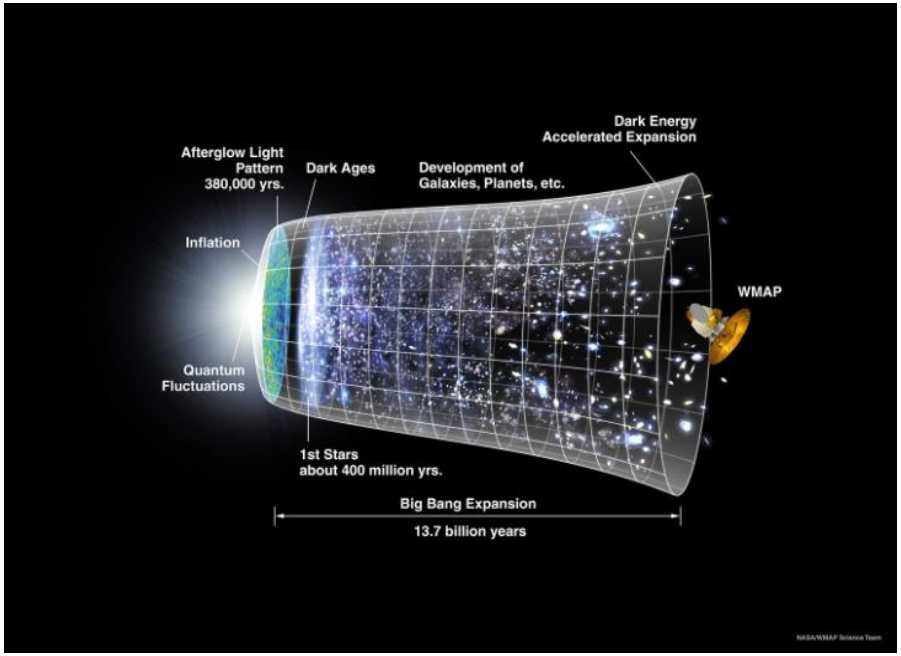
Preheating with Fermions

Daniil Gelfand | University of Heidelberg |



J. Berges, D. G., J. Pruschke, PRL 107 (2011) 061301

J. Berges, D. G., D. Sexty, in progress

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Introduction

Inflation:

- Early period of exponential expansion A. H. Guth, Phys. Rev. D 23 (1981) 347
- Explains:
  - Spatial flatness of space-time
  - Homogeneity and isotropy on large scales
  - Seeds for structure formation from quantum fluctuations
- Homogeneous inflaton field with potential $V(\phi)$

$$E = T + V \qquad p = T - V \qquad E \approx -p$$

Friedmann equations →

$$a(t) \sim e^{Ht} \qquad H = \frac{\dot{a}}{a}$$

Connection to Hot Big Bang evolution?

- Energy densities: $\varepsilon_m \sim \frac{1}{a^3}$ $\varepsilon_r \sim \frac{1}{a^4}$ $\varepsilon_\phi \sim$ const.

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Introduction

Reheating: L. Kofman, A. D. Linde, and A. A. Starobinski, Phys. Rev. Lett. 73 (1994) 3195

- Particle production
- Scattering and decay of produced matter
- Approach to thermal equilibrium

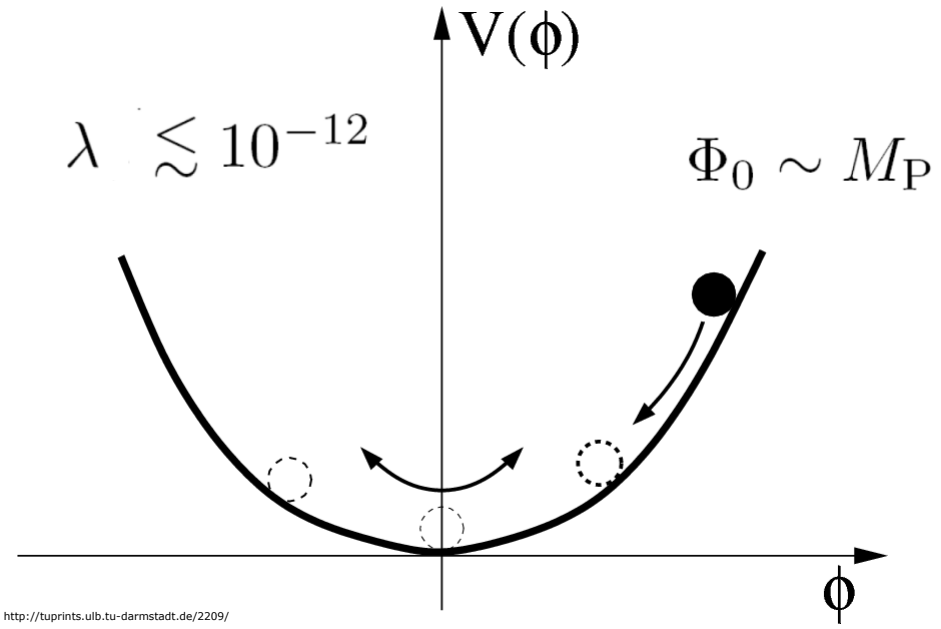
Preheating: P. B. Greene, L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D 56, 6175 (1997)

- First phase of reheating
- Non-perturbative and non-equilibrium particle production

Parametric resonance

(e.g. in chaotic inflation)

