Chiral transition & deconfinement in the presence of a magnetic background

Eduardo S. Fraga

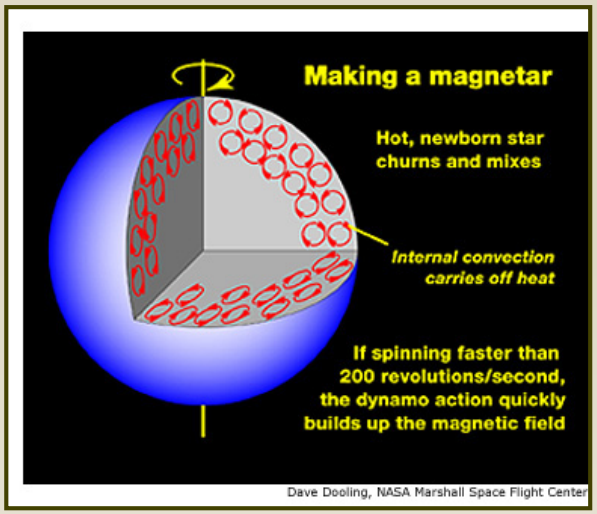


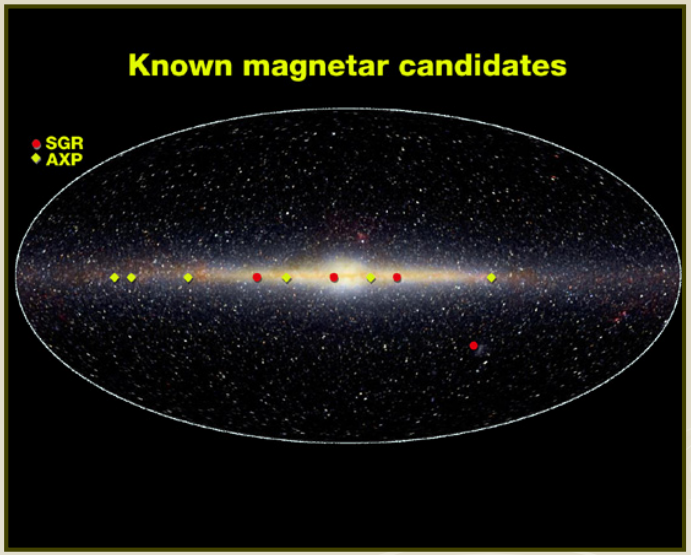
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Motivation

Strong interactions under intense magnetic fields can be found, in principle, in a variety of systems:

- **High density and low temperature**
  - "Magnetars": B ~ 10^{14}–10^{15} G at the surface, much higher in the core [Duncan & Thompson (1992/1993)]



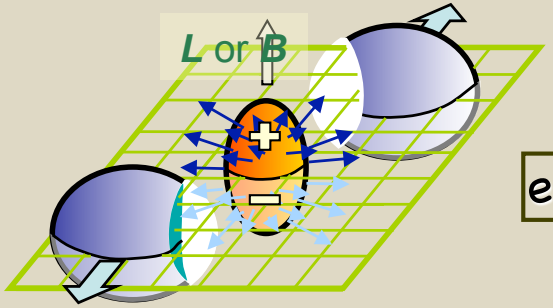


  - Stable stacks of π^0 domain walls or axial scalars (η,η') domain walls in nuclear matter: B ~ 10^{17}–10^{19} G [Son & Stephanov (2008)]

❖ High temperature and low density

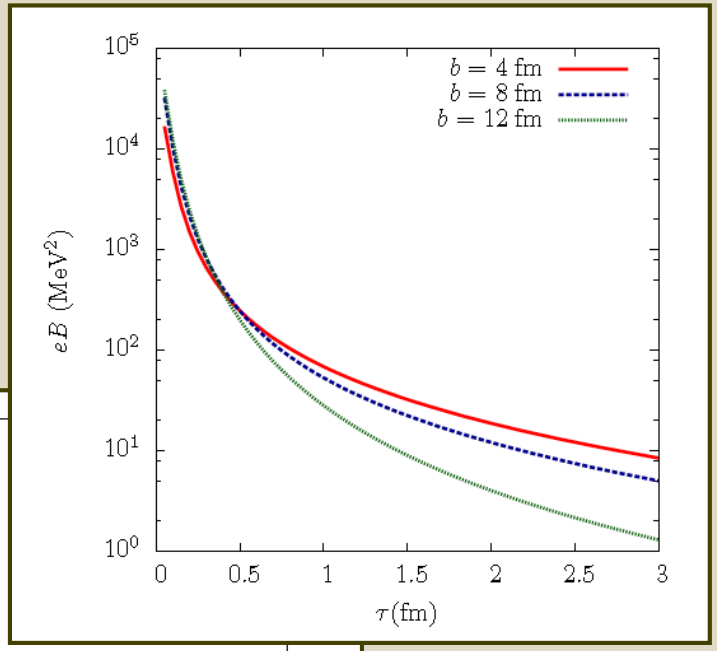
high magnetic fields in non-central RHIC collisions

[Kharzeev, McLerran & Warringa (2008)]

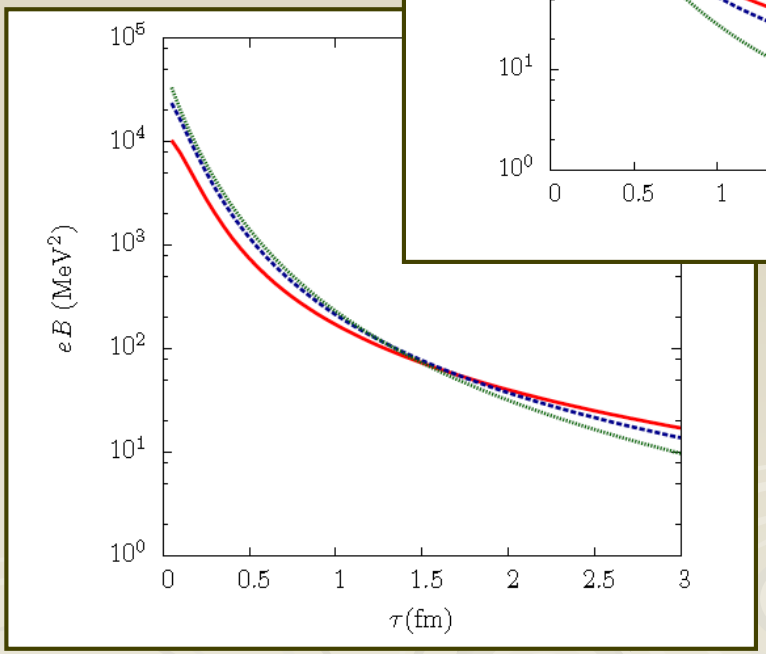


[Voloshin, QM2009]

$eB \sim 10^4$–10^5 MeV^2 $\sim 10^{19}$ G



[Au-Au, 200 GeV]



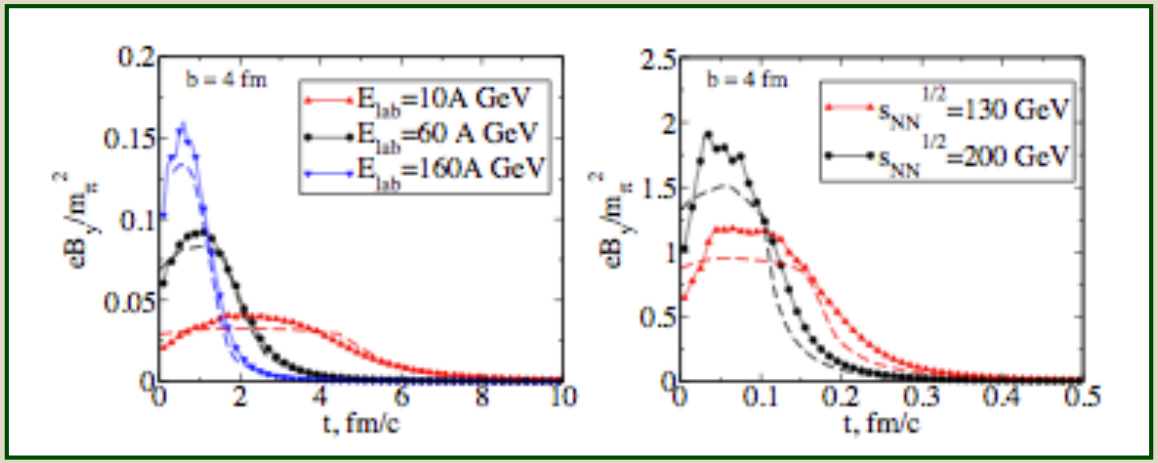
[Au-Au, 62 GeV]

