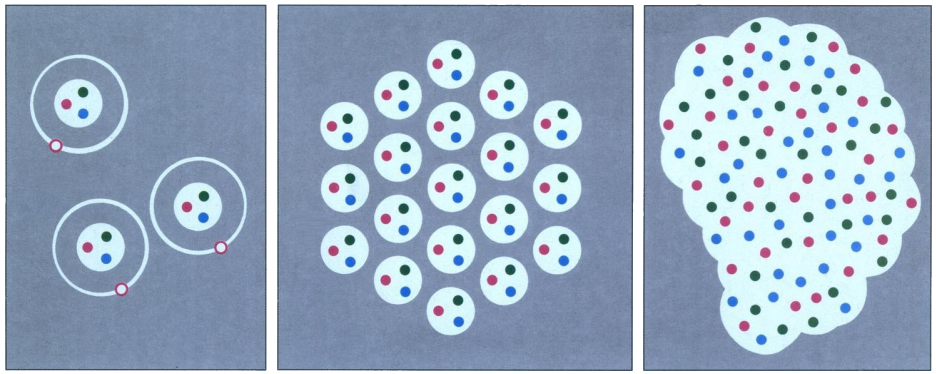
A surprise in the phase diagram of many-flavor staggered fermions

Philippe de Forcrand, ETH & CERN
with Seyong Kim, Sejong University and Wolfgang Unger, ETH

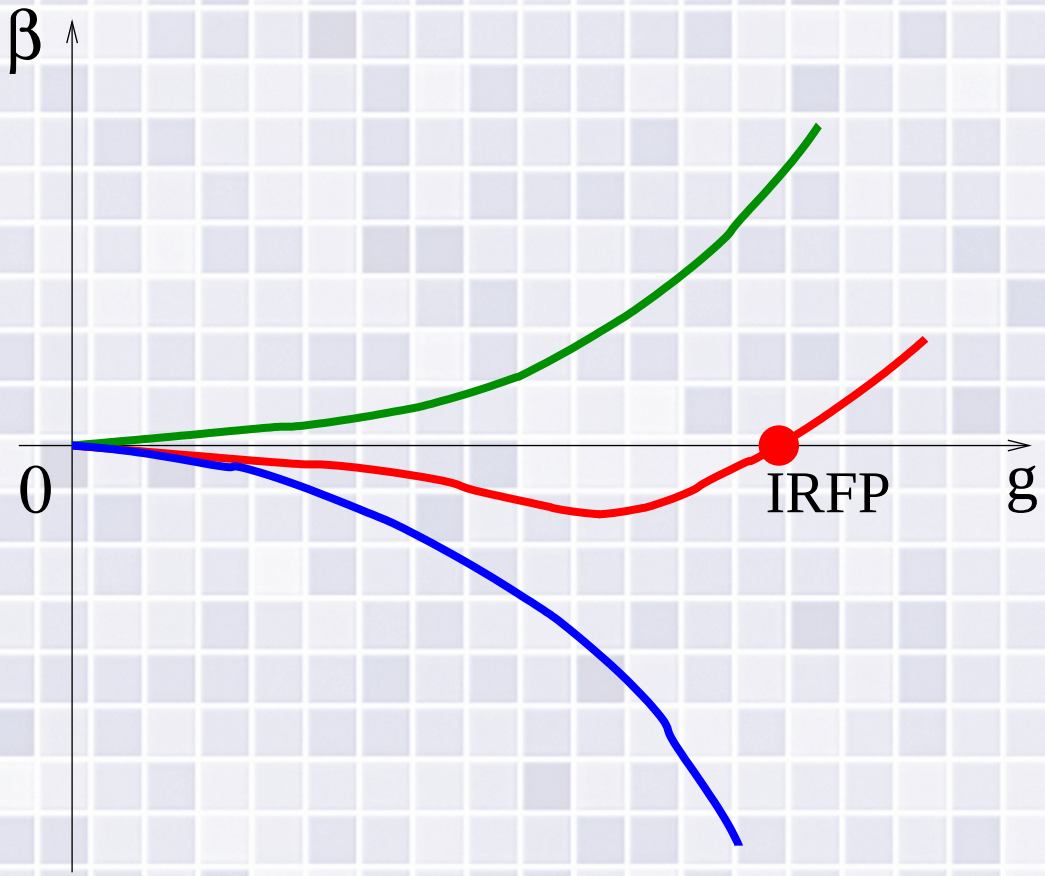
SEWM 2012, Swansea
July 2012

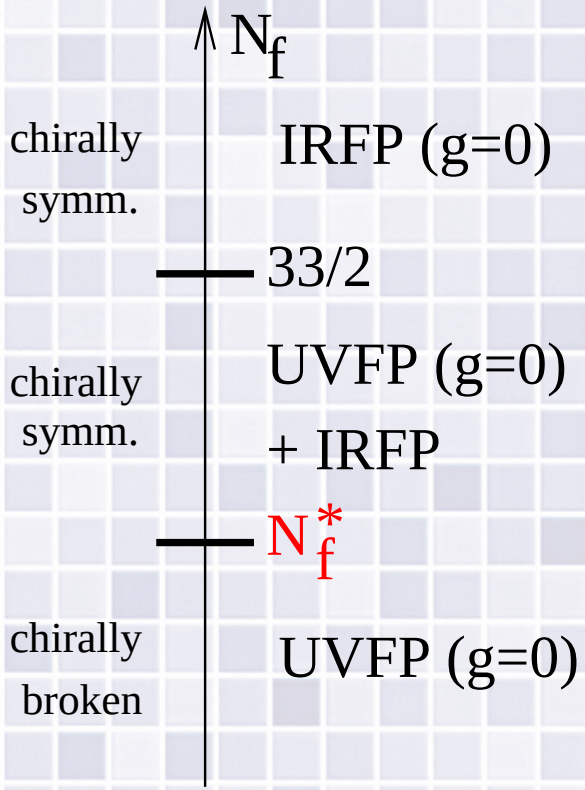


Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Motivation

- Classification of *continuum* QCD-like $SU(3)$ theories with N_f fundamental quarks

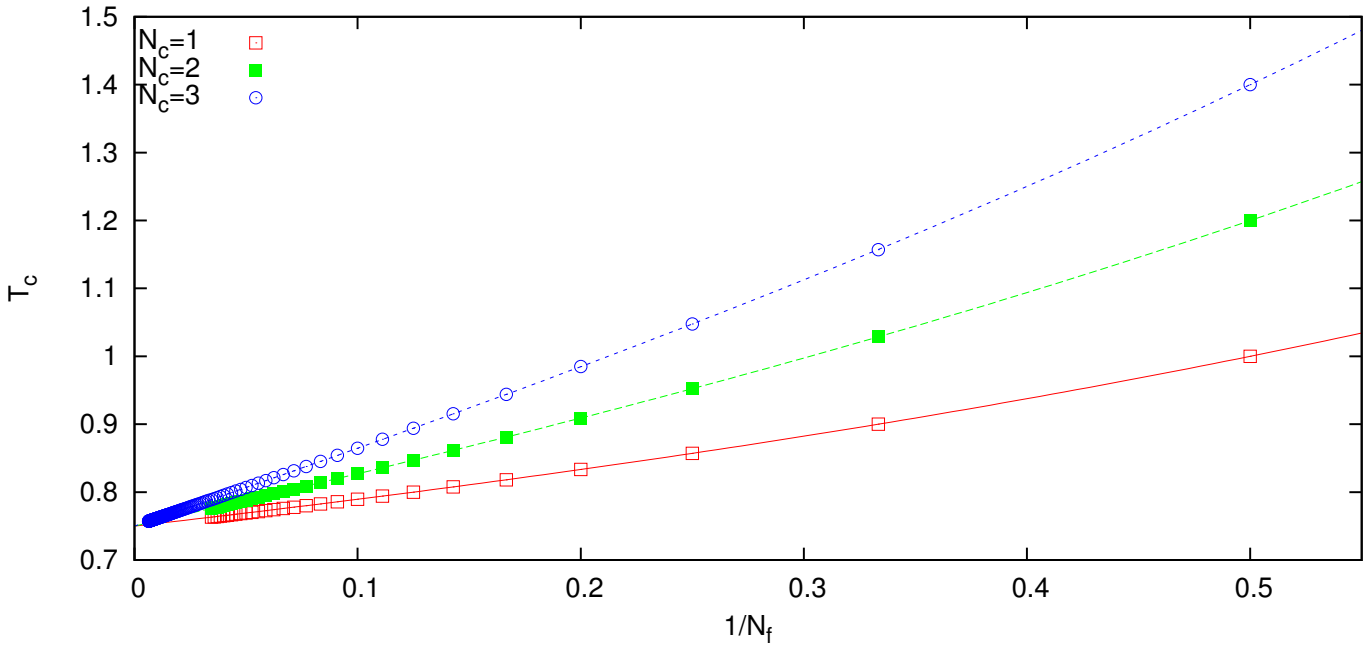




Is there any remnant of this classification in the *lattice strong-coupling limit*?