L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D 56, 3258 (1997); T. Prokopec, T. G. Roos, Phys.Rev. D55 (1997) 3768–3775



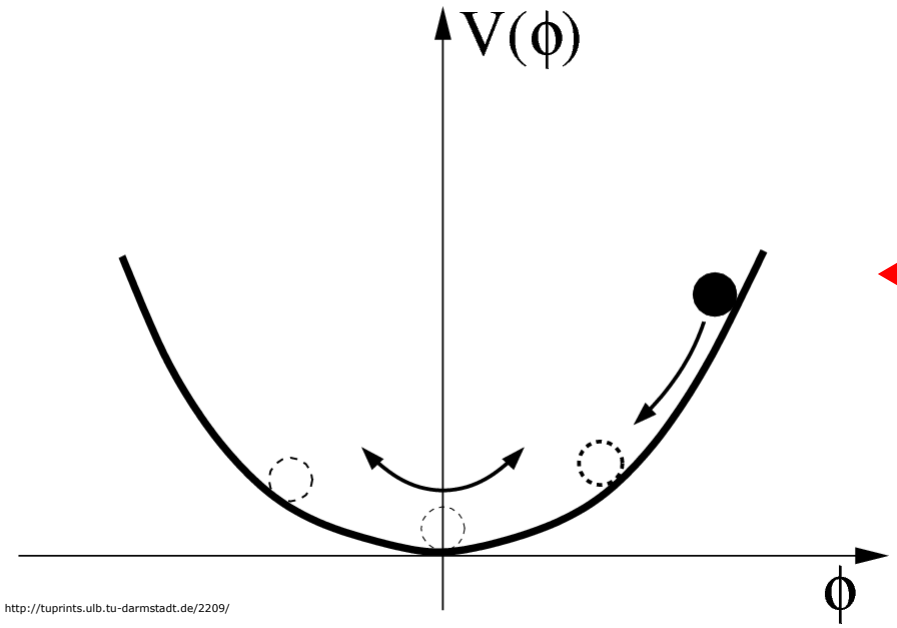
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Introduction

Parametric resonance:

$$\ddot{x} + \omega^2(t)x = 0$$

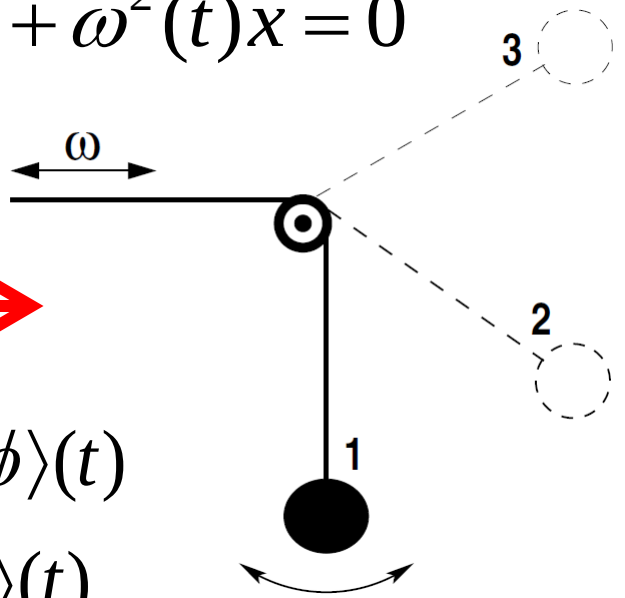


http://tuprints.ulb.tu-darmstadt.de/2209/

analogue

$$x \leftrightarrow \langle\phi\phi\rangle(t)$$

$$\omega \leftrightarrow \langle\phi\rangle(t)$$



http://tuprints.ulb.tu-darmstadt.de/2209/

For massless fields equivalence between EoM in FRW and Minkowski space:

$$\tau = \int dt/a(t)$$

$$\bar{\phi} = \phi a(\tau)/a(0)$$

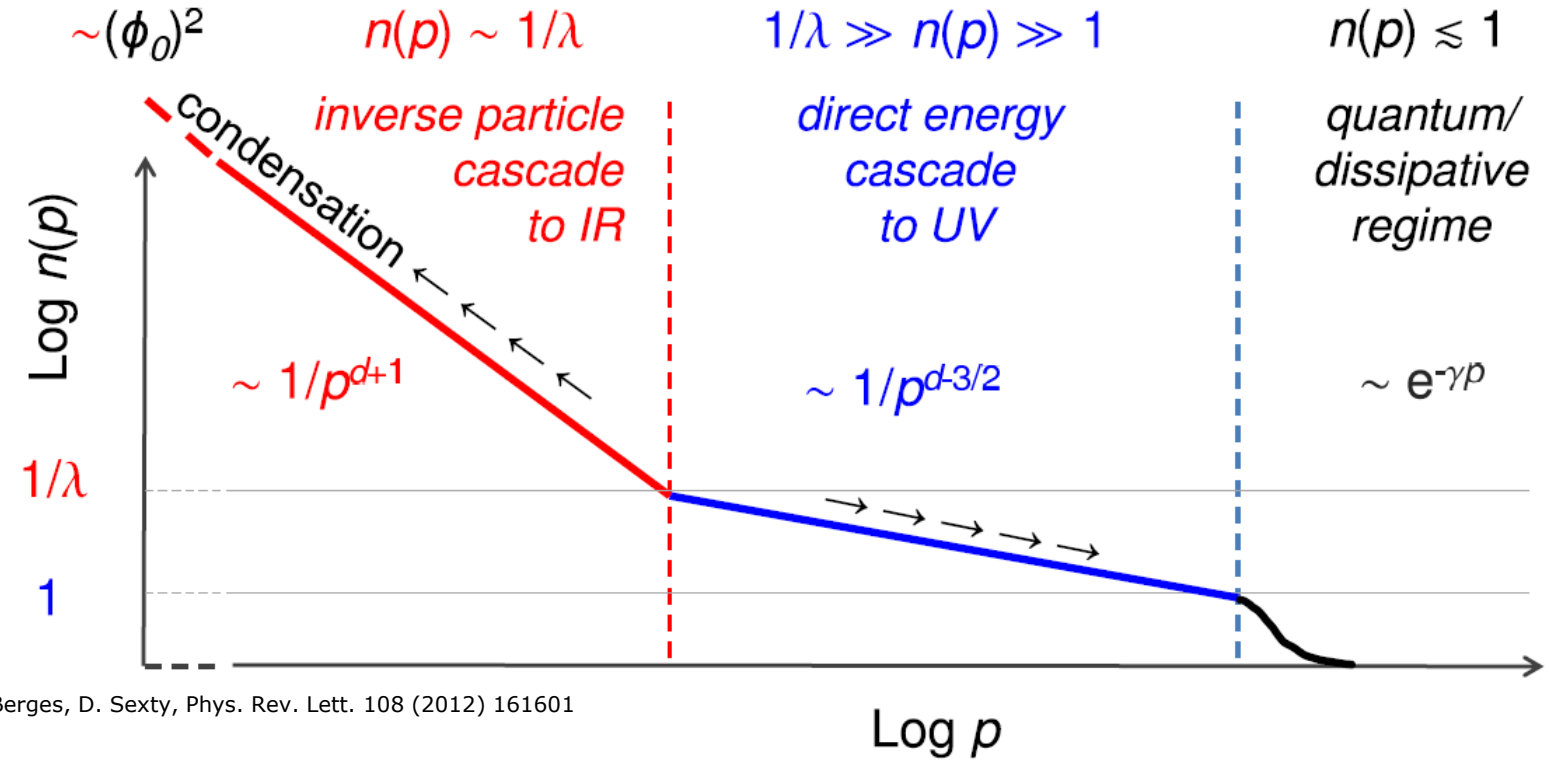
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Introduction

Turbulence:

R. Micha, I. I. Tkachev, Phys.Rev. D70 (2004) 043538; J. Berges, A. Rothkopf, J. Schmidt, Phys.Rev.Lett. 101 (2008) 041603; J. Berges, D. Sexty, Phys. Rev. Lett. 108 (2012) 161601

- Dual cascade



J. Berges, D. Sexty, Phys. Rev. Lett. 108 (2012) 161601

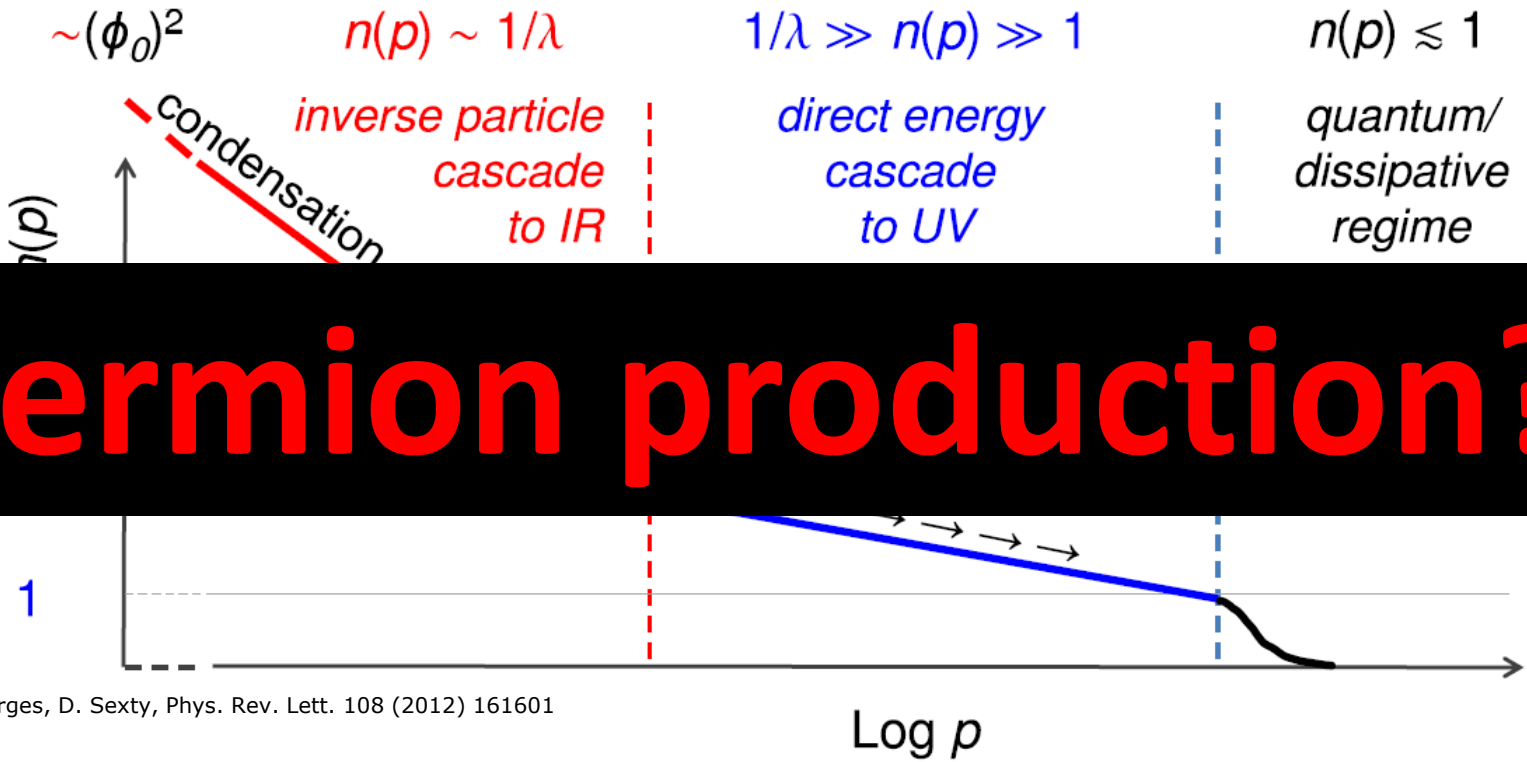
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Turbulence:

- Dual cascade

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J. Berges, D. Sexty, Phys. Rev. Lett. 108 (2012) 161601

Fermion production?

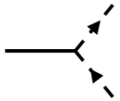
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Implementation

Our model:

- 2 flavours of Dirac fermions coupled to inflaton

$$\mathcal{L} = \bar{\psi}\left(i\partial_\mu\gamma^\mu\right)\psi + \frac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a - \frac{1}{2}m^2\phi_a\phi_a - \frac{\lambda}{4!\cdot N_s}(\phi_a\phi_a)^2 - \frac{g}{N_f}\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\vec{\pi})\psi$$

  - Yukawa coupling  leads to dynamical mass generation:

$$m(t) = \frac{g}{N_f}\phi(t)$$

  - O(4) self-interacting  scalar field $\phi = \{\sigma, \pi^1, \pi^2, \pi^3\}$
  - Macroscopic field $\phi(t) = \langle\sigma(t, \mathbf{x})\rangle$
  - 3+1 dimensions

- Start from initial conditions leading to parametric resonance!

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Implementation

Using two techniques (lattice & 2PI)!

Lattice approach:

- Classical-statistical scalar fields
  - Equations of motion including backreaction of fermions

$$\left(\Box_x + m^2\right)\sigma(x) + \frac{\lambda}{4!}\left(\phi_a(x)\phi_a(x)\right)\sigma(x) + \frac{g}{2}\langle\bar{\psi}(x)\psi(x)\rangle = 0$$

$$\left(\Box_x + m^2\right)\vec{\pi}(x) + \frac{\lambda}{4!}\left(\phi_a(x)\phi_a(x)\right)\vec{\pi}(x) + \frac{ig}{2}\langle\bar{\psi}(x)\gamma_5\vec{\tau}\psi(x)\rangle = 0$$

  - Fermion backreaction from statistical propagator G. Aarts and J. Smit, Nucl. Phys. B 555 (1999) 35

$$D(x,y) = \frac{1}{2}\langle[\psi(x),\bar{\psi}(y)]\rangle$$

$$\langle\bar{\psi}(x)\psi(x)\rangle = -tr\left[D(x,x)\right]$$

$$\langle\bar{\psi}(x)\gamma_5\psi(x)\rangle = -tr\left[D(x,x)\gamma_5\right]$$

- Quantum fermions using ♂/♀ - method S. Borsanyi and M. Hindmarsh, Phys. Rev. D 79 (2009) 065010

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Implementation

♂/♀ **- fermions:**

- Set of auxiliary spinor fields $\psi_{M/F}(x)$ is introduced
- Equations of motion: $i\partial_\mu \gamma^\mu \psi_g(x) - \frac{g}{2}\left(\sigma(x) + i\gamma_5 \vec{\tau}\vec{\pi}(x)\right)\psi_g(x) = 0$
- Initialization:

$$\psi_{M,F}(t=0,\mathbf{x}) = \int \frac{d^3p}{(2\pi)^3} e^{-i\mathbf{px}} \frac{1}{\sqrt{2}} \sum_s \left(\xi_s(\mathbf{p})u_s(\mathbf{p}) \pm \eta_s(\mathbf{p})v_s(\mathbf{p})\right)$$

  - With random numbers

$$\langle \xi_s(\mathbf{p}), \xi^*_{s'}(\mathbf{q})\rangle = (2\pi)^3 \delta_{ss'}\delta(\mathbf{p}-\mathbf{q})\left(1 - 2n^s_+(\mathbf{p})\right)$$
$$\langle \eta_s(\mathbf{p}), \eta^*_{s'}(\mathbf{q})\rangle = (2\pi)^3 \delta_{ss'}\delta(\mathbf{p}-\mathbf{q})\left(1 - 2n^s_-(\mathbf{p})\right)$$

  - Average over all pairs of $\psi_{M/F}(x)$ for observables/backreaction

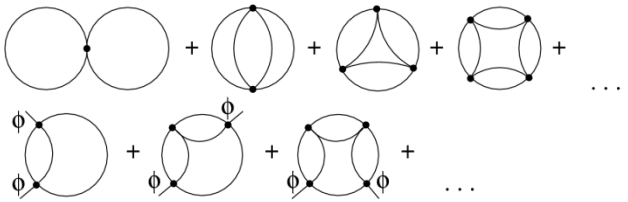
$$D(x,y) = \langle \psi_M(x)\bar{\psi}_F(y)\rangle = \langle \psi_F(x)\bar{\psi}_M(y)\rangle$$