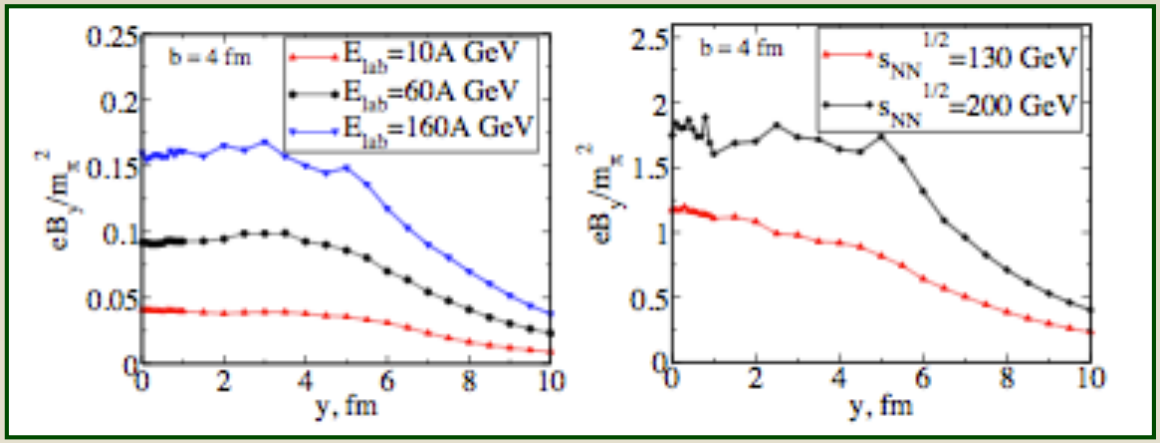
For comparison:

- "Magnetars": B ~ 10^{14}–10^{15} G at the surface, higher in the core
  [Duncan & Thompson (1992/1993)]
- Early universe (relevant for nucleosynthesis): B~ 10^{24} G for the EWPT epoch [Grasso & Rubinstein (2001)]

Plus: mechanism based on separation of charge for the detection of the Chiral Magnetic Effect and P-odd effects
[Voloshin (2000,2004), Kharzeev (2006); Kharzeev & Zhitnitsky (2007); Kharzeev, McLerran & Warringa (2008); Fukushima, Kharzeev & Warringa (2008)]

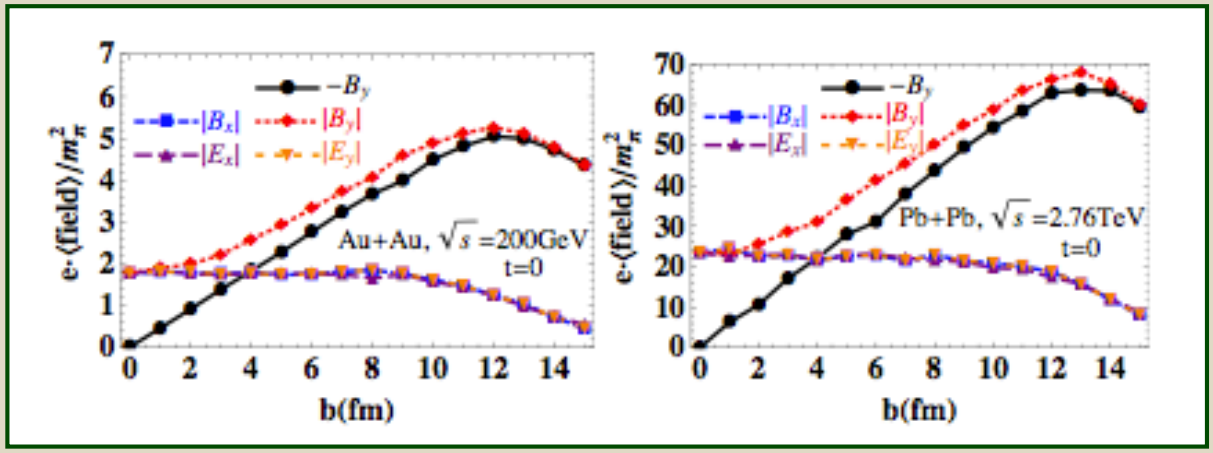
High magnetic fields in heavy-ion collisions have been computed...





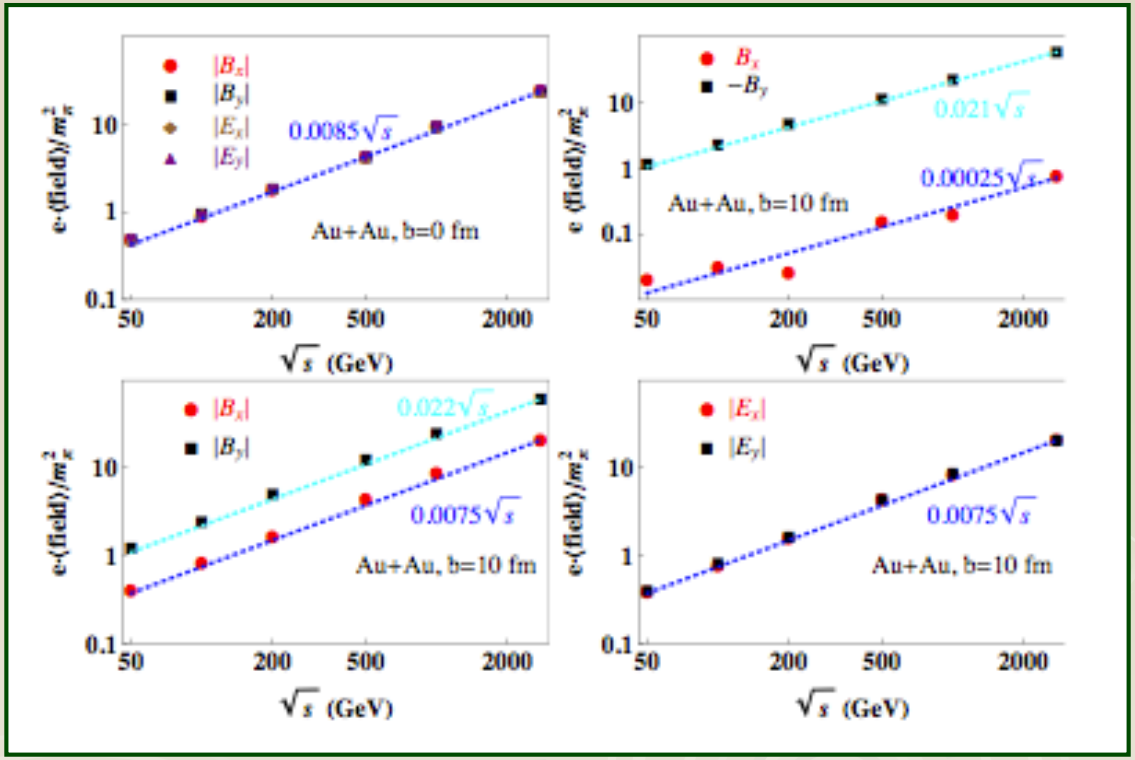
- RHIC energies, higher fields at the LHC
- Fields very flat in the central region (system may be deconfined/chiral)
- semi-analytic estimates & UrQMD agree well

[Skokov, Illariunov & Toneev (2009)]