Mean Field: chiral symmetry is **always broken** in the strong-coupling limit of staggered fermions at $T = 0$ **for all values of** N_f **and** N_c

- chiral condensate well known to be independent of N_f and N_c, i.e. in d spatial dimensions: [Kluberg-Stern *et al.*, 1983]
$$\langle \bar{\psi}\psi \rangle (T=0) = \frac{\left((1+d^2)^{1/2}-1)/2\right)^{1/2}}{d}$$
- we also found, following [Damgaard *et al.*, 1985]: chiral restoration temperature is $T_c = \frac{d}{4} + \frac{d}{8}\frac{N_c}{N_f} + \mathcal{O}(\frac{1}{N_f^2})$



- mean field expected to work well for large number of d.o.f. per site, e.g. exact results in the Gross-Neveu model for $N_f \to \infty$

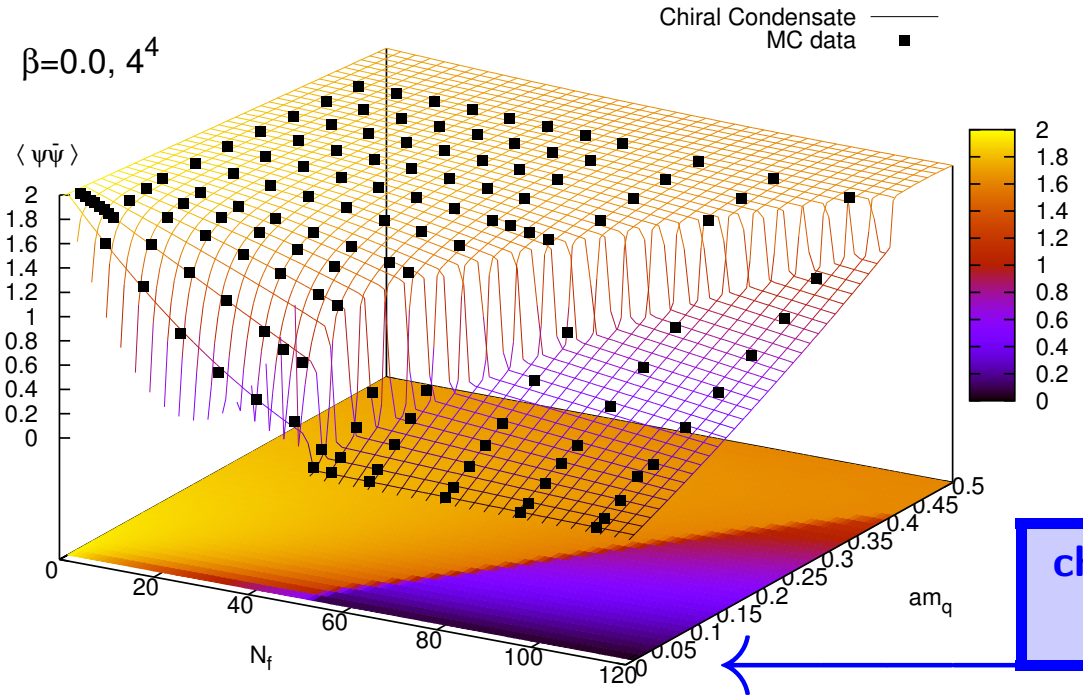
On the other hand: loop expansion of the determinant shows that dynamical fermions induce a plaquette coupling $\propto N_f/m_q^4$, as studied numerically by [A. Hasenfratz, T. DeGrand PRD49 (1994)]: fermions have ordering effect

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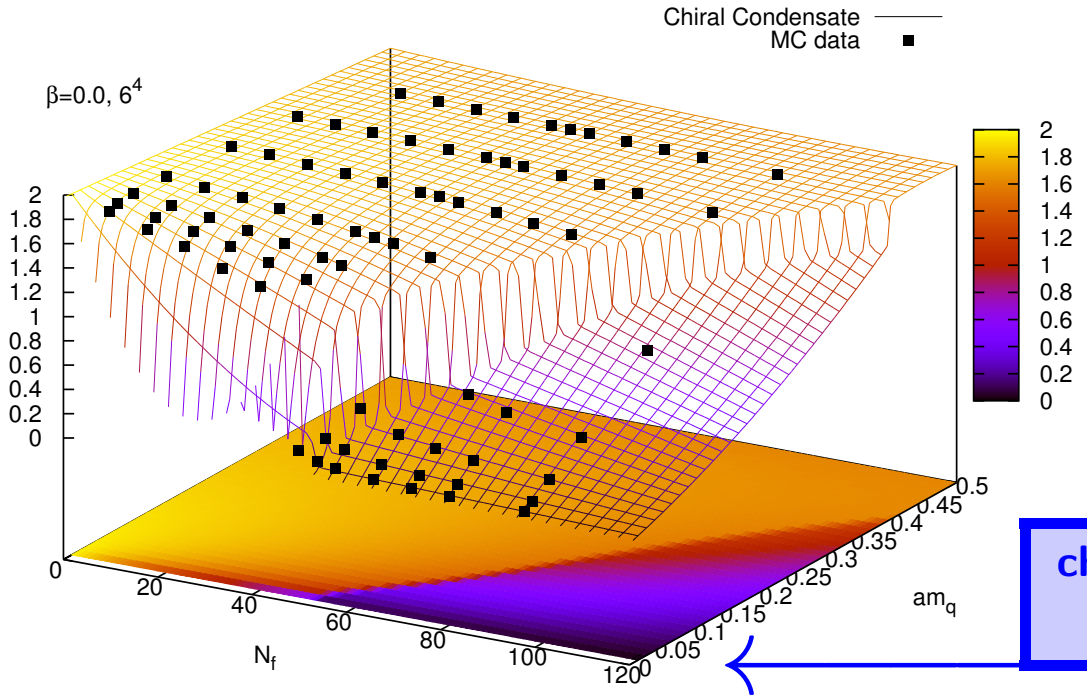
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- $N_f^c \simeq 52$ *continuum* flavors for $m_q = 0$, N_f^c increases with m_q
  (heavy fermions $\rightarrow$ less ordering)