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Implementation

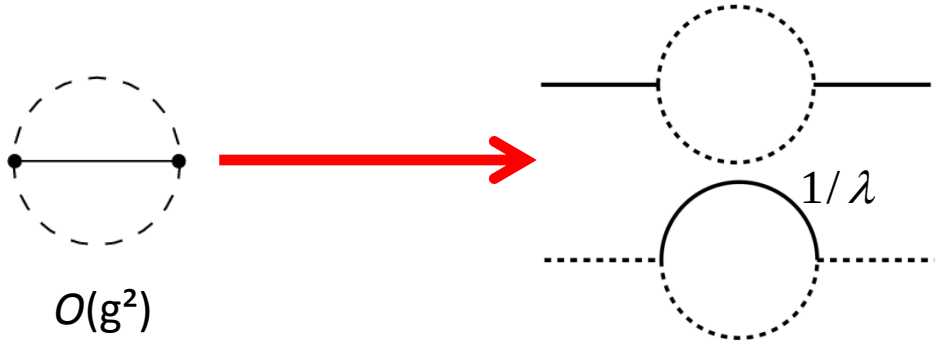
2PI effective action: J. Berges, AIP Conf. Proc. 739, 3 (2005)

- Different truncation schemes for the action
  - 1/N expansion to NLO in the number of scalar fields



J. Berges, Nucl. Phys. A 699 (2002) 847
G. Aarts et al, Phys. Rev. D 66 (2002) 045008

  - Coupling expansion to NLO in the Yukawa coupling



Contribution to scalar self-energy

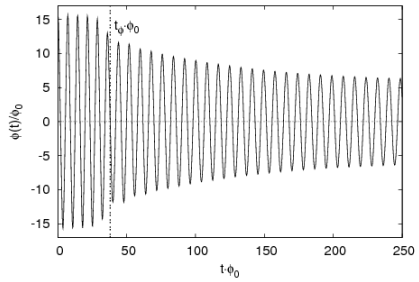
Fermion self-energy

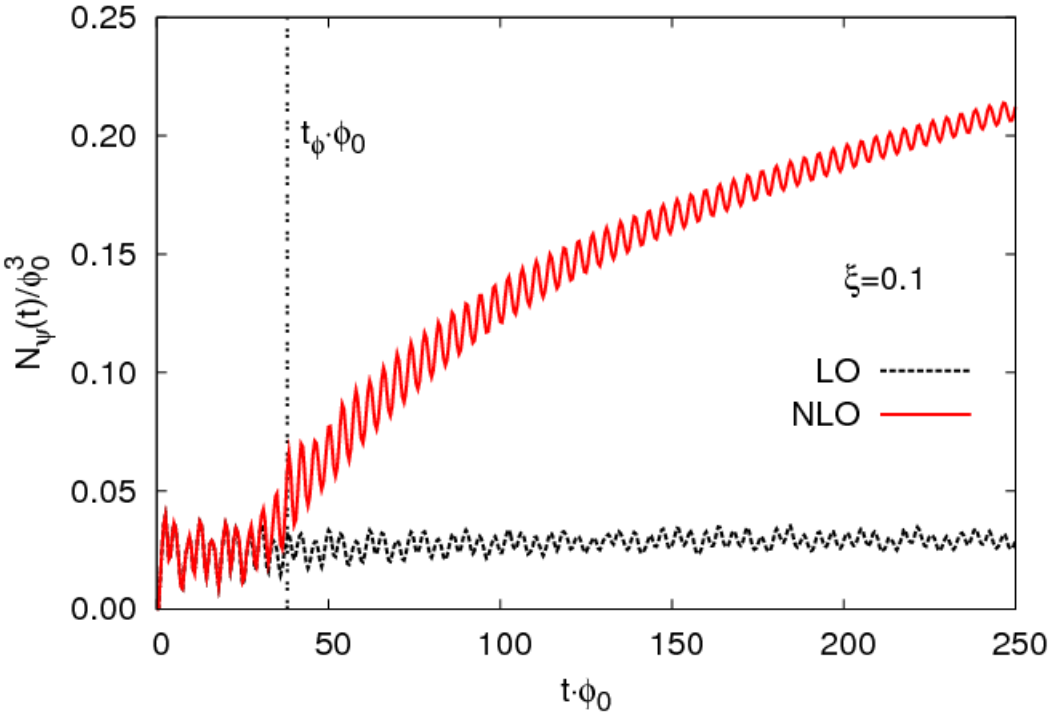
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Results

Weak coupling:

- Total number of produced particles strongly enhanced
- Quantum effects important, even at weak couplings
- Fermion production rate $\sim \xi = g^2/\lambda$





LO: $$\left[i\partial_\mu \gamma^\mu - \frac{g}{N_f}\phi(t) \right] D(x,y) = 0$$

No fluctuations!