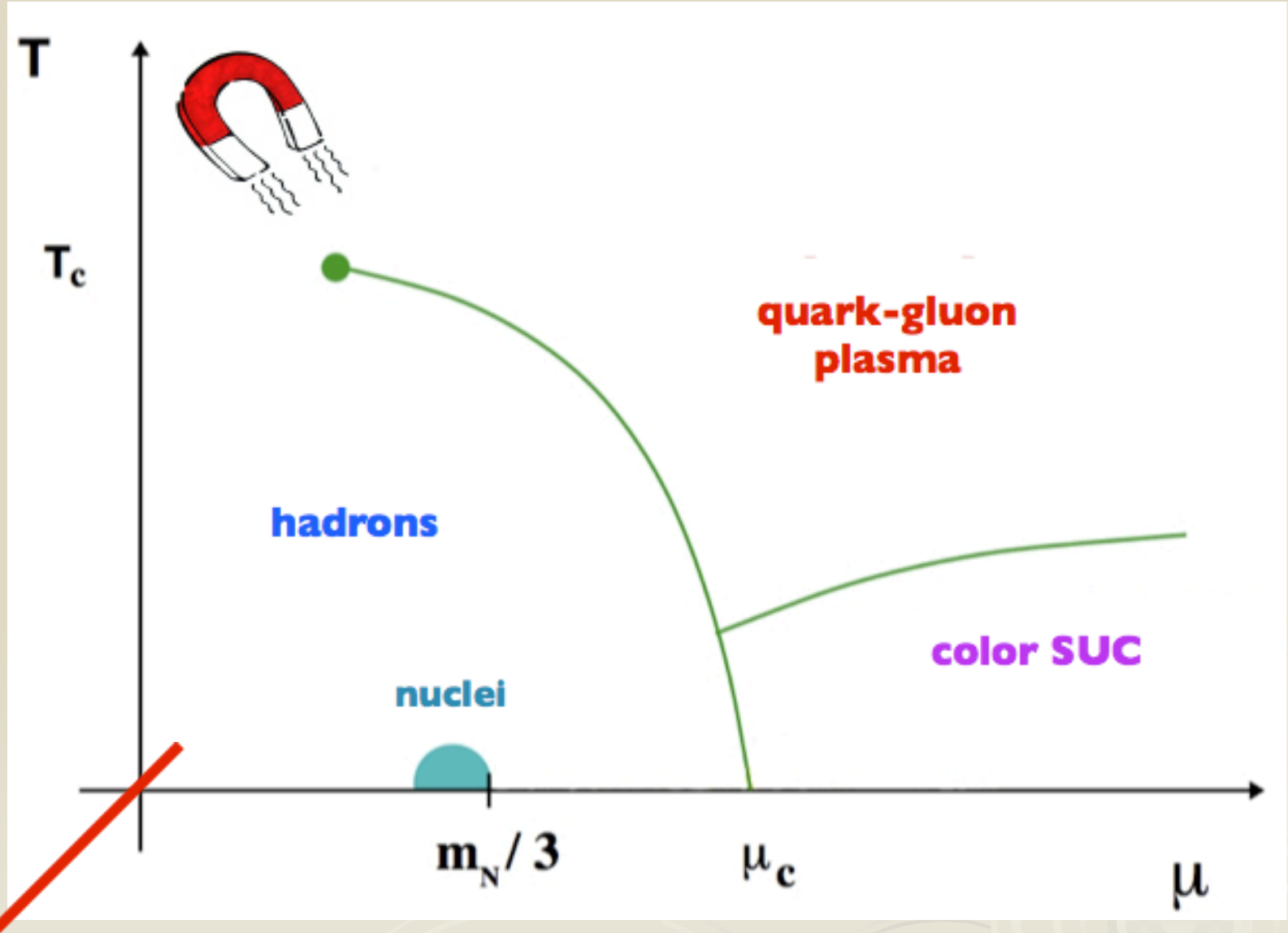
- HIJING computation
- RHIC vs LHC energies

- Huge fields for ultra-peripheral collisions due to event-by-event fluctuations
- Possible vacuum SUC via ρ meson condensation [Chernodub (2010)]
- Possible building of spin-charge correlation for quarks



[Deng & Huang (2012)]

Pictorially:



Strong motivation: in-medium strong interactions under extreme magnetic fields are:

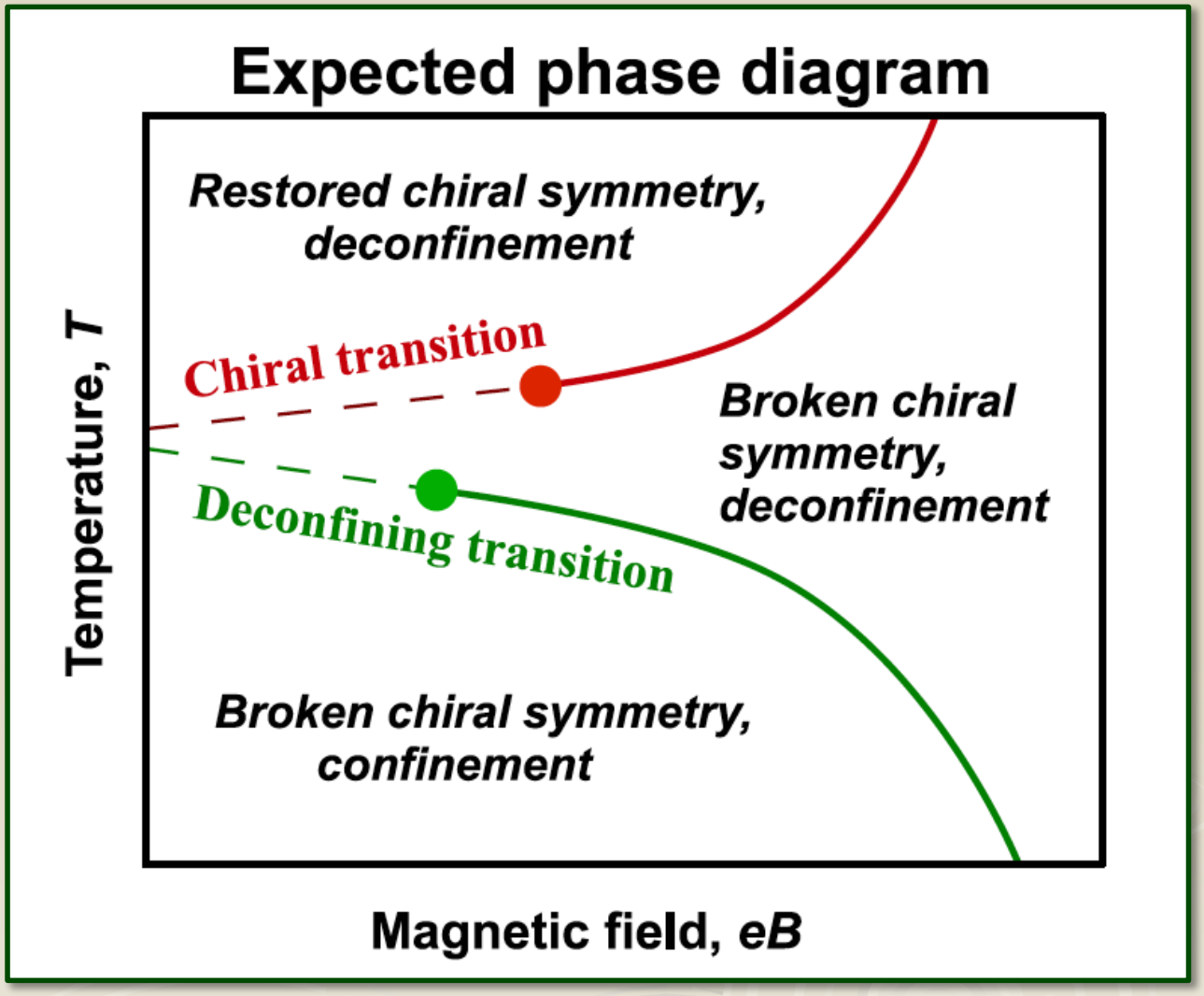
- of experimental relevance
  - HICs, early universe, magnetars
- rich in new phenomenology
  - Chiral magnetic effect, new QCD phase diagram, vacuum SUC
- amenable to lattice simulations: new open channel for comparison!
  - model constraining, tests for pQCD and nonpert. methods, ...

Outline

- Expected phase diagram and some questions
- Incorporating a magnetic background in loop integrals
- LSM coupled to the Polyakov loop (previous results)
- MIT Bag model
- pQCD in a nonperturbative magnetic background (preliminary)
- Final remarks

From first results, one could expect:

- Deconfining:
  Agasian & Fedorov (2008)
- Chiral:
  ESF & Mizher (2008)



[Mizher, Chernodub & ESF (2010)]

Several approaches (oldest usually concerned with vacuum properties)

NJL/LSM:

- Klevansky & Lemmer (1989)
- Gusynin, Miransky & Shovkovy (1994/1995)
- Klimenko et al. (1998-2008)
- Hiller, Osipov, … (2007-2008)
- Rojas, Ayala, Bashir & Raya (2008)
- Boer & Boomsma (2009)
- Menezes et al (2009)
- Fukushima, Ruggieri & Gatto (2010-2011) – PNJL
- Andersen & Khan (2011)
- …

χPT:

- Shushpanov & Smilga (1997)
- Agasian & Shushpanov (2000)
- Cohen, McGady & Werbos (2007)
- Agasian & Fedorov (2008)
- …

Large-N QCD:

- Miransky & Shovkovy (2002)

Quark model:

- Kabat, Lee & Weinberg (2002)

FRG:

- Skokov (2011)
- Fukushima & Pawlowski (2012)
- Andersen & Tranberg (2012)

Lattice:

- D'Elia, Mukherjee & Sanfilippo (2010)
- D'Elia & Negro (2011)
- Bali et al (2011)

Holographic:

- Callebaut, Dudal & Verschelde (2011)

Incorporating a magnetic background in loop integrals

Let us assume the system is in the presence of a strong magnetic field background that is constant and homogeneous:

$$\vec{B} = B\hat{z} \quad \xrightarrow{\text{choice of gauge}} \quad A^\mu = (A^0, \vec{A}) = (0, -By, 0, 0)$$