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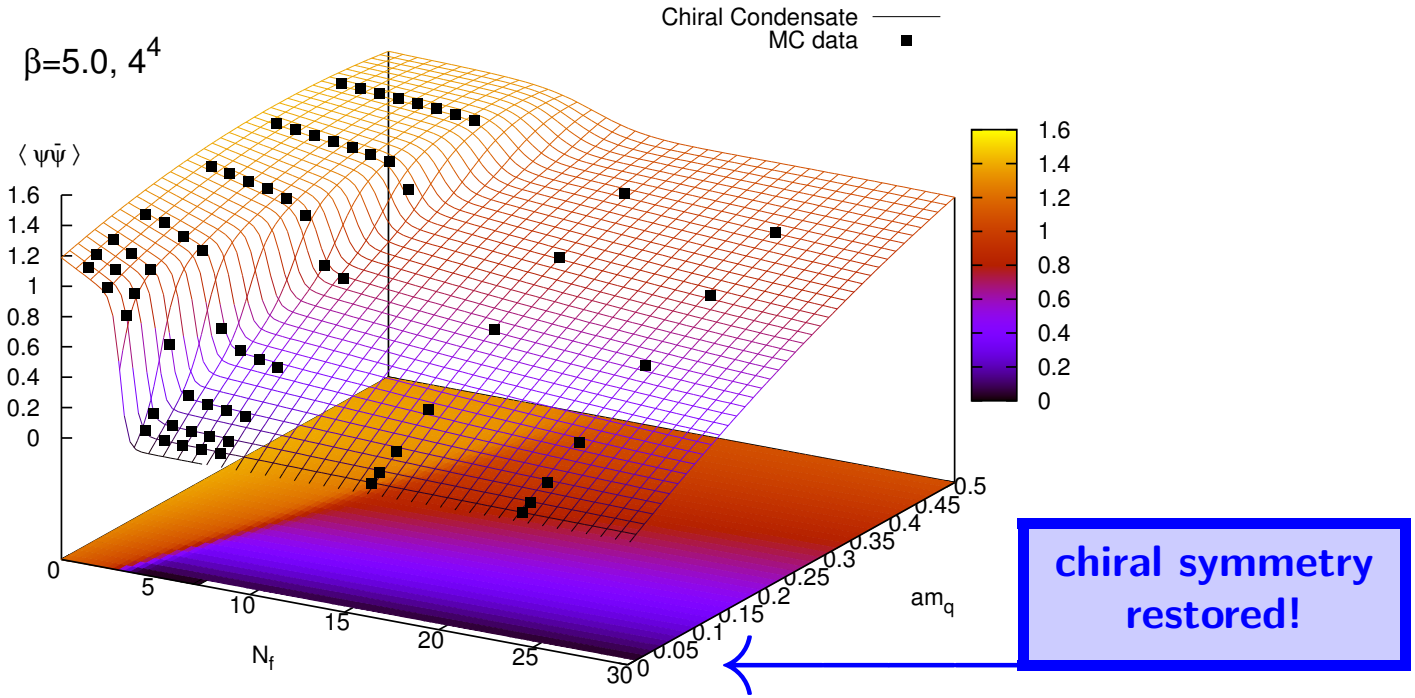
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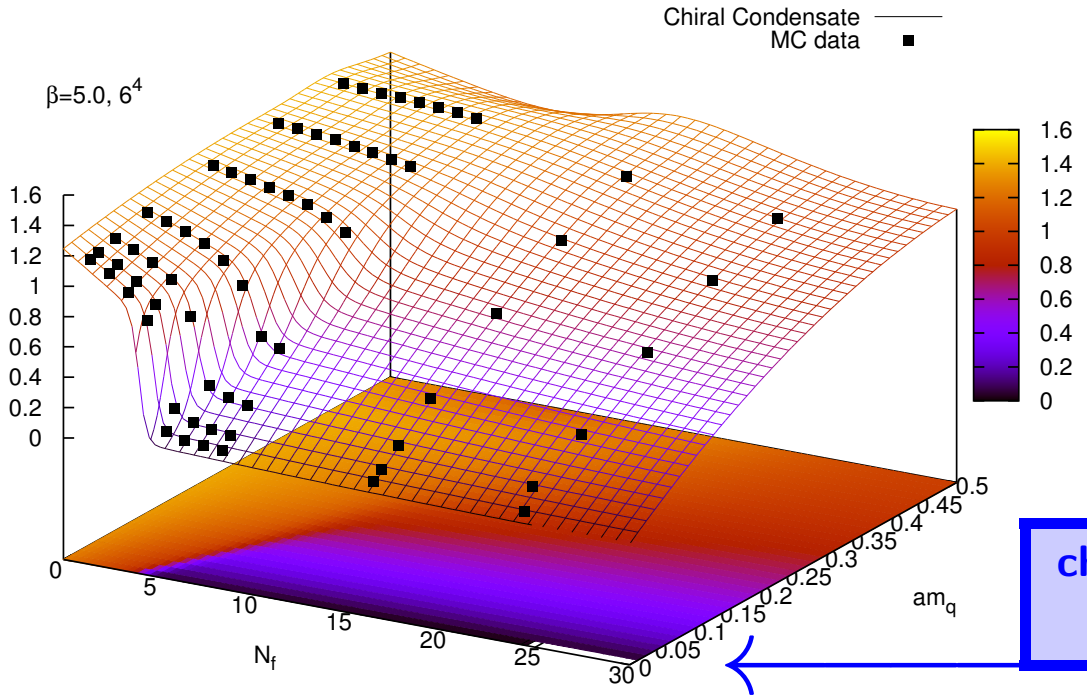


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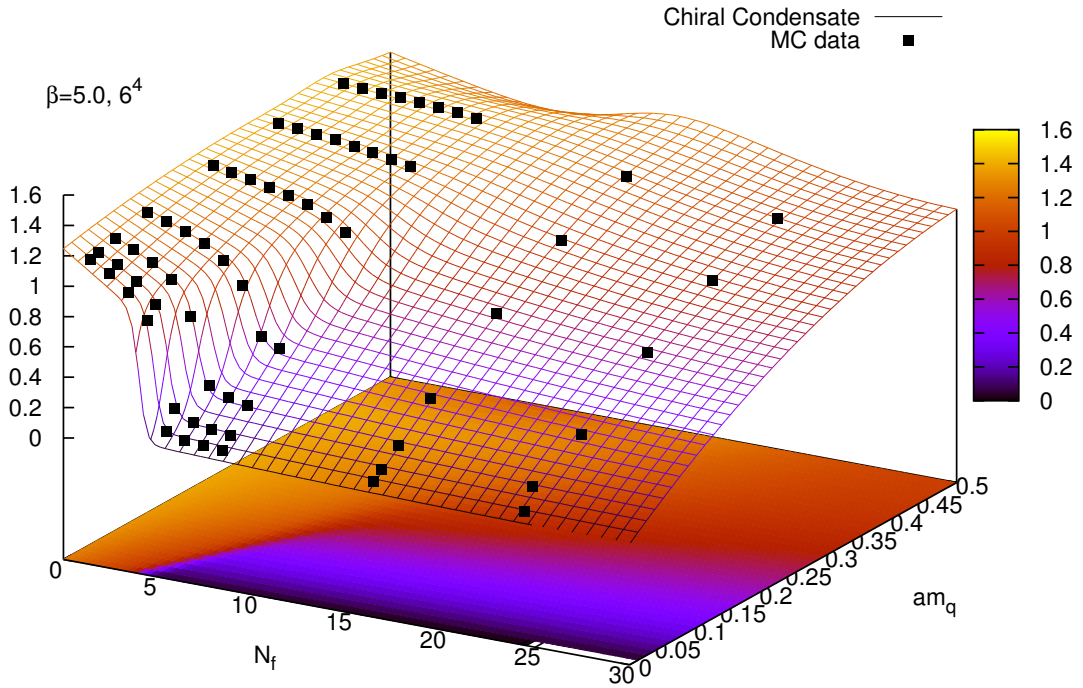
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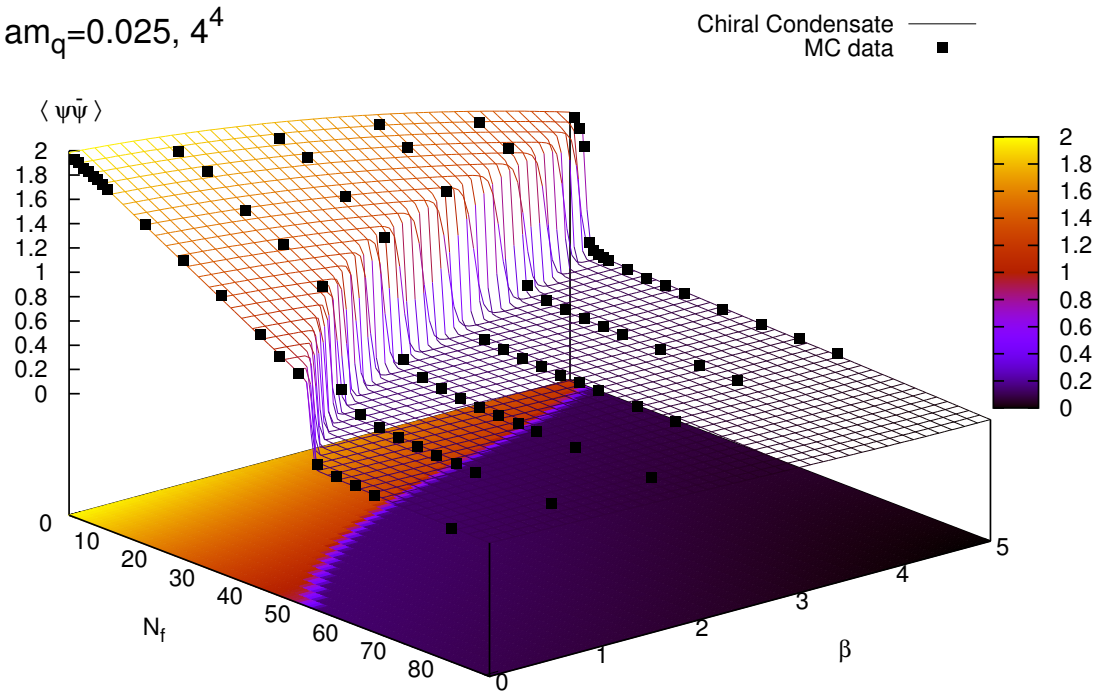
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- Explanation for **failure of mean field:** terms of $\mathcal{O}(\frac{N_f}{N_c}, \frac{N_f}{d^2})$ are neglected

The Chirally Restored Phase for large β

- smooth variation with $\beta \rightarrow$ N_f-driven transition extends to weak coupling
- $N_f^c \simeq \mathcal{O}(10)$ at weaker coupling
- connection with N_f-driven transition to **conformal window?**



Characterizing the chirally restored phase

Chirally symmetric yet "confining" ($\beta = 0$)

Conformal or not ? If conformal, trivial (IRFP $g = 0$) or not?

