The QCD Phase Transition at High Isospin Asymmetry

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Why? & What? & How?

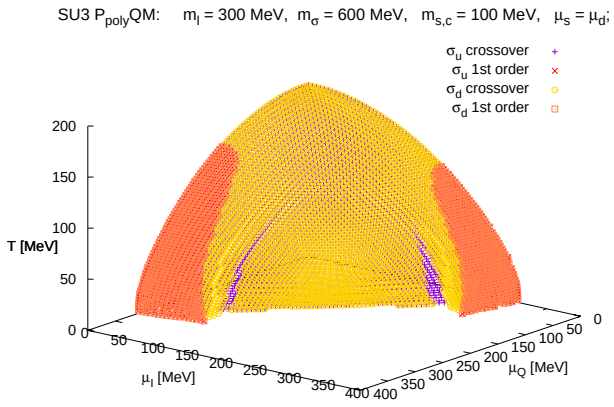
Dense matter as produced in core collapse supernovae or with Pb or Au beams are highly asymmetric in isospin, in the early universe the poorly constrained lepton asymmetry might be orders of magnitude larger than the baryon asymmetry allowing for a large isospin asymmetry.

Hence, for the search of signatures of the QCD phase transition under these extreme conditions, the dependence on finite net isospin densities should be taken into account.

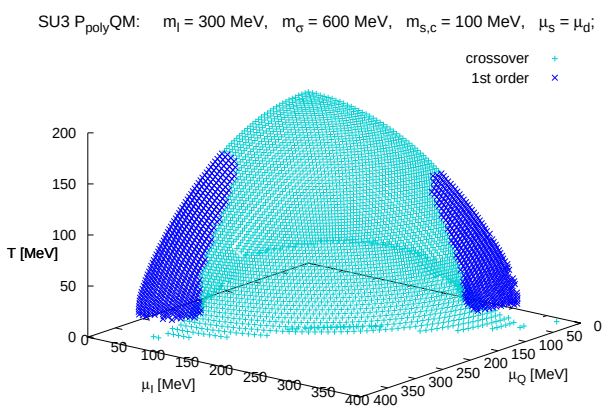
We investigate the phase structure of QCD within a Polyakov-Quark-Meson model that includes chiral symmetry breaking and restoration, as well the confinement-deconfinement phase transition.

We present our results on the phase diagram and equation of state for different asymmetries in isospin and discuss implications.

The phase diagram at finite Isospin



Phase diagram of chiral symmetry in the up and down quark sector. We defined the phase transition by the strongest change of the order parameter.

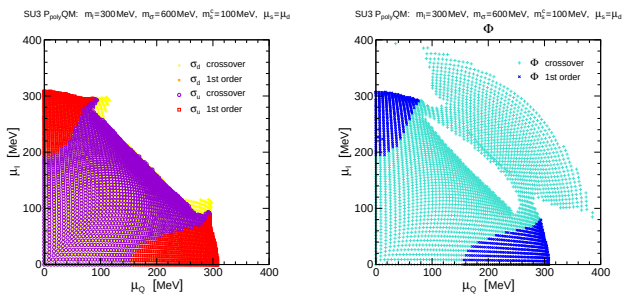


Phase diagram of (de)confinement according to the Polyakov loop variable as order parameter for deconfinement.

Two 'islands' were the order parameters of chiral symmetry and (de)confinement jump, so where the phase transition is of first order.

The common one at small isospin and large quark densities and another one where the QCD phase transition is of first order at small quark densities when the isospin asymmetry is large enough.