- charged mesons (new dispersion relation):

$$(\partial^2 + m^2)\phi = 0 \qquad \partial_\mu \to \partial_\mu + iqA_\mu$$

$$\longrightarrow \quad \varphi''(y) + 2m\left[\left(\frac{p_0^2 - p_z^2 - m^2}{2m}\right) - \frac{q^2B^2}{2m}\left(y + \frac{p_x}{qB}\right)^2\right]\varphi(y) = 0$$

Landau levels:

$$\varepsilon_n \equiv \left(\frac{p_{0n}^2 - p_z^2 - m^2}{2m}\right) = \left(n + \frac{1}{2}\right)\omega_B \qquad \omega_B \equiv \frac{|q|B}{m}$$

$$\longrightarrow \quad p_{0n}^2 = p_z^2 + m^2 + (2n+1)|q|B$$

- quarks (new dispersion relation):

$$(i\gamma^\mu\partial_\mu - m)\psi = 0 \qquad \partial_\mu \to \partial_\mu + iqA_\mu$$

$$u''(y) + 2m\left[\left(\frac{p_0^2 - p_z^2 - m^2 + qB\sigma}{2m}\right) - \frac{q^2B^2}{2m}\left(y + \frac{p_x}{qB}\right)^2\right]u(y) = 0$$

$$p_{0n}^2 = p_z^2 + m^2 + (2n + 1 - \sigma)|q|B \qquad \sigma = \pm 1$$

- integration measure:

T = 0:

$$\int \frac{d^4k}{(2\pi)^4} \mapsto \frac{|q|B}{2\pi}\sum_{n=0}^{\infty}\int \frac{dk_0}{2\pi}\frac{dk_z}{2\pi}$$

T > 0:

$$T\sum_\ell \int \frac{d^3k}{(2\pi)^3} \mapsto \frac{|q|BT}{2\pi}\sum_\ell\sum_{n=0}^{\infty}\int \frac{dk_z}{2\pi}$$

l: Matsubara index
n: Landau level index

Linear Sigma Model coupled to Polyakov Loops

[Mizher, Chernodub & ESF (2010)]

A. Degrees of freedom and approximate order parameters

O(4) chiral field: $\phi = (\sigma, \vec{\pi})\,, \qquad \vec{\pi} = (\pi^+, \pi^0, \pi^-)$

quark spinors: $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$

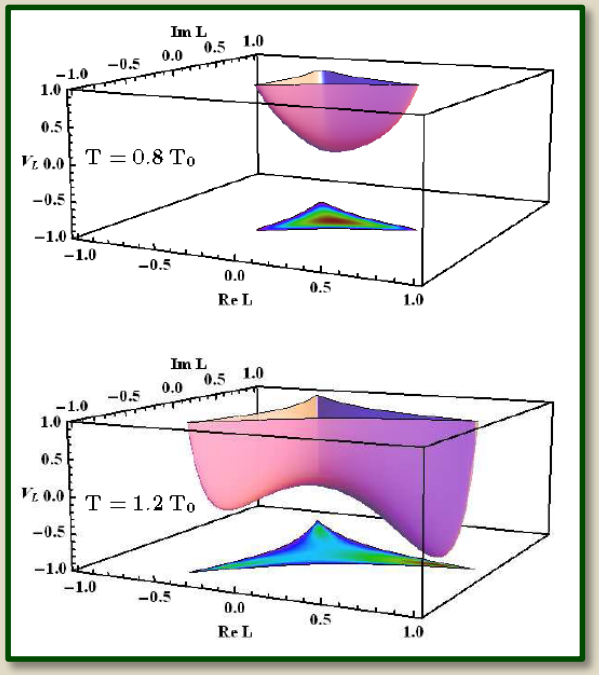
Polyakov loop: $L(x) = \frac{1}{3}\mathrm{Tr}\,\Phi(x)\,, \quad \Phi = \mathcal{P}\exp\Big[i\int_0^{1/T} \mathrm{d}\tau A_4(\vec{x},\tau)\Big]$

$$\text{Chiral symmetry}: \quad \begin{cases} \langle\sigma\rangle \neq 0\,, & \text{low } T \\ \langle\sigma\rangle = 0\,, & \text{high } T \end{cases}$$

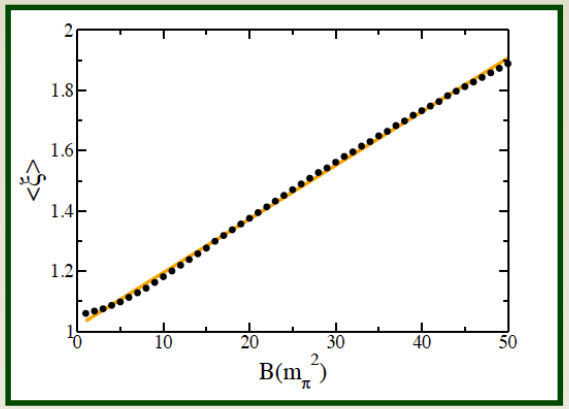
$$\text{Confinement}: \quad \begin{cases} \langle L\rangle = 0\,, & \text{low } T \\ \langle L\rangle \neq 0\,, & \text{high } T \end{cases}$$

B. Summary of results

Paramagnetically-increased breaking of Z(3)

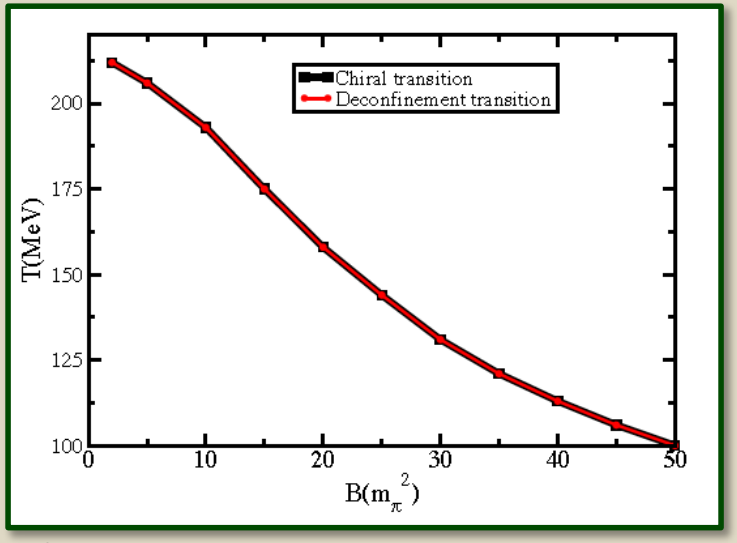


Linear chiral condensate

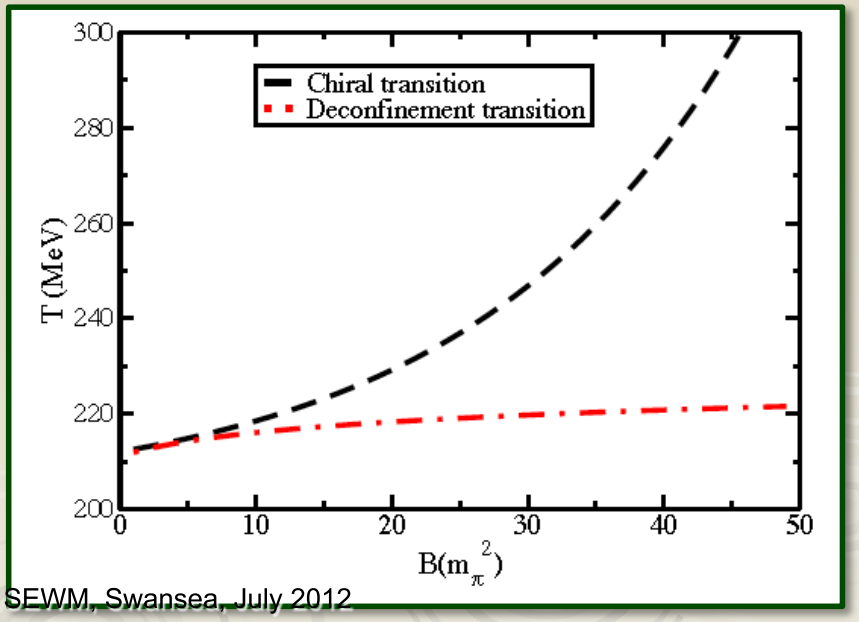


Phase diagram

(without vacuum corrections)