Baacke, Heitmann, Pätzold, PRD 58 (1998) 125013; Greene, Kofman, PLB 448 (1999) 6; Giudice, Peloso, Riotto, Tkachev, JHEP 9908 (1999) 014; Garcia-Bellido, Mollerach, Roulet, JHEP 0002(2000) 034; …

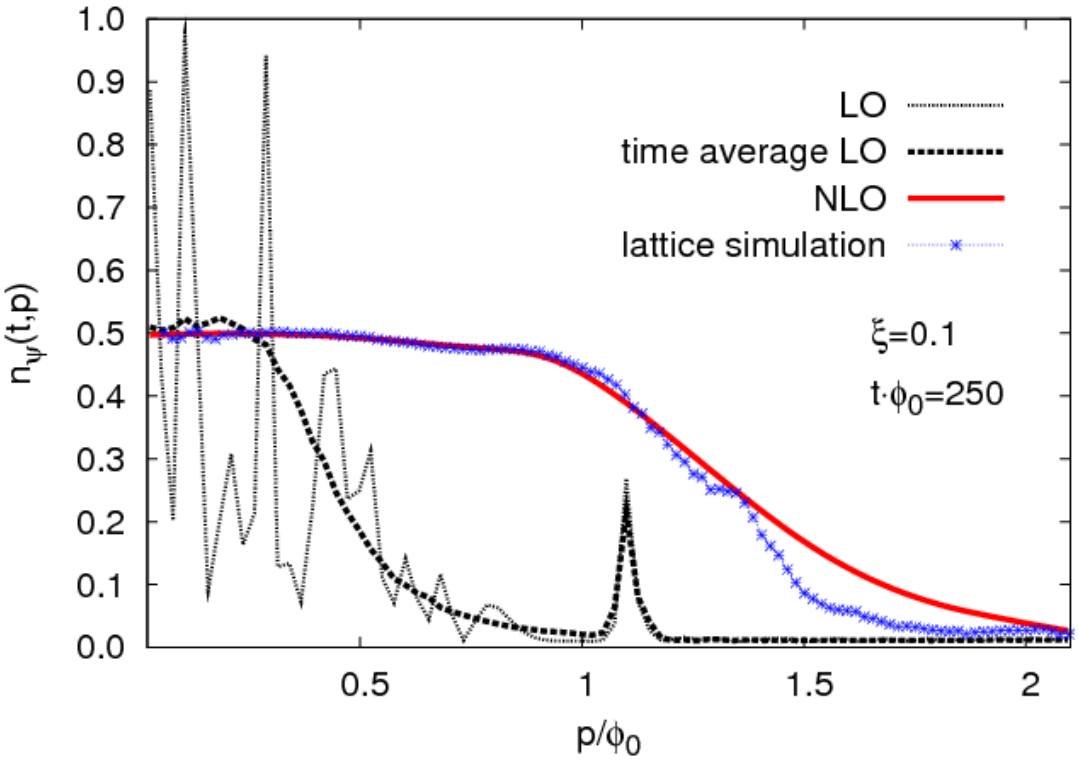
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Results

Weak coupling:

- Qualitative and quantitative difference between LO and NLO
- Good agreement between lattice and 2PI
- Particle numbers drop at the rescaled initial field amplitude:

$$\phi_0 = \phi(t=0)/\sqrt{6N_s/\lambda}$$



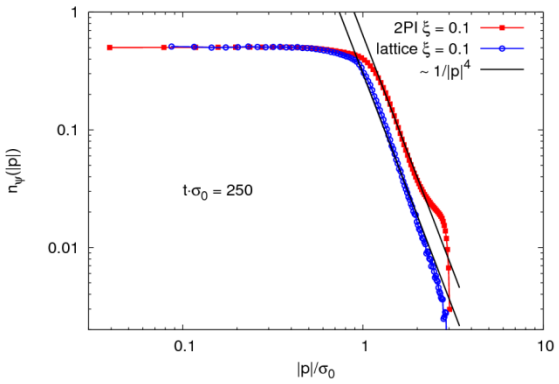
LO: $$\left[i\partial_\mu\gamma^\mu - \frac{g}{N_f}\phi(t)\right] D(x,y) = 0$$

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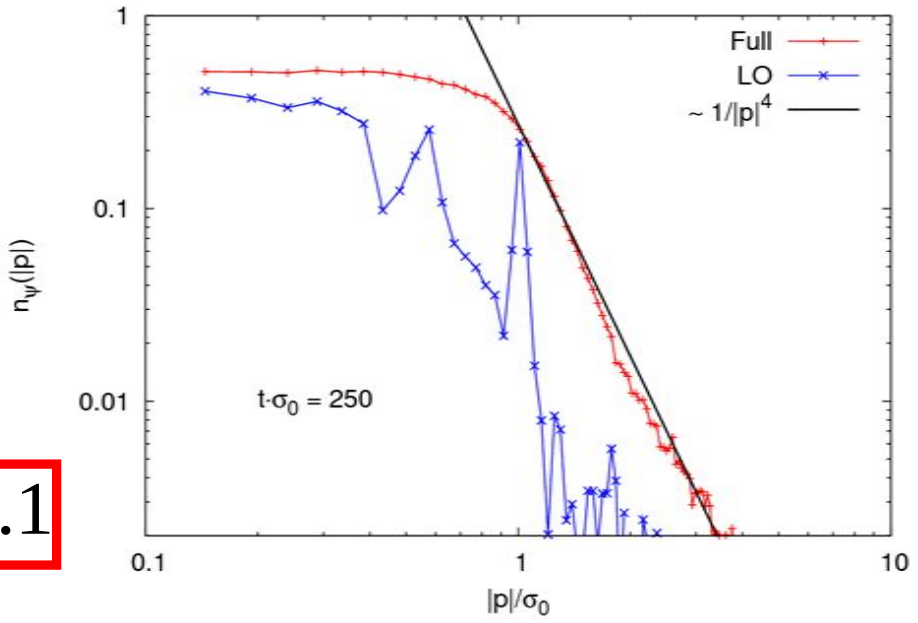
Results