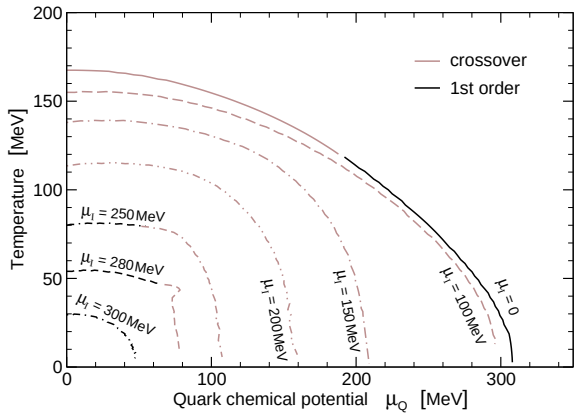
In between, there is a region at finite isospin where the phase transition is a crossover for all quark densities.



Projection of the phase diagrams of chiral symmetry and (de)confinement on the plane of quark and isospin chemical potential.

Quark and isospin chemical potentials are defined as below and strange quark chemical potential according to β-equilibrium:

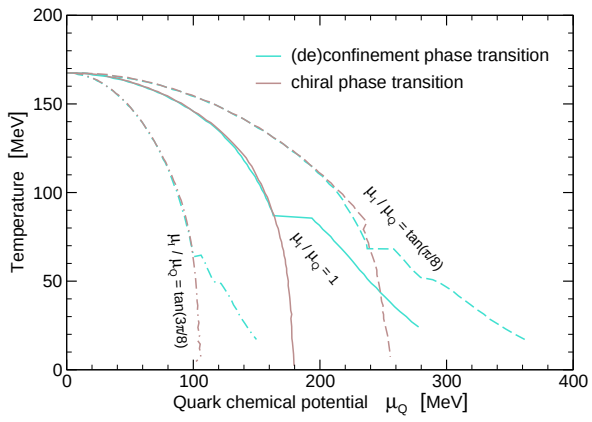
$$\mu_\mathrm{Q} = \frac{\mu_\mathrm{u} + \mu_\mathrm{d}}{2} \qquad \mu_\mathrm{I} = \frac{\mu_\mathrm{u} - \mu_\mathrm{d}}{2} \qquad \mu_\mathrm{s} = \mu_\mathrm{d}$$



Phase diagram of the chiral phase transition in the plane of quark chemical potential and temperature for different isospin chemical potentials.

The phase transition line moves to smaller quark chemical potentials and temperatures with increasing isospin asymmetry.

At large isospin asymmetries the common order of crossover and first order phase transition is inverted and the phase transition is of first order at small quark chemical potentials and becomes a crossover at quark densities larger than at the critical endpoint.

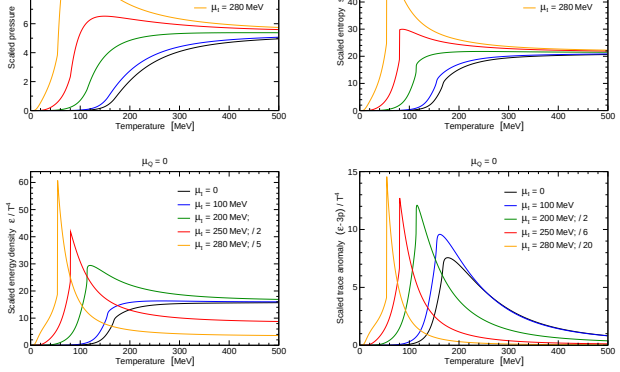


Phase diagram of the chiral and (de)confinement phase transition for different ratios of isospin and quark chemical potentials. For these the phase transition is a crossover everywhere.

At intermediate isospin asymmetries where the phase transition is crossover everywhere, the phase transition lines for chiral and (de)confinement phase transition split for lower temperatures which can be interpreted as the appearing of quarkyonic phase.

Equation of state & Thermodynamics

Below we show results for thermodynamic quantities and the equation of state for several isospin chemical potentials.



Pressure, entropy, energy density and trace anomaly for several isospin chemical potentials.

References

[1] J. T. Lenaghan, D. R. Rischke, J. Schaffner-Bielich, *Phys. Rev. D* 62, 085008, 2000.

[2] A. Dumitru, R. D. Pisarski, *Phys. Lett. B* 504, 282Ð290, 2001. O. Scavenius, A. Dumitru, J. T. Lenaghan, *Phys. Rev. C* 66, 034903, 2002.