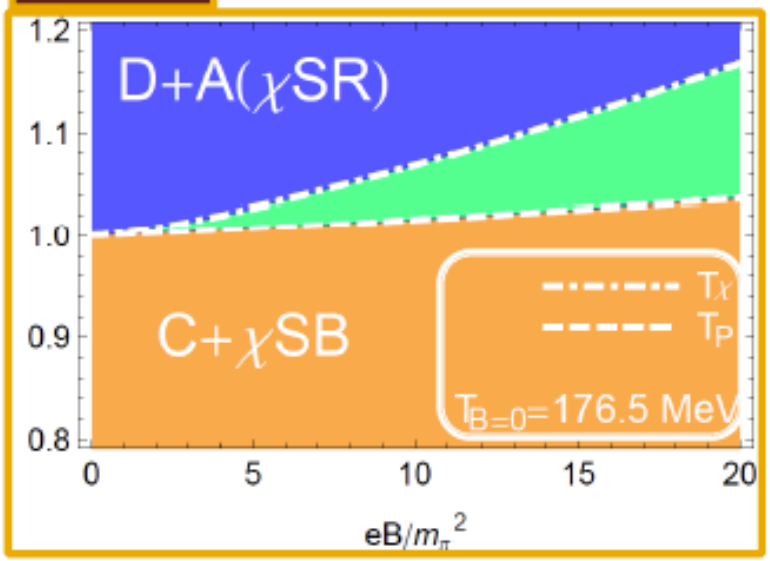
(with vacuum corrections)



Comparison with PNJL results

[Ruggeri & Gatto (2010)]

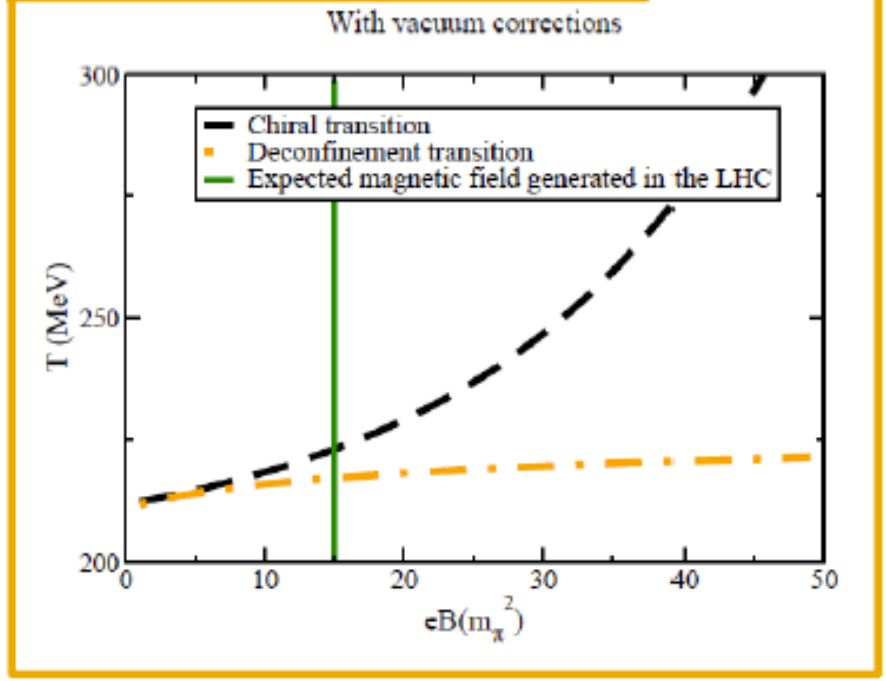
P-NJL



Excellent agreement

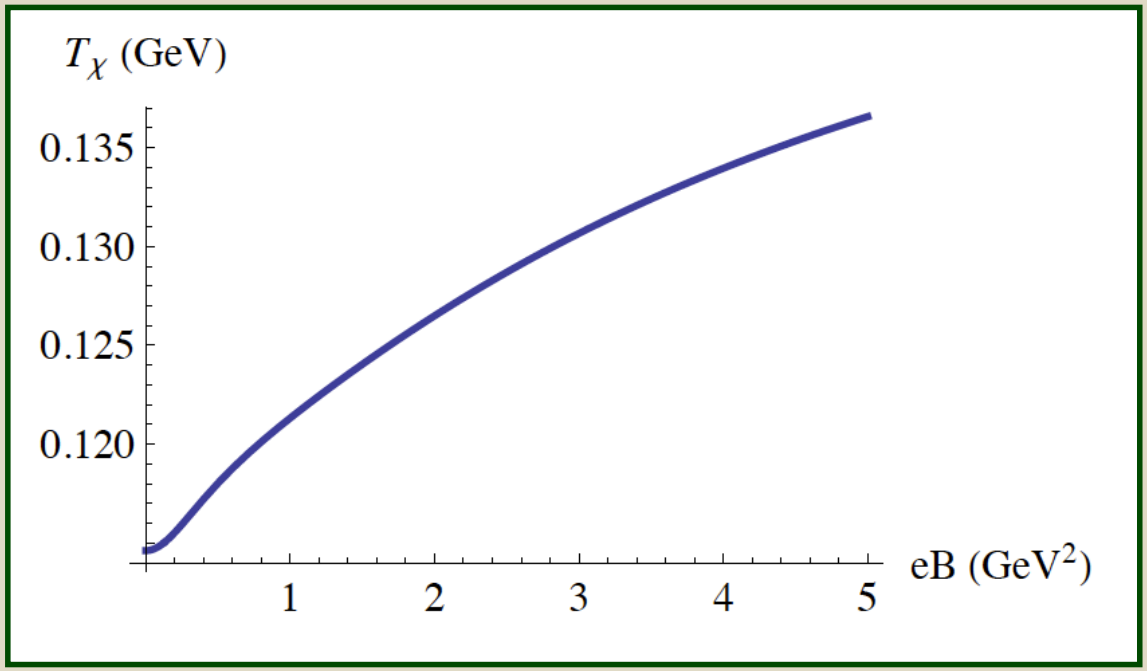
P-QM

A. J. Mizher *et al*, **arXiv:1004.2712**



Comparison with holographic results

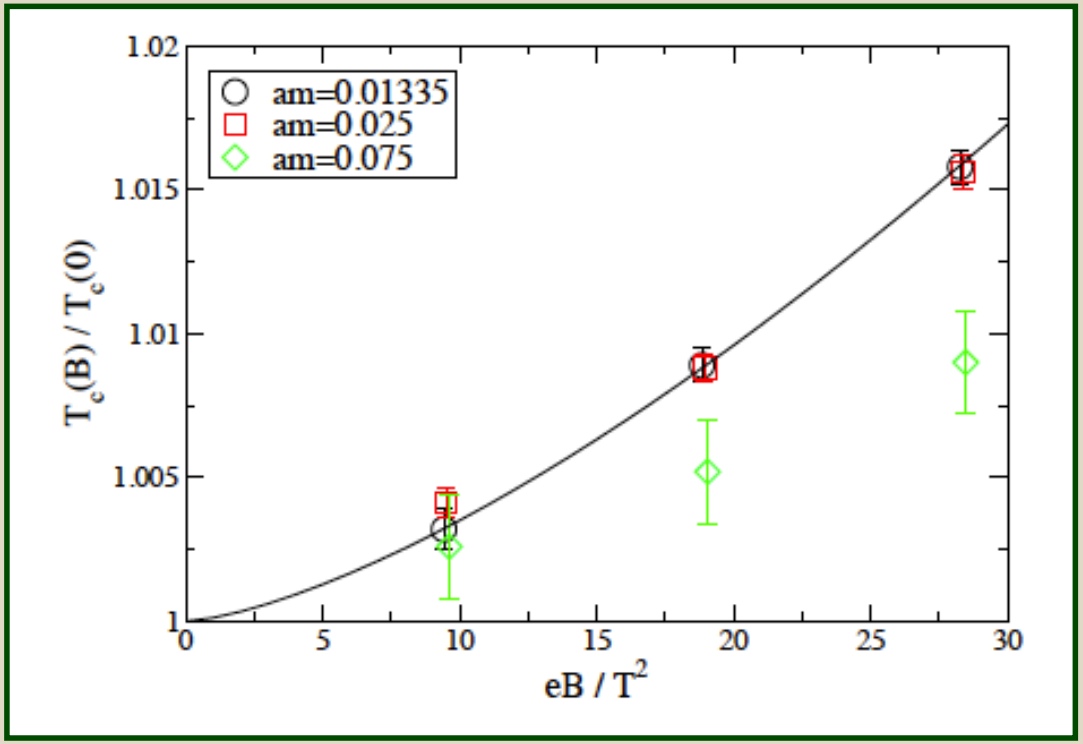
[Callebaut, Dudal & Verschelde (2011)]



Deconfining critical temperature independent of B by construction in the Sakai-Sugimoto model, so there is a splitting.