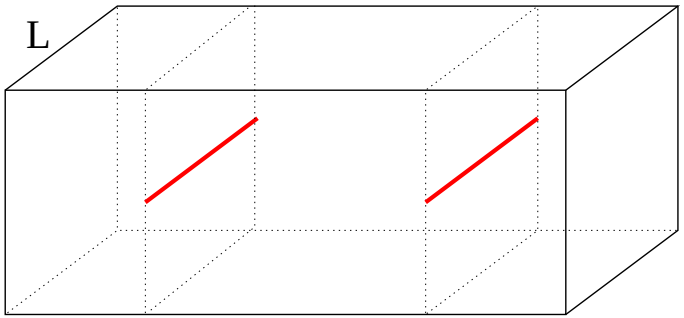
- Numerical simulations: $N_f = 56$ & 96, $\beta = 0$, $m_q = 0$, max. $12^3 \times 24$
- Observables: - torelon mass (gluon flux tube)
  - Dirac eigenvalue spectrum
  - meson masses

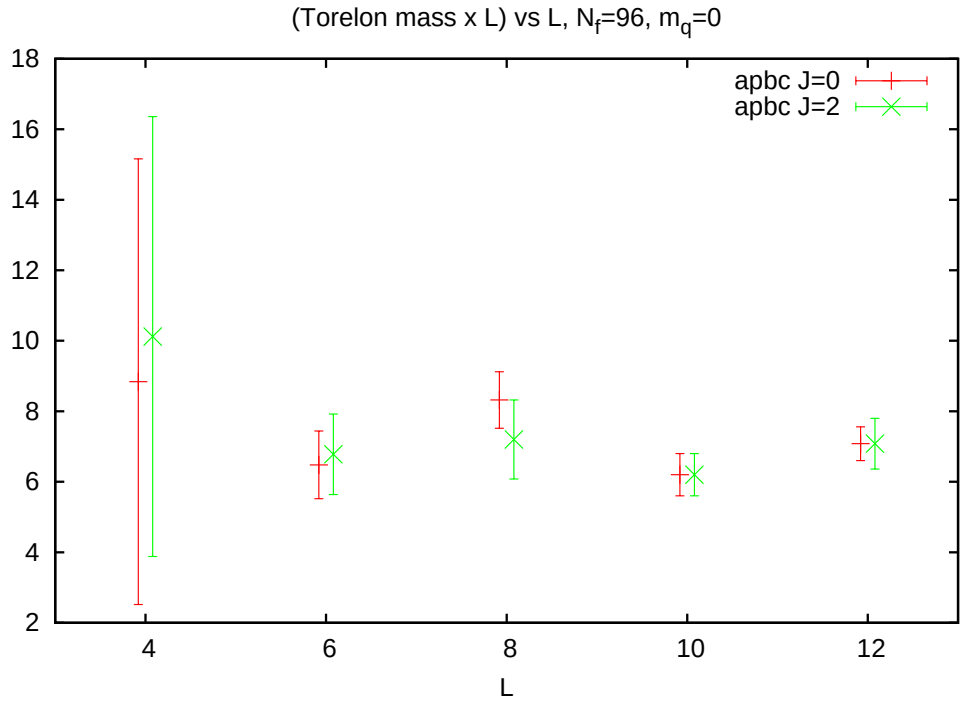
Characterizing the chirally restored phase: I. Torelon masses

Energy of spatially-wrapping loop $E(L) \propto \sigma L$ in confining theory

Here:

- $E(L) \times a$ **decreases as** L **increases**
- $E(L) \times L \sim$ constant



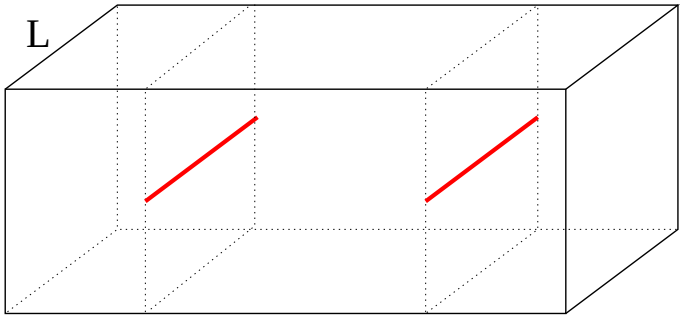


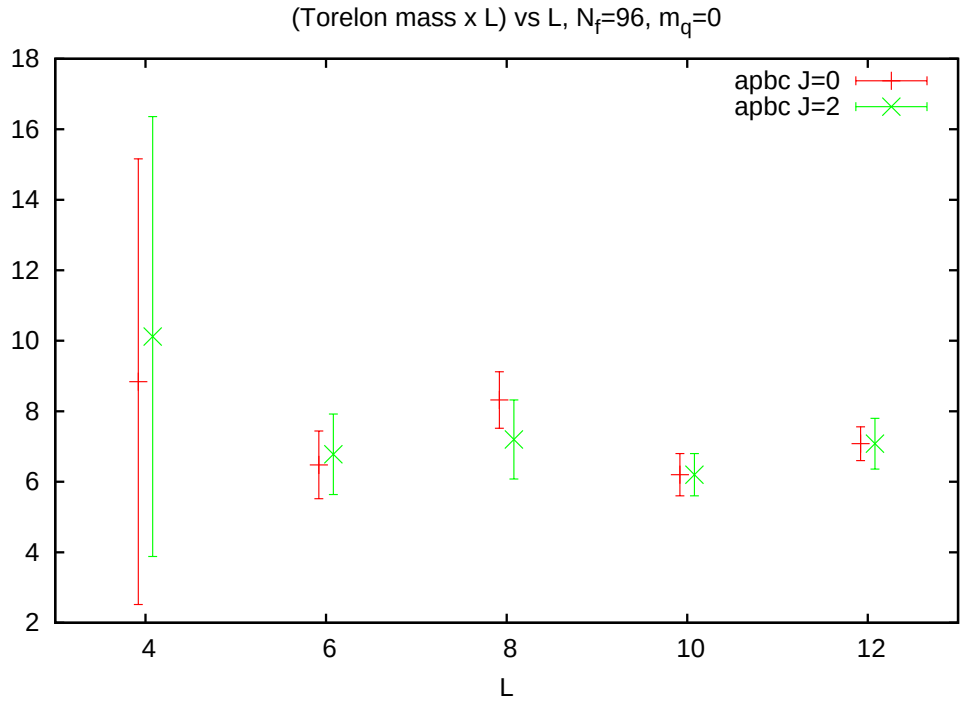
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Explanation: the reference length scale is L, not a

Consider L constant ($\Rightarrow E(L) =$ constant) and increase $L/a \Rightarrow a$ shrinks

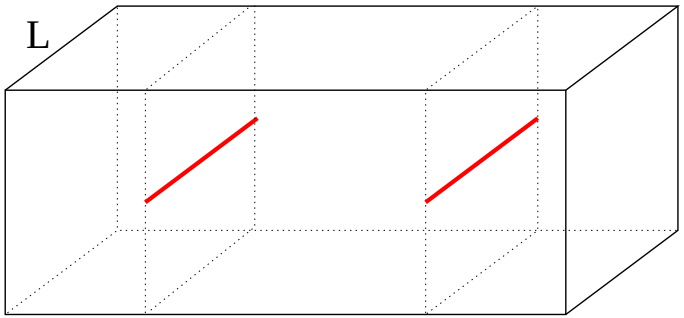
$\longrightarrow \quad E(L) \times a \propto a/L$