[3] C. Ratti, M. A. Thaler, W. Weise, *Phys. Rev. D* 73, 014019, 2006.

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Polyakov Quark Meson model

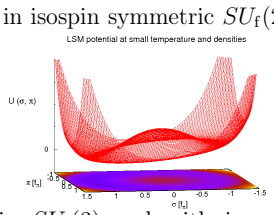
We are working with the grandcanonical potential

$$\Omega\left(\sigma_\mathrm{u}, \sigma_\mathrm{d}, \Phi, \bar{\Phi}, T, \mu_f\right) = U\left(\sigma_\mathrm{u}, \sigma_\mathrm{d}, \sigma_\mathrm{s}\right) + \mathcal{U}\left(\Phi, \bar{\Phi}, T, \mu_f\right) + \\ + \Omega_{\mathrm{q\bar{q}}}\left(\sigma_u, \sigma_d, \sigma_s, \Phi, \bar{\Phi}, T, \mu_f\right)$$

that results from the Lagrangian $\mathcal{L}$ in the common way via the partition function: $\mathcal{L} \rightarrow \mathcal{Z} \rightarrow \Omega = -T \log \mathcal{Z}/V$

The potential of the mesons U describes spontaneous and explicit symmetry breaking. Rather trivial in isospin symmetric $SU_\mathrm{f}(2)$

$$U\left(\sigma, \vec{\pi}\right) = \tfrac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 - h\sigma$$



it becomes much more involved in $SU_\mathrm{f}(3)$ and with isospin asymmetry [1]:

$$U\left(\sigma_\mathrm{u}, \sigma_\mathrm{d}, \sigma_\mathrm{s}\right) = \frac{2\lambda_1 + \lambda_2}{8}\sigma_\mathrm{ns}^4 + \frac{\lambda_1 + \lambda_2}{4}\sigma_\mathrm{s}^4 + \frac{2\lambda_1 + \lambda_2}{8}\sigma_3^4 + \\ + \frac{\lambda_1}{2}\sigma_\mathrm{ns}^2\sigma_\mathrm{s}^2 + \frac{\lambda_1}{2}\left(\sigma_\mathrm{ns}^2 + \sigma_\mathrm{s}^2\right)\sigma_3^2 + \frac{3}{4}\lambda_2\,\sigma_\mathrm{ns}^2\sigma_3^2 - \\ - \frac{c}{2\sqrt{2}}\left(\sigma_\mathrm{ns}^2 - \sigma_3^2\right)\sigma_\mathrm{s} + \frac{m^2}{2}\left(\sigma_\mathrm{ns}^2 + \sigma_\mathrm{s}^2 + \sigma_3^2\right) - h_{ns}\sigma_\mathrm{ns} - h_s\sigma_\mathrm{s}$$

$$\text{with} \quad \sigma_\mathrm{u} = \sigma_\mathrm{ns} + \sigma_3 \quad \text{and} \quad \sigma_\mathrm{d} = \sigma_\mathrm{ns} - \sigma_3$$

The red highlighted terms are those which emerge at finite isospin.

A quantity that combines information of the chiral condensates of all quark flavours is the subtracted condensate $\Delta_{\mathrm{l,s}}$:

$$\Delta_{\mathrm{l,s}} = \frac{\frac{1}{2}\left(\sigma_\mathrm{u} + \sigma_\mathrm{d}\right) - \frac{h_\mathrm{ns}}{h_\mathrm{s}}\sigma_\mathrm{s}\Big|_{T,\mu_f}}{\frac{1}{2}\left(\sigma_\mathrm{u} + \sigma_\mathrm{d}\right) - \frac{h_\mathrm{ns}}{h_\mathrm{s}}\sigma_\mathrm{s}\Big|_{T=0,\ \mu_f=0}}$$