Comparison with first lattice results [N_f=2]

[D'Elia, Mukherjee & Sanfilippo (2010)]

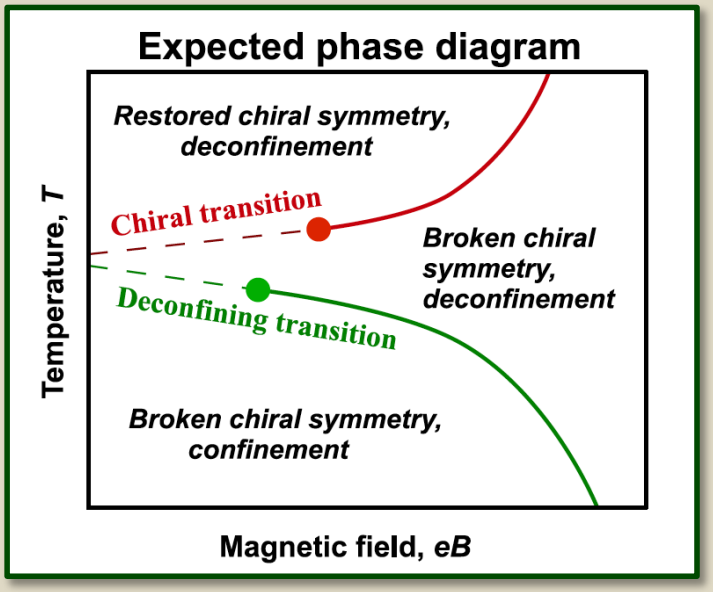


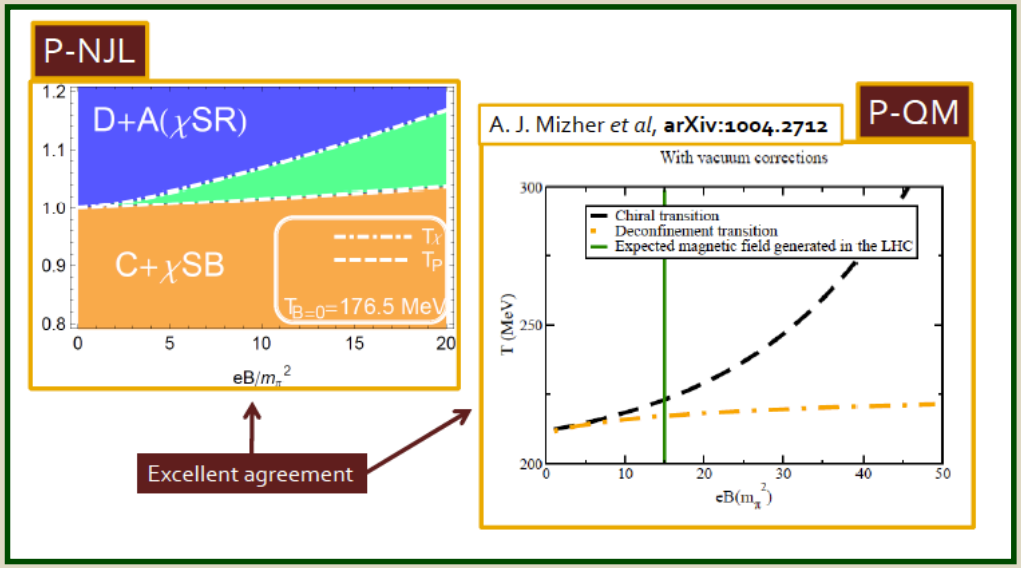
- Very small effect

« the deconfinement and chiral restoring temperatures both increase, even if we do not see any sign for a faster grow and splitting of the chiral transition till |eB| ~ 20 m_π^2 ».

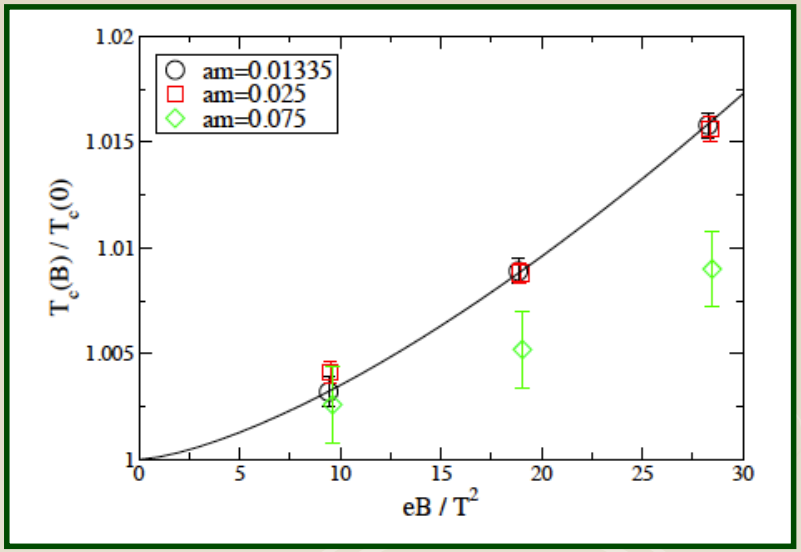
However, large pion masses: may have to go to higher B to see a splitting, if there is one.

An incomplete set of results

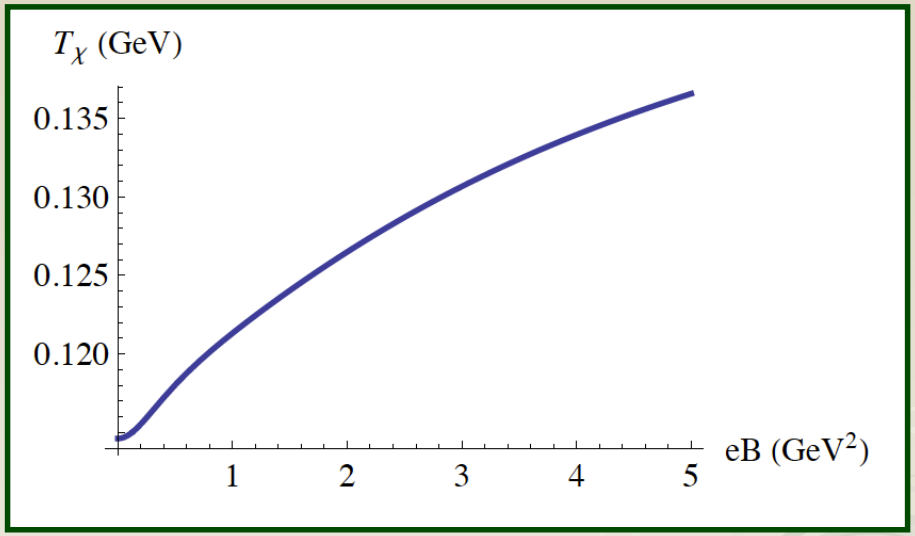




Lattice (1st results)



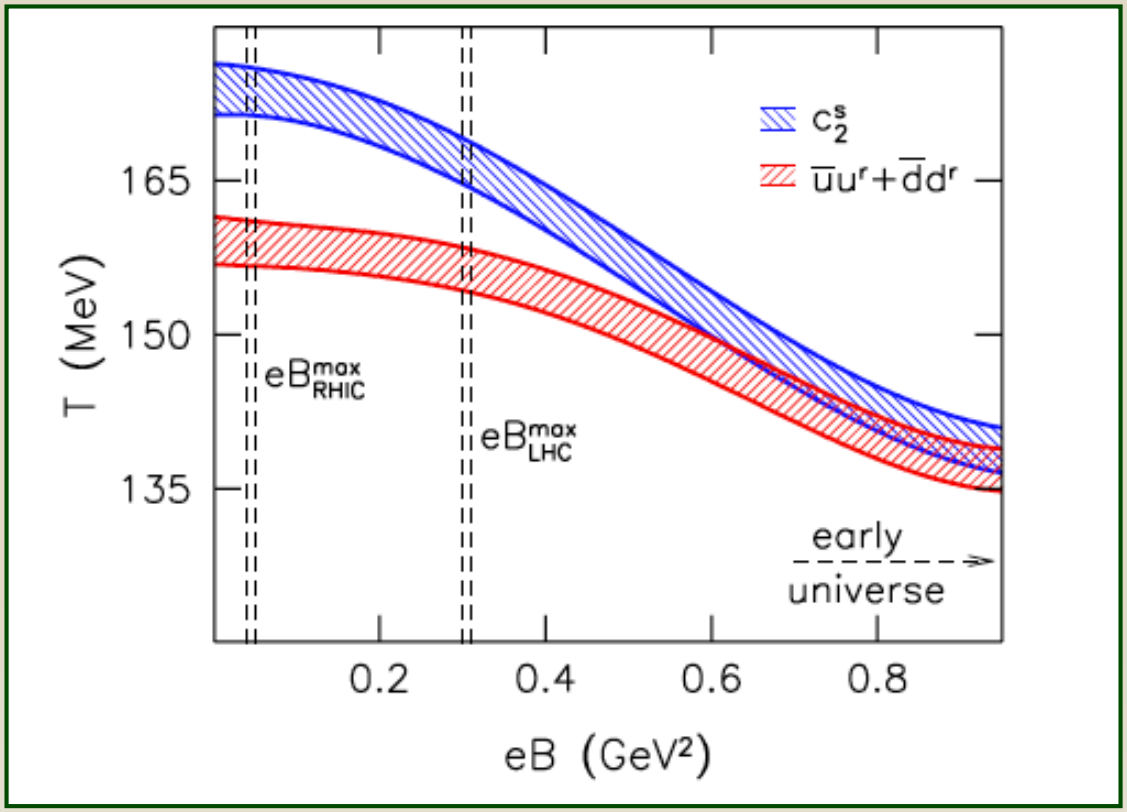
Holographic



Everything looking good and consistent in 2011, but...

