2v^2}$$



"Dual Meissner Effect"

- Abelian duality[4]

Effective U(1) Theory

$$S_{\mathrm{U}(1)} = \int_{R^3} \frac{1}{4}(F_{ij})^2 + \mathrm{Monopoles}$$

(Distance >> M_w^{-1})

↔

Dual Scalar Theory

$$S_{\mathrm{dual}} = \int_{R^3}\left[\frac{g^2}{32\pi^2 L}(\partial_i\sigma)^2 - \sum_a \xi_a e^{iq_a\sigma}\right]$$

Mass Gap = $\frac{4\pi}{g}\sqrt{L\sum_a \xi_a}$ String Tension = $\frac{2g}{\pi}\sqrt{\frac{1}{L}\sum_a \xi_a}$

where $\xi_a = \mathrm{Fugacity\ of\ monopole} \sim f_a(\theta)e^{-S_a}$ and $a = \{BPS, KK, \overline{BPS}, \overline{KK}\}$

- Poisson duality[5]

Sum over BPS and KK monopoles

$$\sum \xi_a = \xi_{BPS}(\theta) + \xi_{KK}(\theta)$$

↔

A gas of Julia-Zee dyons (provided LM >>1)

$$2\sum_{n\in\mathbb{Z}} \tilde{\xi}(2n)\, e^{i2n\theta} \sim \sum_{n\in\mathbb{Z}} e^{-LM + i2n\theta}$$

$M = \mathrm{Mass\ of\ dyon} = v\sqrt{\left(\frac{4\pi}{g}\right)^2 + (gn)^2}$

Conclusions

1. In this model, the Higgs and confined phases are not compatible. (Agreement with the 't Hooft argument[6].)
2. Instead, we found a new phase where the behavior of the scalar field and the Polyakov loop are entwined.
3. The mass gap and the string tension can be calculated in the large portion of the phase diagram.

Future work

1. Generalizing to SU(N). A richer phase structure is expected.
2. Testing the overall phase structure with lattice simulations.
3. More exploration of the duality between Euclidean-space monopoles and Minkowski-space dyons.

Bibliography / Acknowledgments

1. J.C. Myers and M.C. Ogilvie, Phys. Rev. D **77**, 125030 (2008); J. High Energy Phys. 07 (2009) 095
   M. Unsal, Phys. Rev. Lett. **100**, 032005 (2008); Phys. Rev. D **80** 065001 (2009)
   M. Unsal and L. Yaffe, Phys. Rev. D **78**, 065035 (2008)
2. T.C. Kraan and P. van Baal, Phys. Lett. B **428**, 268 (1998); Nucl. Phys. B **533**, 627 (1998)
   K.-M. Lee and C.-h Lu, Phys. Rev. D **58**, 025011 (1998)
3. H. Nishimura and M.C. Ogilvie, Phys. Rev. D **85**, 065021 (2012)
4. A.M. Polyakov, Nucl. Phys. B **120**, 429 (1977)
5. E. Poppitz and M. Unsal, J. High Energy Phys. 07 (2011) 082
6. G. 't Hooft, Nucl. Phys. B **138**, 1 (1978); Nucl. Phys. B **153**, 141 (1979)

We thank the U.S. Department of Energy for support.