Weak coupling:

- Unexpected power-law dependence in the UV
- Exact value of exponent varies in time, stays ≈ 4



$$\xi = 0.1$$



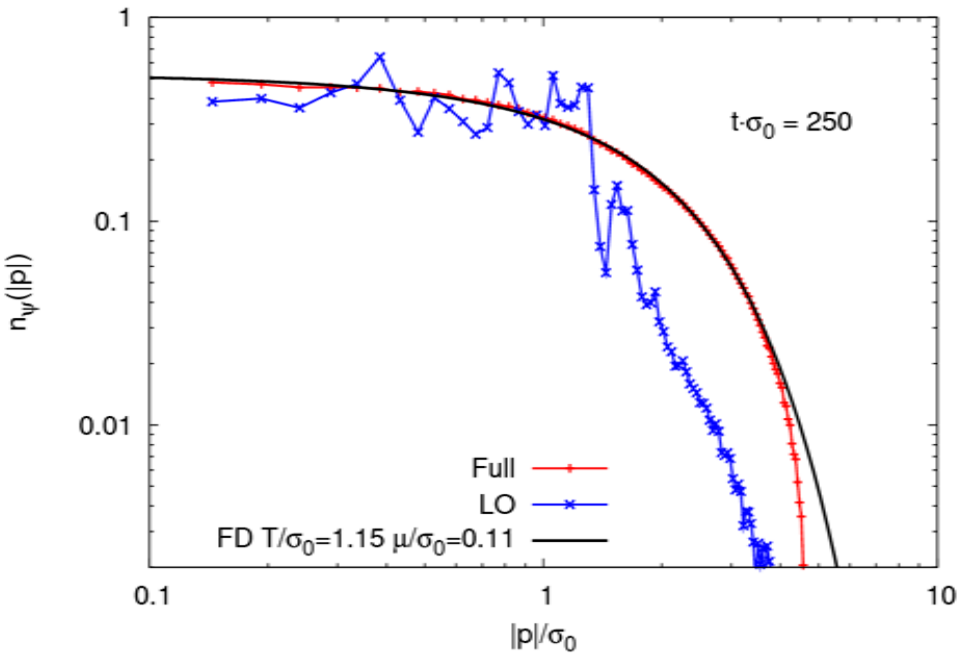
Connection to bosonic turbulence?
Work in progress!

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Results

Strong coupling:

- Full dynamics shows higher occupancy in UV
- Fermions seem to take on a Fermi-Dirac distribution
- „Temperature" and „chemical potential" are fit parameters



$$\xi = 1.0$$

Stefan-Boltzmann limit: $$T_{eq} = \sigma_0 \left(\frac{45 N_s}{\pi^2 \left(N_s + \frac{7}{2} N_f \right) \lambda} \right)^{\frac{1}{4}} \simeq 2.02 \sigma_0$$

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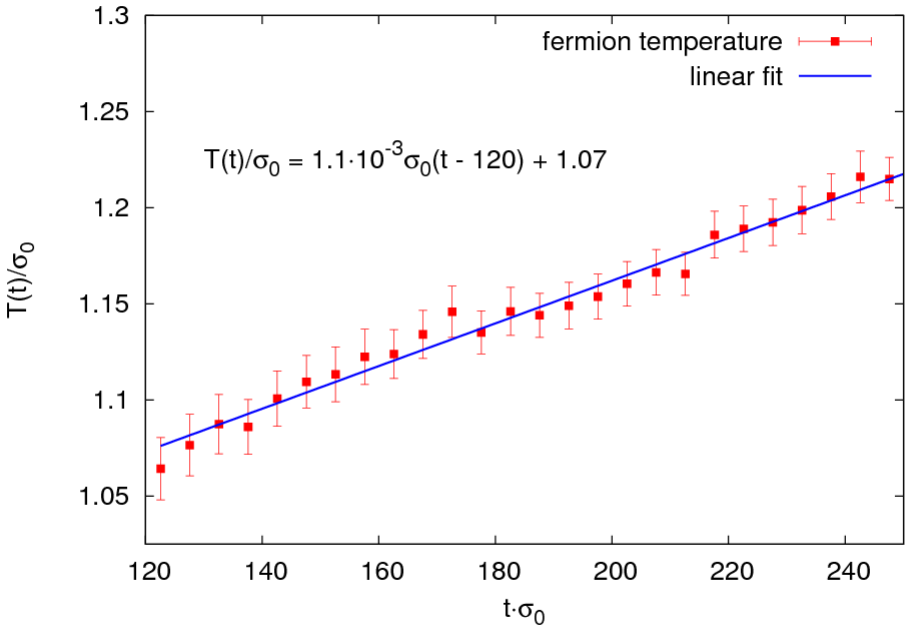
Results

Strong coupling:

- Similar quasi-equilibration already observed in a strongly coupled Abelian Higgs model in 1+1d

  G. Aarts, J. Smit, Phys. Rev. D61 (2000) 025002

- Time dependent non-equilibrium temperature and chemical potential



General feature of strongly interacting fermions out of equilibrium?