... then came the newest lattice results [N_f=2+1, physical masses]:

T_c decreases and saturates!



[Bali et al (2012)]

???

Magnetic MIT Bag Model [ESF & Palhares (2012)]

- Simplest phenomenological approach: MIT bag pressure for the QGP + pion gas for the hadronic sector

- Simple setup to discuss subtleties of vacuum & thermal contributions in each phase (no extra model complications)

- Presumably reasonable qualitative description of the behavior of $T_c(B)$

  - QGP sector:

$$P^B_{\rm QGP} = 2(N_c^2-1)\frac{\pi^2 T^4}{90} + P_q - \mathcal{B}$$

$$\frac{P_q}{N_c} \overset{\text{large } B}{=} \sum_f \frac{(q_f B)^2}{2\pi^2}\left[x_f \ln\sqrt{x_f}\right] + T\sum_f \frac{q_f B}{2\pi^2}\int dk_z \ln\left[1+e^{-\sqrt{k_z^2+m_f^2}/T}\right]$$

  - pion sector:

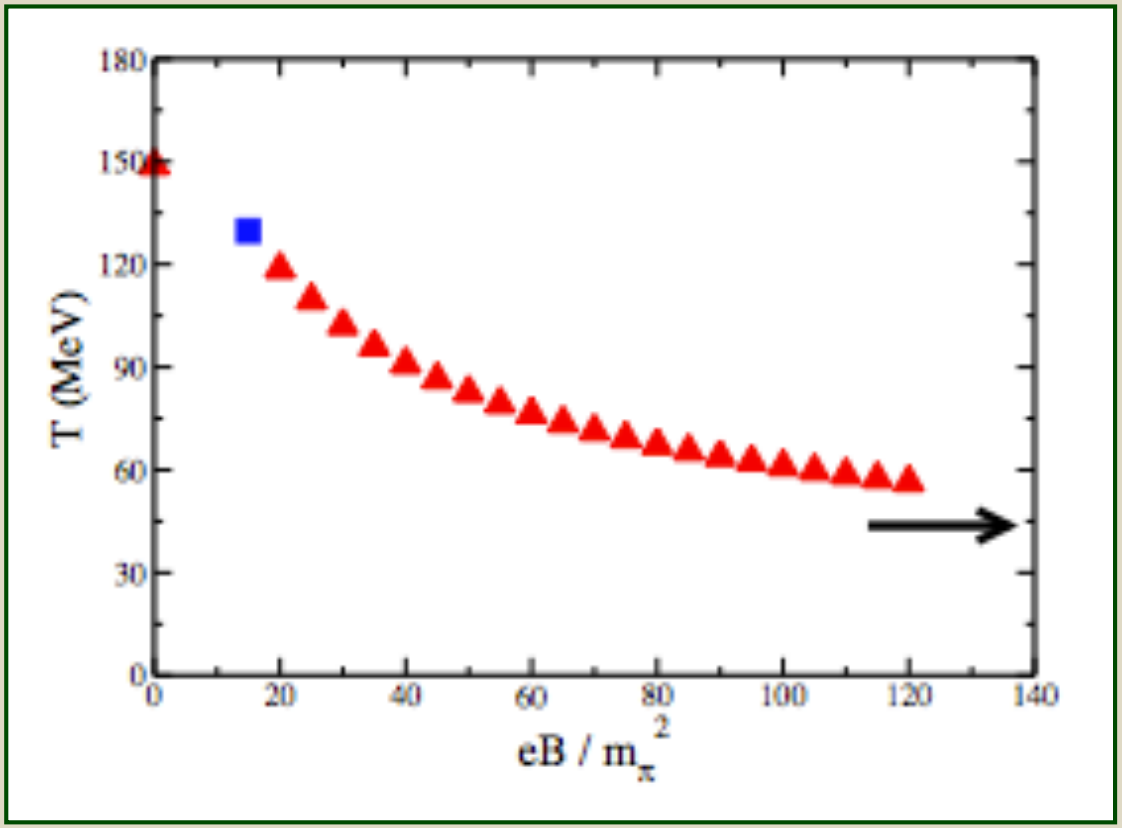
$$P_{\pi^0} = -\frac{T}{2\pi^2}\int dk k^2 \ln\left[1-e^{-\sqrt{k^2+m_\pi^2}/T}\right]$$

$$P_{\pi^+} + P_{\pi^-} \overset{\text{large } B}{=} -\frac{(eB)^2}{4\pi^2}\zeta^{(1,1)}(-1,1/2)\, x_\pi$$

$$x_i \equiv \frac{m_i^2}{2q_i B}$$

Phase diagram

[ESF & Palhares (2012)]

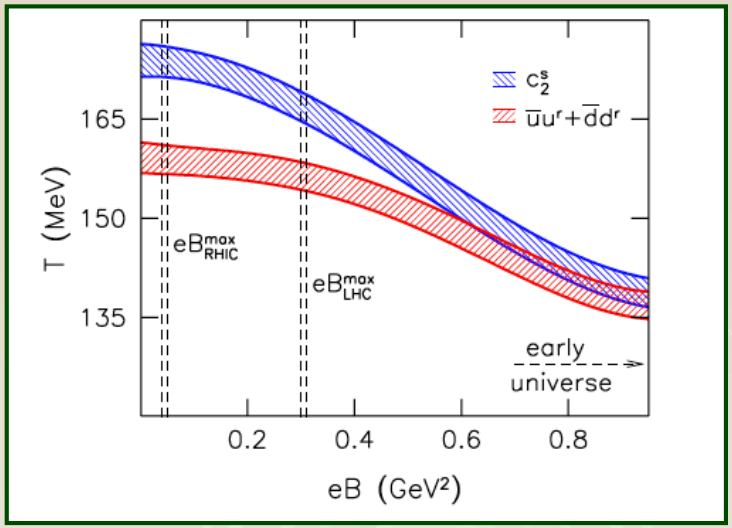


- Excellent qualitative agreement with recent lattice results

- Caveats (as in usual finite T): misses nature of the phase transition, quantitatively off

Q: So, what's the catch?
A: Renormalization.

A constant & uniform B field has sick divergences (purely magnetic pressure) that are not cured by the vacuum counterterm. Subtraction is subtle due to finite terms ! -> physical guidance for choice !



[Bali et al (2012)]

Subtleties of renormalization

Quark pressure after subtraction of P(T=0):

$$P_q^V = \frac{N_c}{2\pi^2}\sum_f (q_f B)^2 \left[\zeta'(-1,x_f) + \frac{1}{2}(x_f - x_f^2)\ln x_f + \frac{x_f^2}{4} - \frac{1}{12}\left(2/\epsilon + \ln(\Lambda^2/2q_f B) + 1\right)\right]$$

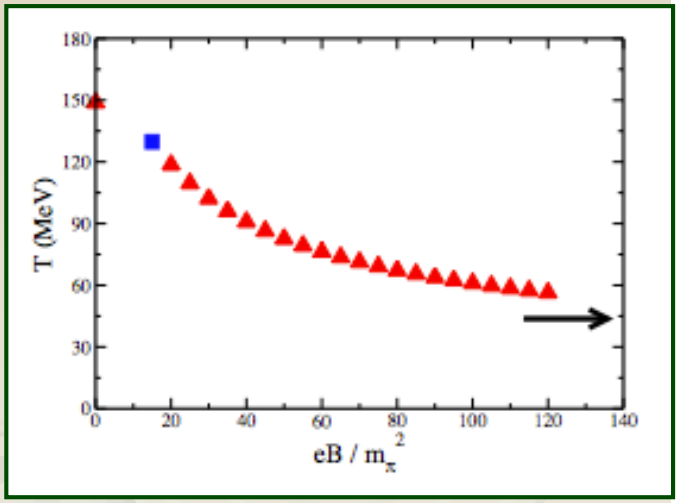
- ***Reminiscent divergence: pure magnetic pressure spread throughout the (infinite) space.***
  - related to wavefunction renormalization of the (classical) F_{em}^2 term.
  - Modification of the permeability of vacuum, μ_0 (=1 in QFT). ['t Hooft; Nielsen (1980)]

How to fix the finite part? Subtract all matter-independent T=0 pressure.