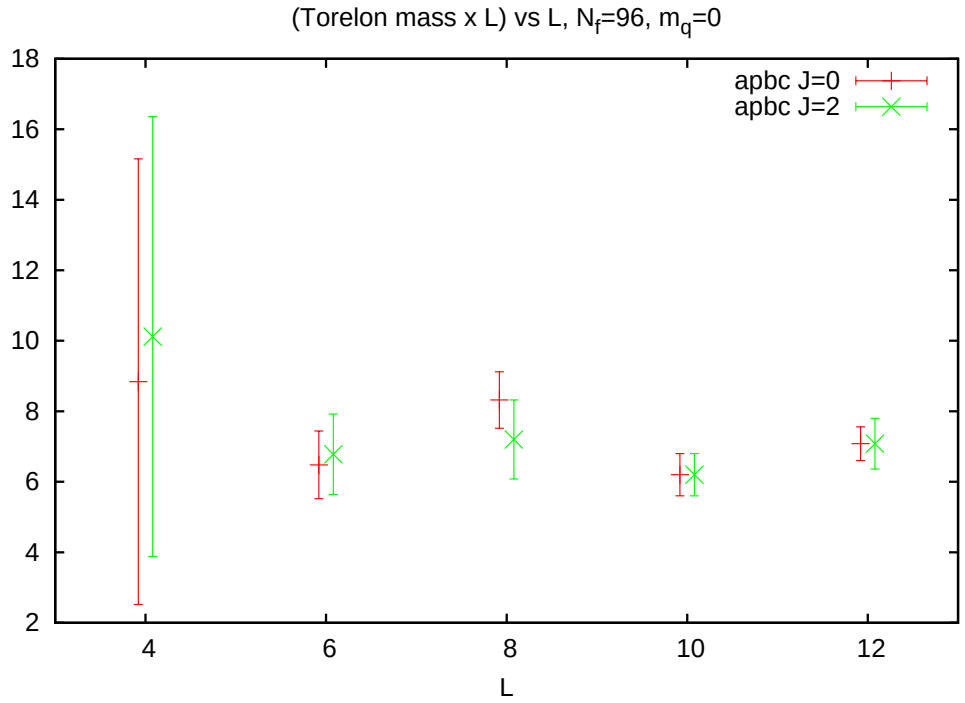
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Torelon mass is Debye mass m_D after relabeling axes

Remember $m_D = gT\sqrt{\frac{N_c}{3} + \frac{N_f}{6}}$ $\rightarrow$ Define running coupling $g(L) = m_D(L)L/\sqrt{\frac{N_c}{3} + \frac{N_f}{6}}$

In that scheme, $m_D(L)L = \text{const.} \Longrightarrow$ **non-trivial IRFP !**

Characterizing the chirally restored phase: II. Dirac Spectrum

Dirac eigenvalue spectrum, measured at **zero quark mass**, $\beta = 0$:

- **integrated eigenvalue density:** $\int_0^\lambda \rho(\bar{\lambda}) d\bar{\lambda} = \frac{\text{rank}(\lambda)}{\text{rank(Dirac matrix)}} \in [0,1]$
- measures the fraction of eigenvalues smaller than λ
- derivative gives $\rho(\lambda)$

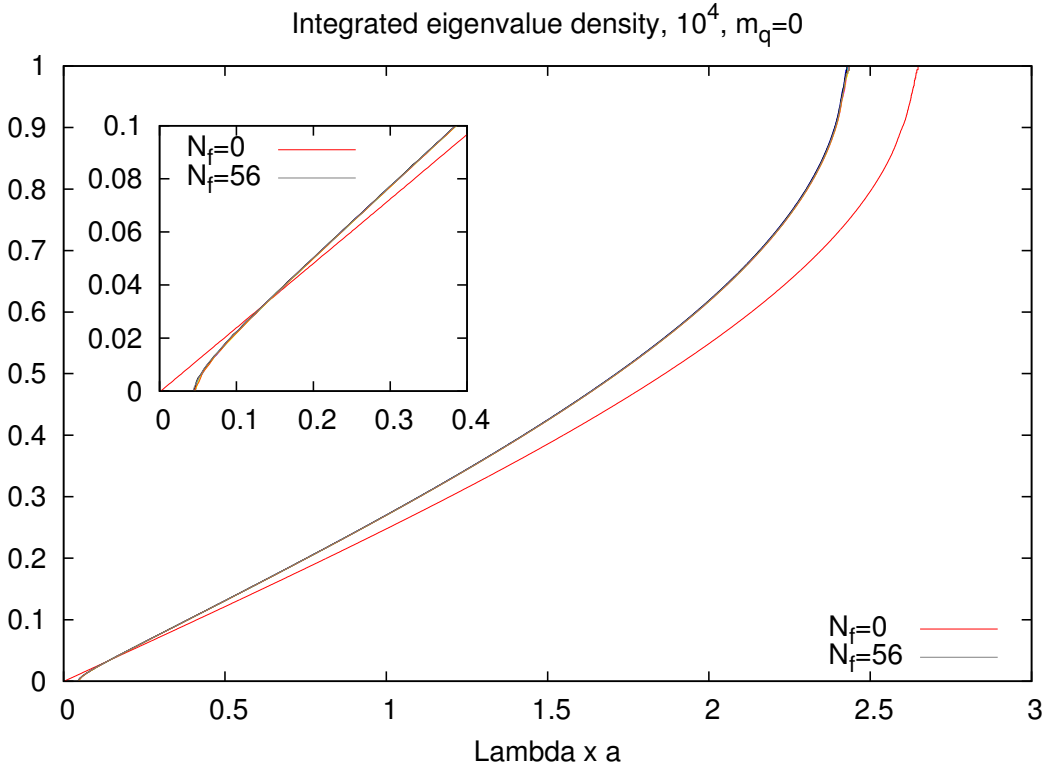
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Compare $N_f = 0$ (quenched configurations) and $N_f = 56$ (chirally symmetric phase)

- similar for large eigenvalues (UV)
- the $N_f = 56$ curve shows a **gap** for small eigenvalues (IR), consistent with chiral symmetry restoration: $\rho(0) = 0$



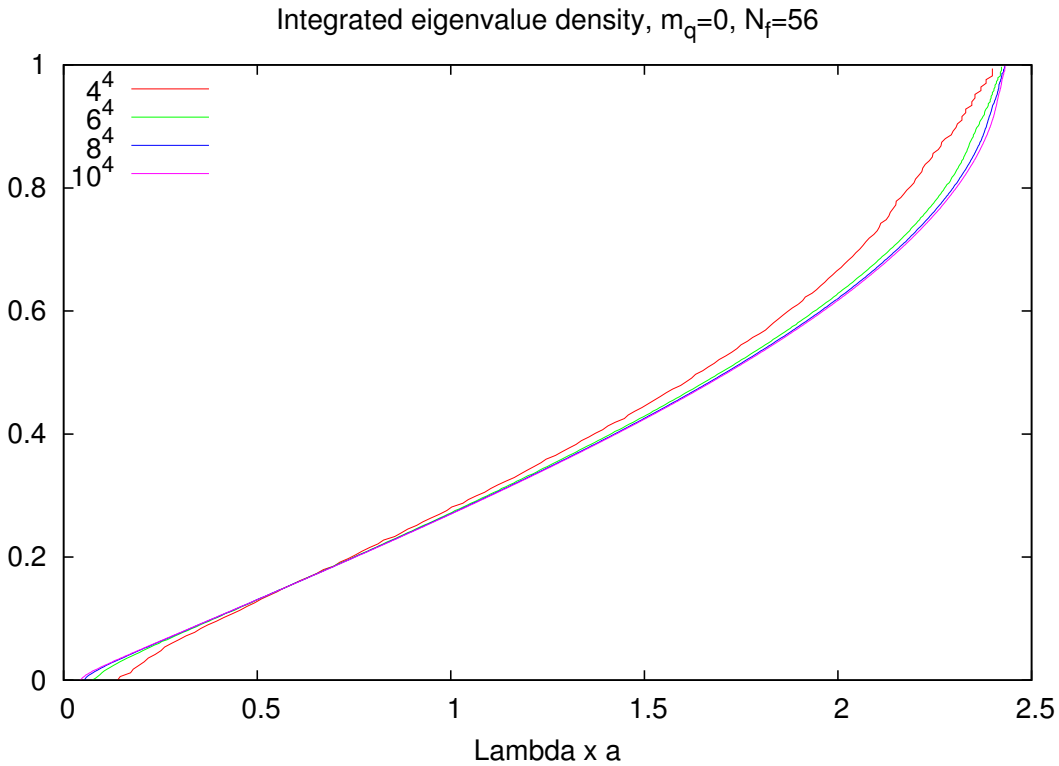
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Compare different volumes for $N_f = 56$:

- large eigenvalues (UV) are L-independent,
- the IR spectral gap shrinks as L increases



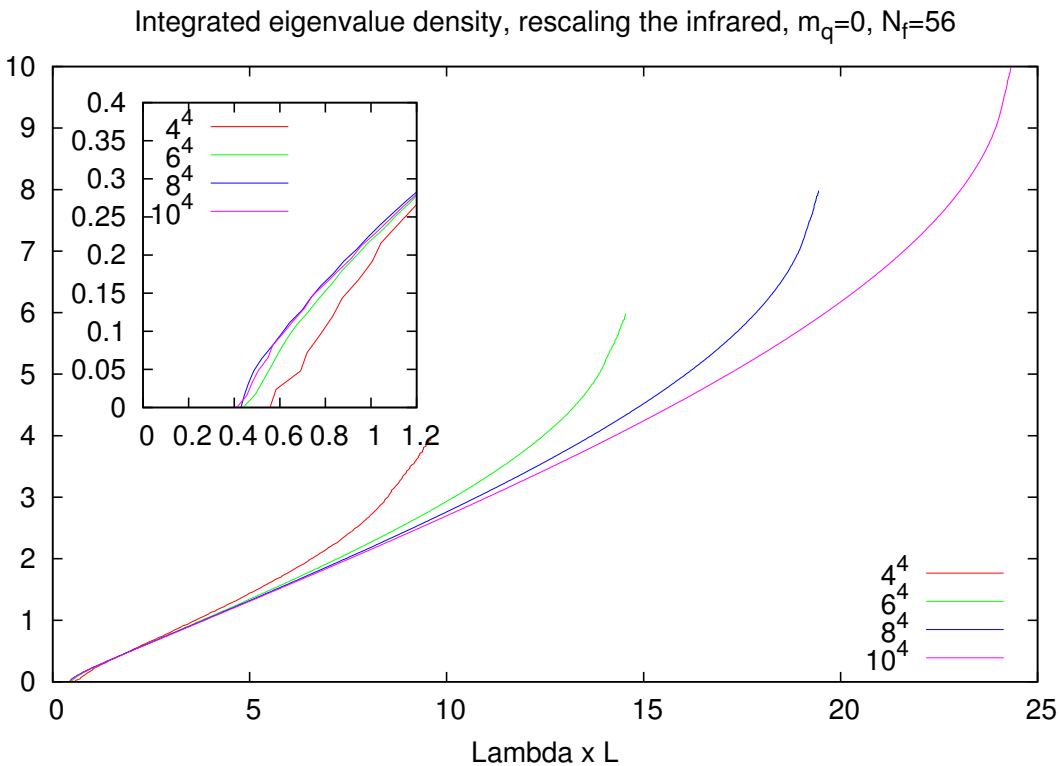
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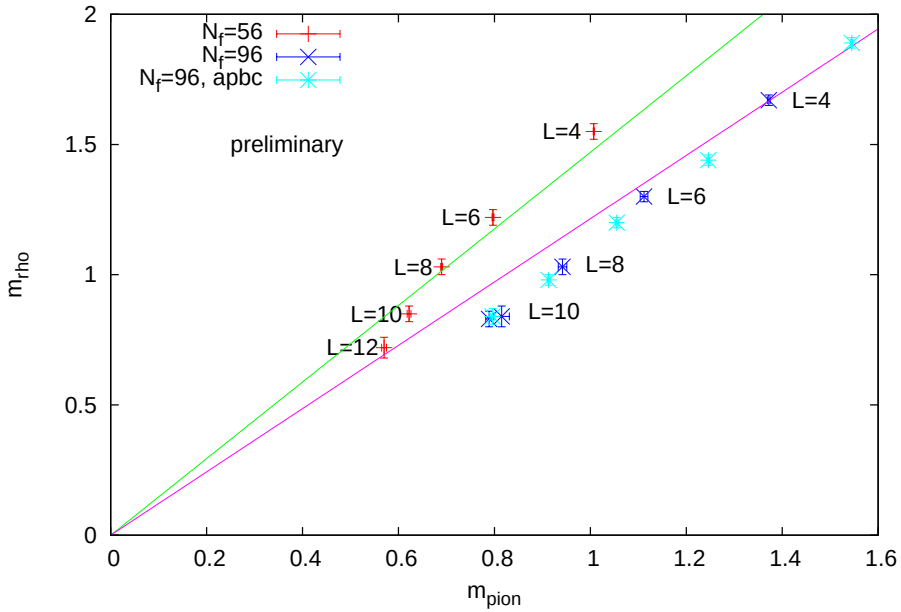
- IR spectrum invariant after rescaling by L: spectral gap $\propto 1/L$
- IR physics only depends on L, while the UV physics depends on a
- no other scale in the system $\Rightarrow$ Dirac spectrum consistent with **IR-conformal theory!**

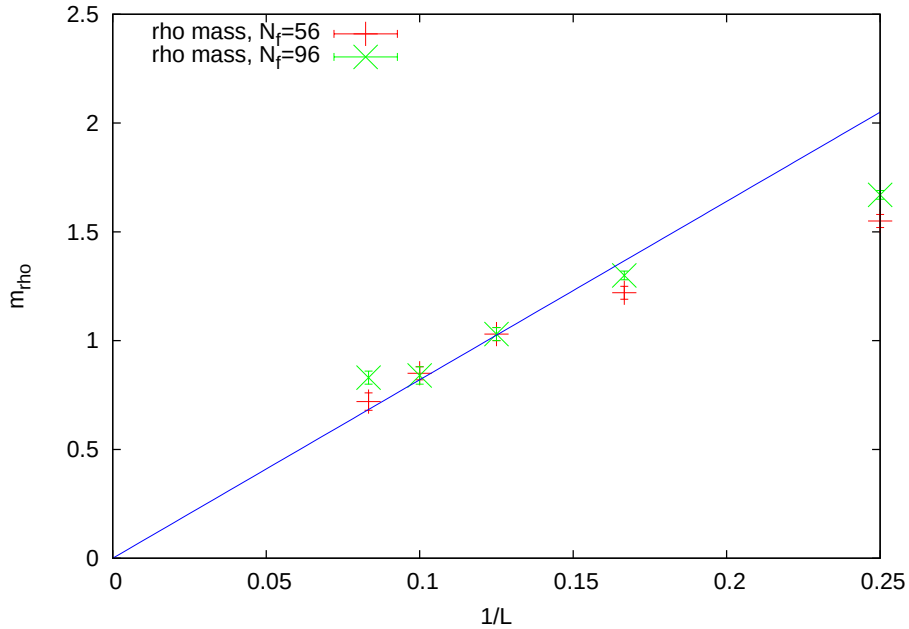


Characterizing the chirally restored phase: III. hadron masses

Hadron spectrum obtained from simulations with $N_f = 56$ and $N_f = 96$ at **zero quark mass**

- hadron masses measured for $m_q = 0$ are non-zero
- but masses decrease (a lot) as the lattice size L is increased
- parity partners degenerate (c.f. chiral symmetry restoration)
- mass ratios $\sim$ **independent of L ?**:

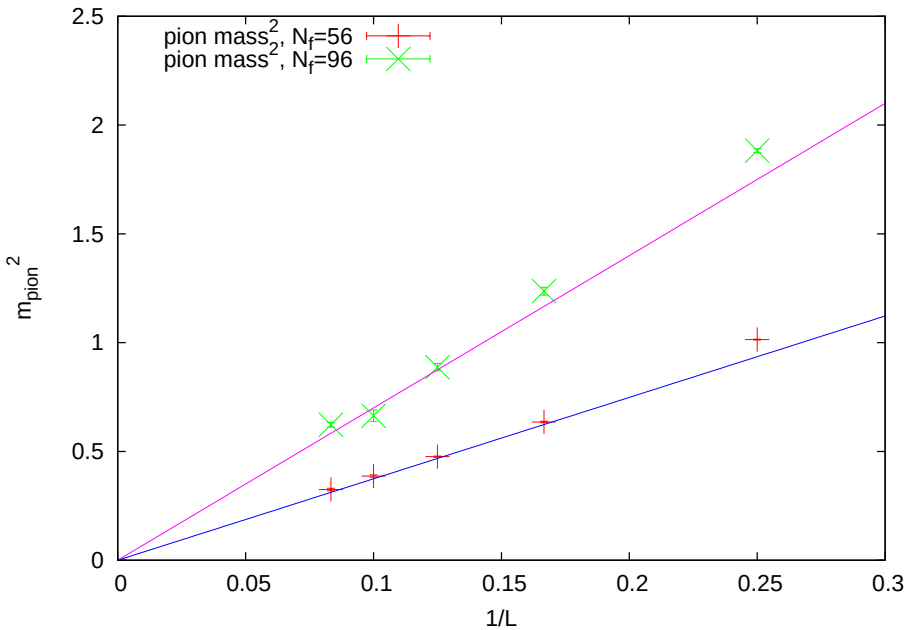




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Conjecture: $\beta = 0$ IR-conformal phase is analytically connected with the weak-coupling, continuum IR-conformal phase