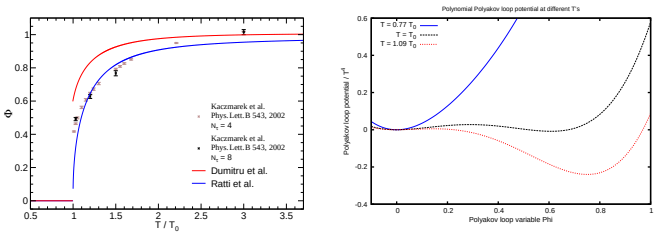
The gauge potential $\mathcal{U}$ can be parametrised as a polynomial in the Polyakov loop variables Φ and $\bar{\Phi}$ [2, 3]

$$\frac{\mathcal{U}\left(\Phi, \bar{\Phi}, T, \mu_f\right)}{T^4} = -\frac{b_2\left(T, \mu_f\right)}{2}\Phi\bar{\Phi} - \frac{b_3}{6}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{b_4}{4}\left(\Phi\bar{\Phi}\right)^2$$

where the prefactor $b_2\left(T, \mu_f\right)$ is a polynomial in T_0/T:

$$b_2\left(T, \mu_f\right) = \sum_{n=0} b_n \left(\frac{T}{T_0\left(\mu_f\right)}\right)^n$$

The parameters are fitted to reproduce Polyakov expectation value and equation of state of lattice simulations of pure $SU_\mathrm{c}(3)$ gauge theory [2, 3] that is its first order phase transition.



Evolution of the Polyakov loop expectation value in pure gauge theory and Polyakov loop potential at different temperatures relative to the phase transition temperature.

To extend the pure gauge potential towards the glue potential in full QCD, the effect of finite quark chemical potentials is parametrised as a running of the critical temperature in the gauge sector $T_0\left(\mu_f\right)$ [4].

Finally, the constituent quark contribution couples quarks, gauge field and mesons:

$$\Omega_{\mathrm{q\bar{q}}}(\sigma_u, \sigma_d, \sigma_s, \Phi, \bar{\Phi}, T, \mu_f) = -2T \sum_{f=\mathrm{u,d,s}} \int \frac{d^3p}{(2\pi)^3} \times \\ \times \left\{ \ln\left[1 + 3\left(\Phi + \bar{\Phi}e^{-\left(E_f - \mu_f\right)/T}\right)e^{-\left(E_f - \mu_f\right)/T} + e^{-3\left(E_f - \mu_f\right)/T}\right] + \right. \\ \left. + \ln\left[1 + 3\left(\bar{\Phi} + \Phi e^{-\left(E_f + \mu_f\right)/T}\right)e^{-\left(E_f - \mu_f\right)/T} + e^{-3\left(E_f + \mu_f\right)/T}\right] \right\}$$

via the dispersion relation of the quarks

$$E_f = \sqrt{k^2 + m_f^2} \quad \text{with} \quad m_\mathrm{u} = \frac{g}{2}\sigma_\mathrm{u},\ m_\mathrm{d} = \frac{g}{2}\sigma_\mathrm{d},\ m_\mathrm{s} = \frac{g}{\sqrt{2}}\sigma_\mathrm{s}.$$

That this contribution is just related to the quark and antiquark densities is easier to see in a pure Quark Meson model without the coupling to a gauge field:

$$\Omega_{\mathrm{q\bar{q}}} = 2N_c T \sum_f \int_0^\infty \frac{d^3p}{(2\pi)^3}\left\{\ln\left[1 - n_{q,f}\left(T, \mu_f\right)\right] + \ln\left[1 - n_{\bar{\mathrm{q}},f}\left(T, \mu_f\right)\right]\right\}$$

Working with the grand canonical potential, the relations to the thermodynamic quantities are given by:

$$p = -\Omega, \quad s = -\frac{\partial\Omega}{\partial T}, \quad n_f = -\frac{\partial\Omega}{\partial\mu_f}, \quad \epsilon = \Omega + Ts + \sum_{f=\mathrm{u,d,s}} \mu_f n_f$$

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