- For the pion case: guarantees consistency with magnetic catalysis in the vacuum.

- These terms are independent of order parameters and do not affect the effective potential.

- Not seen on the lattice: only 'measures' derivatives with respect to T (or m_q).



pQCD to $O(g^2)$ in a nonperturbative magnetic background

[Blaizot, ESF & Palhares (2012)]

- Exact fermion propagator:

$$S_0(x,y) \;=\; \Phi(x,y)\;\overline{S}_0(x-y)$$

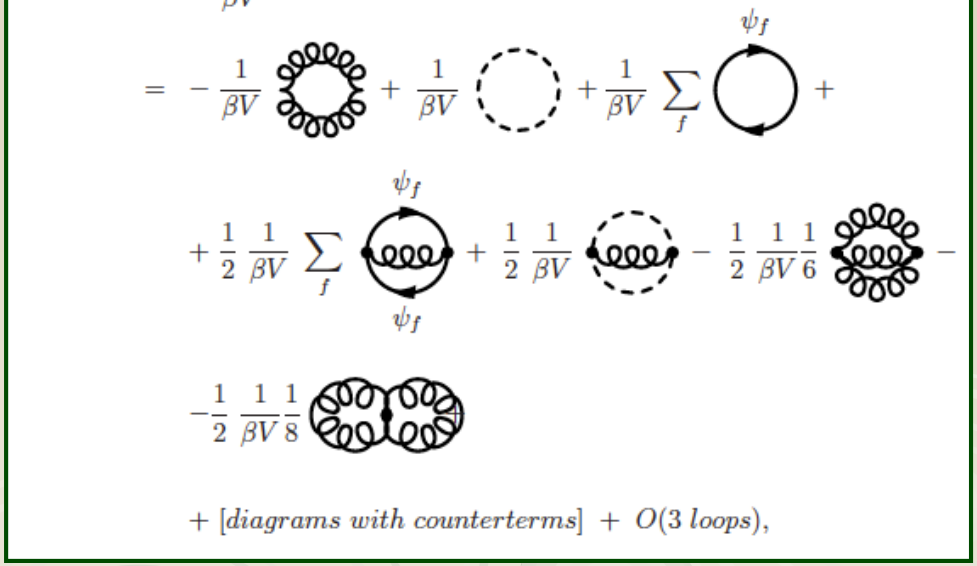
$$\Phi(x,y) \;\equiv\; \exp\left[iq\int_x^y dx'_\mu A^\mu_{\rm cl}(x')\right]$$

$$\overline{S}_0(P) \;=\; i\exp\left[-\frac{\mathbf{p}_T^2}{qB}\right]\sum_{n=0}^{\infty}(-1)^n\frac{D_n(qB,P)}{\mathbf{p}_L^2-m_f^2-2nqB}$$

- Thermodynamic potential:

(gluonic part from usual hot pQCD + magnetically-dressed quarks)

$$\Omega_{QCD} \;\equiv\; -\frac{1}{\beta V}\ln Z_{QCD}$$



Exchange diagram in a magnetic background (in the LLL approx.):

After a very long and subtle computation... [L.F. Palhares (PhD thesis)]



$$\overset{\text{LLL}}{=} \underbrace{\left(\frac{q_f B}{2\pi}\right) \int \frac{dk_1 dk_2}{(2\pi)^2}\, \mathrm{e}^{-\frac{k_1^2+k_2^2}{2q_f B}}}_{\text{Average over gluon "transverse mass"}}$$



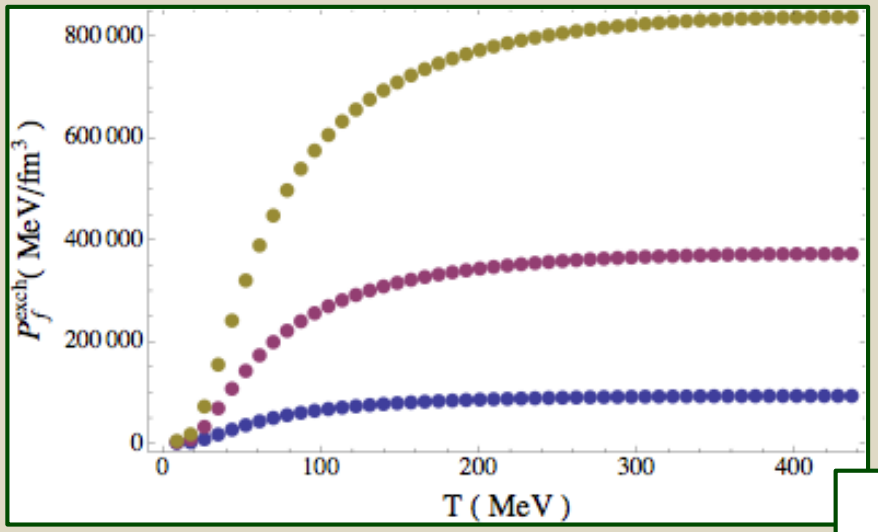
$$\underbrace{\bar{d}=2;\ m_k^2=k_1^2+k_2^2}_{\text{exchange contribution in dim. 2 hot QCD with a "massive gluon"}}$$

- In 2 dimensions the Dirac trace is proportional to the quark mass. Trivial chiral limit ?

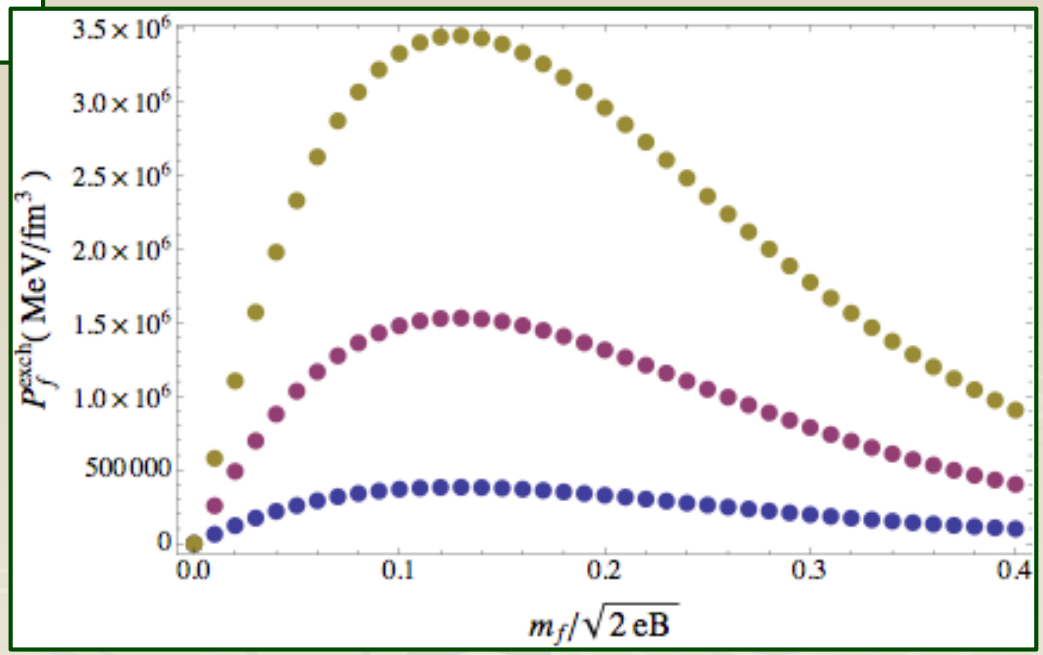
- Regularization and subtraction (renormalization) subtleties expected, even with this effective dimensional reduction...

- Clean first-principle calculation for high T & B (on-going).

Preliminary results for the exchange contribution



- eB = 20, 40 & 60 m_π^2
- Very strong B approximation to perform momentum integrals



- Apparently no IR divergence -> trivial chiral limit!

[maybe due to behavior of Landau orbits as m_f vanishes]

- Non-monotonic behavior with the quark mass

Final remarks

- Strong magnetic fields open new possibilities in the study of the phase diagram of QCD.

- However, the structure of the phase diagram is wide open:

  - Lattice results disagree (probably due to a mass effect)
  - The key issue in model outputs is the renormalization procedure

- The qualitative success of the bag model description presented (compared to recent lattice data) relies on [ESF & Palhares (2012)]

  - the full subtraction of purely B-dependent (matter-independent) contributions to the pressure.
  - its focus on confinement (T_c as a confinement-driven observable).

- Preliminary results [Blaizot, ESF & Palhares, to appear]:

  - first-principle pressure in magnetic pQCD to $O(g^2)$ shows trivial chiral limit & non-monotonic dependence on m_f.
  - Maybe classical behavior induced by large B?

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