Study of **continuum limit** is much more difficult:

- for a given lattice size L^4, the scales are ordered as $a \ll 1/\Lambda_{QCD} \ll L$
- at strong-coupling the hierarchy is $a \simeq 1/\Lambda_{QCD} \ll L$
- range of conformal invariance ($L\Lambda_{QCD}$) maximized at $\beta = 0$ for given lattice size L/a

weak coupling:



strong coupling:



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strong-coupling limit is the laboratory of choice to study a 4d IR-conformal gauge theory

Summary

Shown: for $\beta = 0$, a strong first order **bulk transition** exists which is $N_{\rm f}$**-driven** to a chirally symmetric phase

- in the chiral limit: $N_{\rm f}^c = 52(4)$ *continuum* flavors
- finding in contrast to meanfield prediction
- chirally restored phase extends towards weak coupling

Also shown: for $\beta = 0$, **"large-$N_{\rm f}$ QCD" is IR-conformal with non-trivial IRFP**

- strong-coupling limit allows economical study of a 4d IR-conformal gauge theory
- large $N_{\rm f}, m_q = 0$ simulations can be performed without too much computer effort

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no evidence for additional $T = 0$ phase transition as β is increased

$\Downarrow$

Conjecture: strong coupling chirally symmetric, IR-conformal phase is analytically connected with the **continuum IR-conformal phase**

Summary

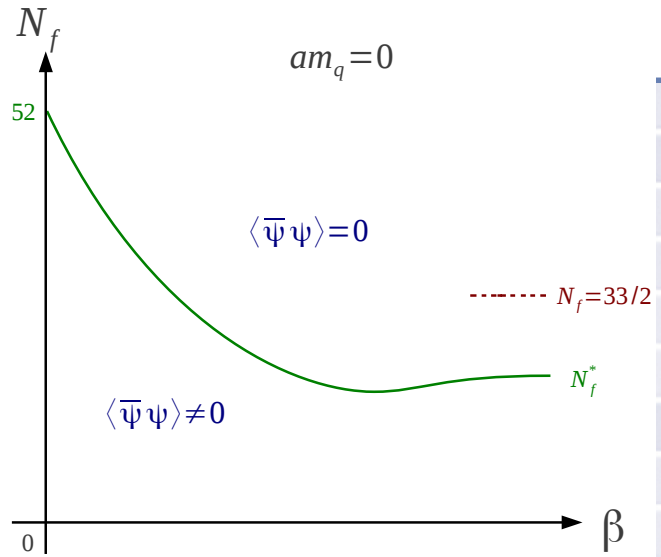
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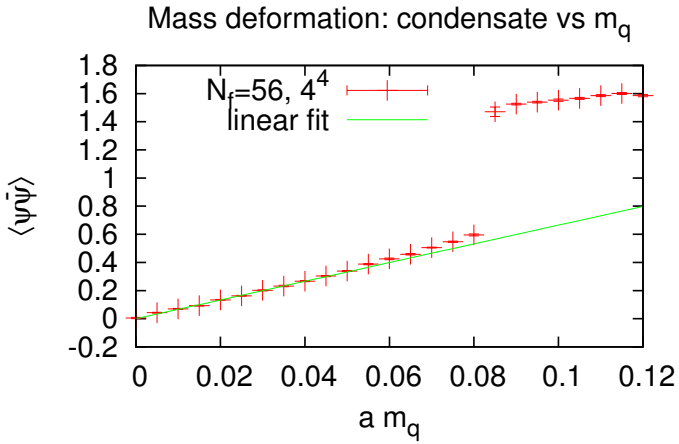
Conjecture:



Discussion

Outlook

- quark mass breaks conformality; chiral condensate is proportional to $m_q \quad \Rightarrow \quad \gamma^* = 1$
- glueballs, static potential, ...



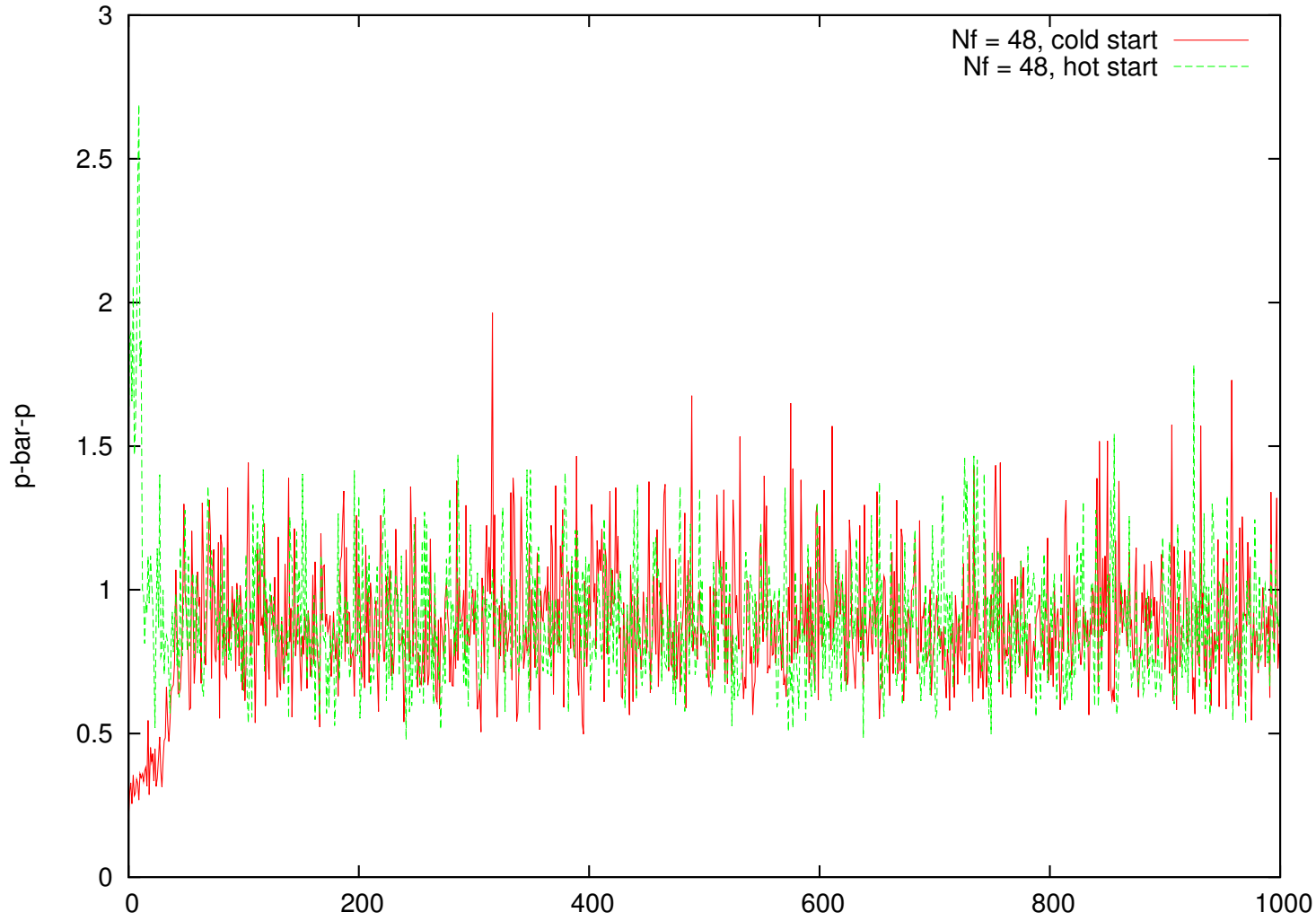
Single parameter of this IR-conformal theory is N_f:

- increasing N_f increases magnitude of the spectral gap – IRFP moves towards 0 ?
- decreasing N_f brings us back to a chirally broken phase, via a first-order transition as in Sannino's "Jumping dynamics" [hep-ph/1205.4246] $\rightarrow$ no walking dynamics

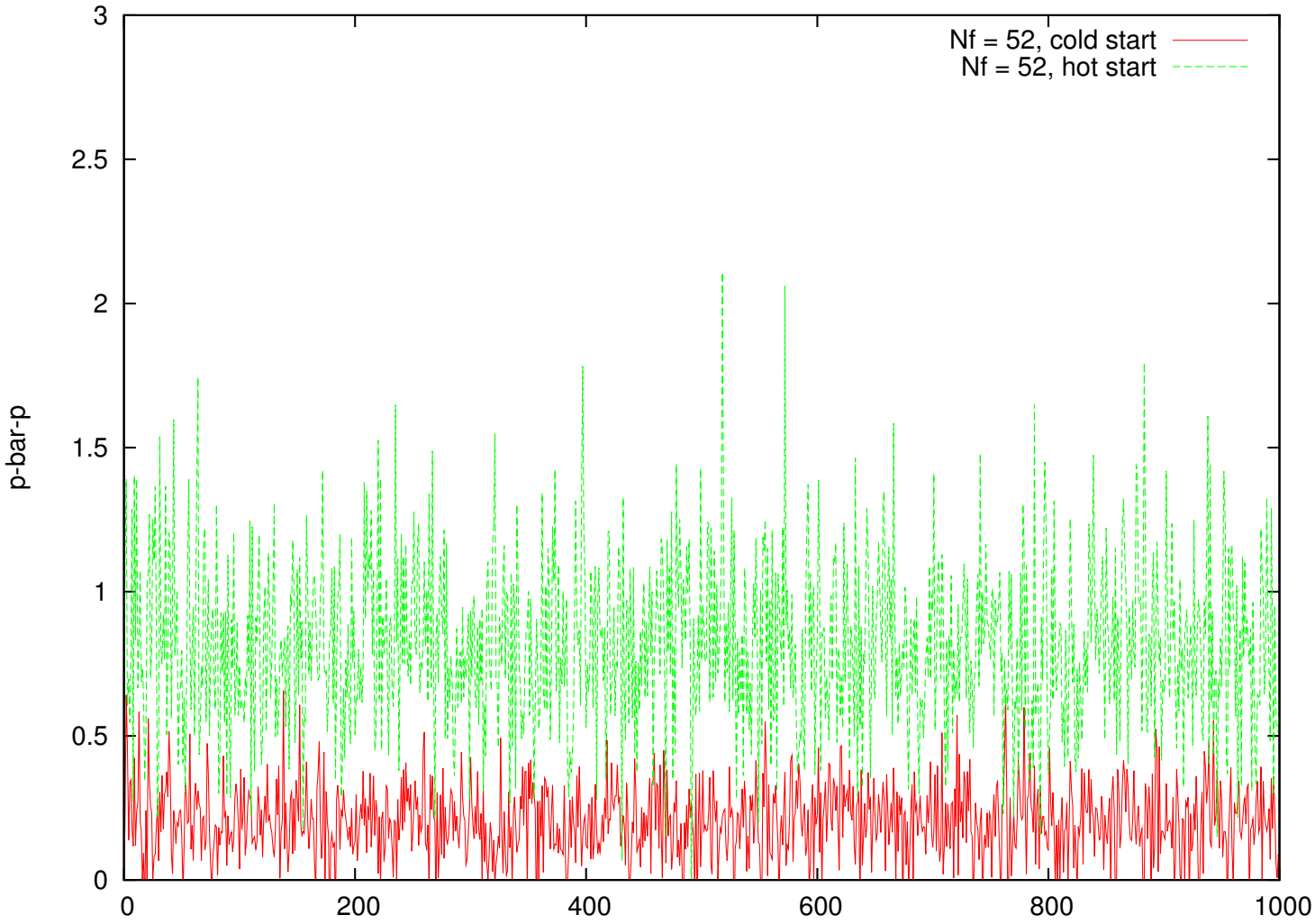
Our findings are **consistent with the literature:**

- Kogut, Sinclair, Nucl. Phys. B295 (1985) bulk transition for $N_f = 12$
- Damgaard, *et al.*, Phys. Lett. B400 (1997): bulk transition for $N_f = 16$
- Deuzeman et al., PRD 82 (2010) bulk transition for $N_f = 12$
- Jin, Mawhinney, PoS Lattice2011: bulk transition for $N_f = 12$ with improved action
- A. Hasenfratz, hep-lat/1111.2317 (2012): bulk transition for $N_f = 8, 12$

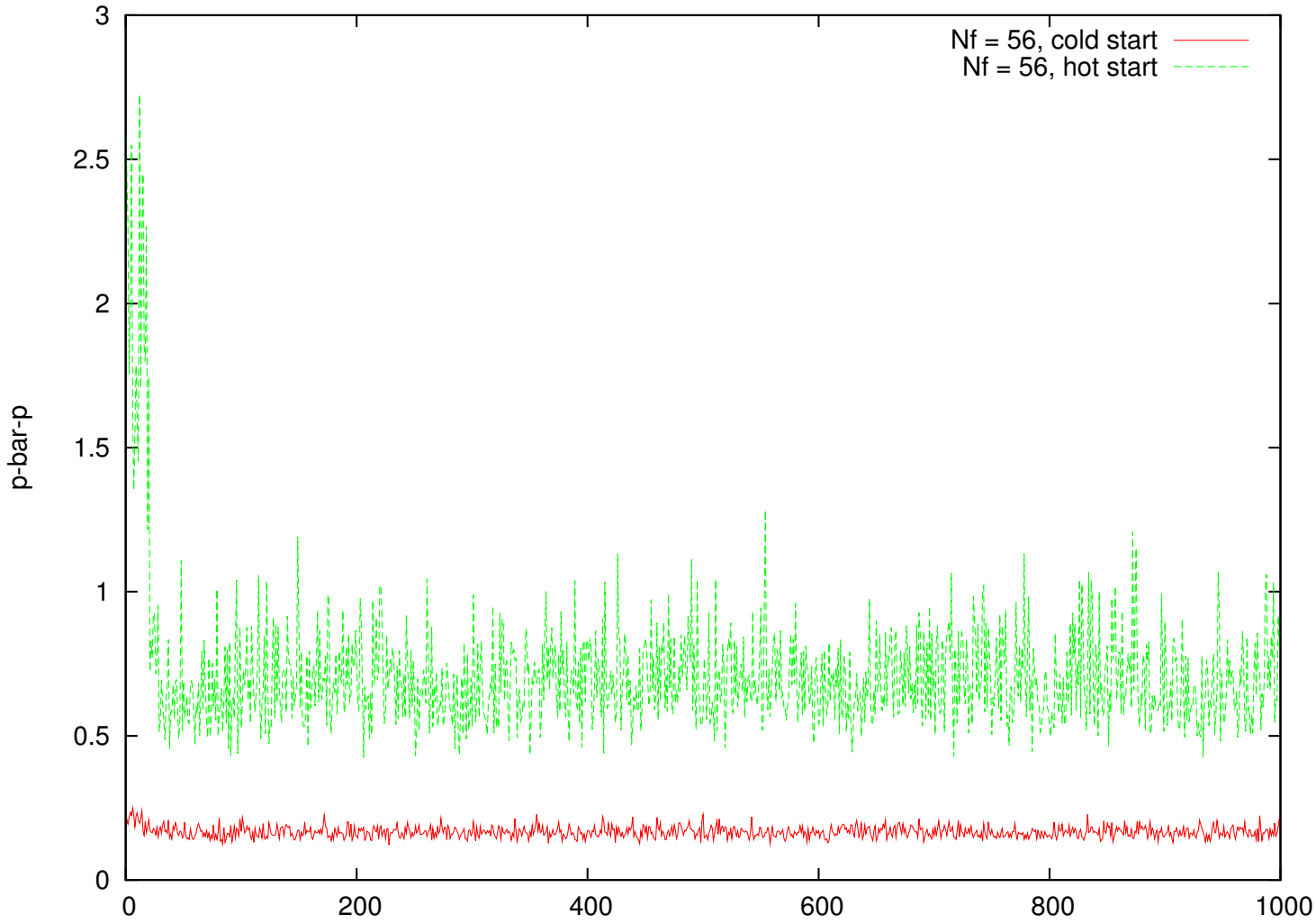
Backup Slide: Time Histories for Chiral Condensate



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