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Summary & Outlook

Summary:

- Real-time simulations of fermion dynamics in 3+1d possible with modern lattice techniques
- Quantum fluctuations strongly enhance fermion production
- For weak coupling a power-law in the UV is found
- In strongly coupled systems a Fermi-Dirac distribution emerges

Outlook:

- Fermion production in isotropically expanding background(FRW)
- Exploring other models, especially gauge theories
- Explanation for the power-law exponent

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The End

Thank you for your attention!

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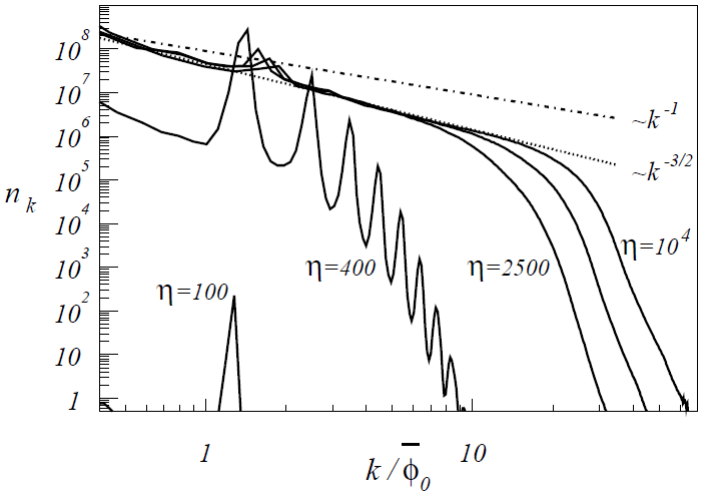
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Supplement

Parametric resonance:

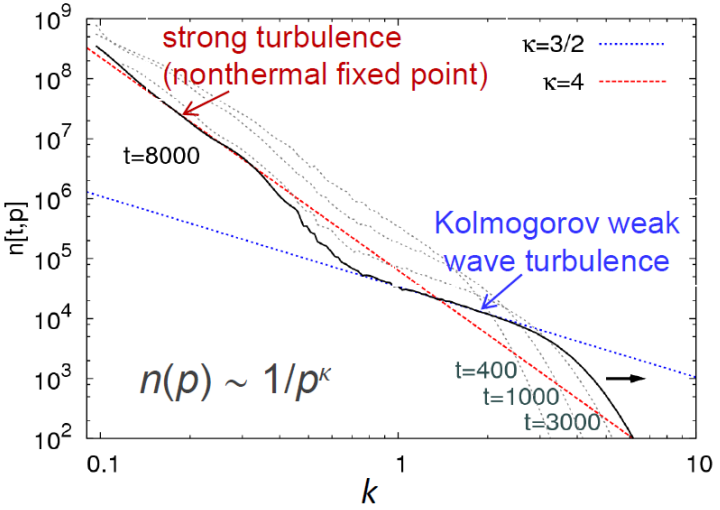
- Main features of produced particle distributions

High occupancy



R. Micha, I. I. Tkachev, Phys.Rev. D70 (2004) 043538

Turbulence



J. Berges, A. Rothkopf, J. Schmidt, Phys.Rev.Lett. 101 (2008) 041603

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Supplement

Initial conditions:

- Large inflaton field expectation value $\phi(t=0) \sim \sqrt{\frac{6N_s}{\lambda}}$
- Fermionic (quantum) vacuum fluctuations
- Bosonic vacuum fluctuations for unstable modes only
  - Otherwise problems with classical Rayleigh-Jeans divergence
- Enhancement of boson fluctuations through instabilities
  - Bosonic two-point function becomes parametrically large $\langle \phi\phi \rangle \sim \frac{1}{\lambda}$
  - IR fixed point prevents early thermalization (**strong turbulence**)

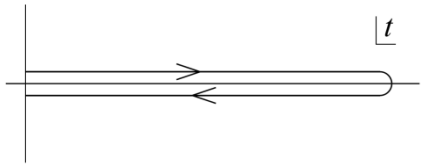
J. Berges, A. Rothkopf and J. Schmidt, Phys. Rev. Lett. 101 (2008) 041603; J. Berges and G. Hoffmeister, Nucl. Phys. B 813, 383 (2009); J. Berges and D. Sexty, Phys. Rev. D 83, 085004 (2011); J. Berges, D. Sexty, arXiv:1201.068

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Supplement

2PI effective action: J. Berges, AIP Conf.Proc. 739 (2005) 3–62

- Functional method to describe time evolution of quantum fields
  - Closed time path (in-in formalism)
  - Time evolution of two-point functions



$$\text{commutator:} \quad \rho(x,y) = i\langle[\Phi(x),\Phi(y)]\rangle\,,$$

$$\text{anti-commutator:} \quad F(x,y) = \frac{1}{2}\langle\{\Phi(x),\Phi(y)\}\rangle\,.$$

  - Kadanoff-Baym equations of motion

$$\left[\Box_x + M^2(x)\right]\rho(x,y) = -\int_{y^0}^{x^0} \mathrm{d}z\,\Sigma_\rho(x,z)\rho(z,y)\,,$$

$$\left[\Box_x + M^2(x)\right]F(x,y) = -\int_{0}^{x^0} \mathrm{d}z\,\Sigma_\rho(x,z)F(z,y) + \int_{0}^{y^0} \mathrm{d}z\,\Sigma_F(x,z)\rho(z,y).$$

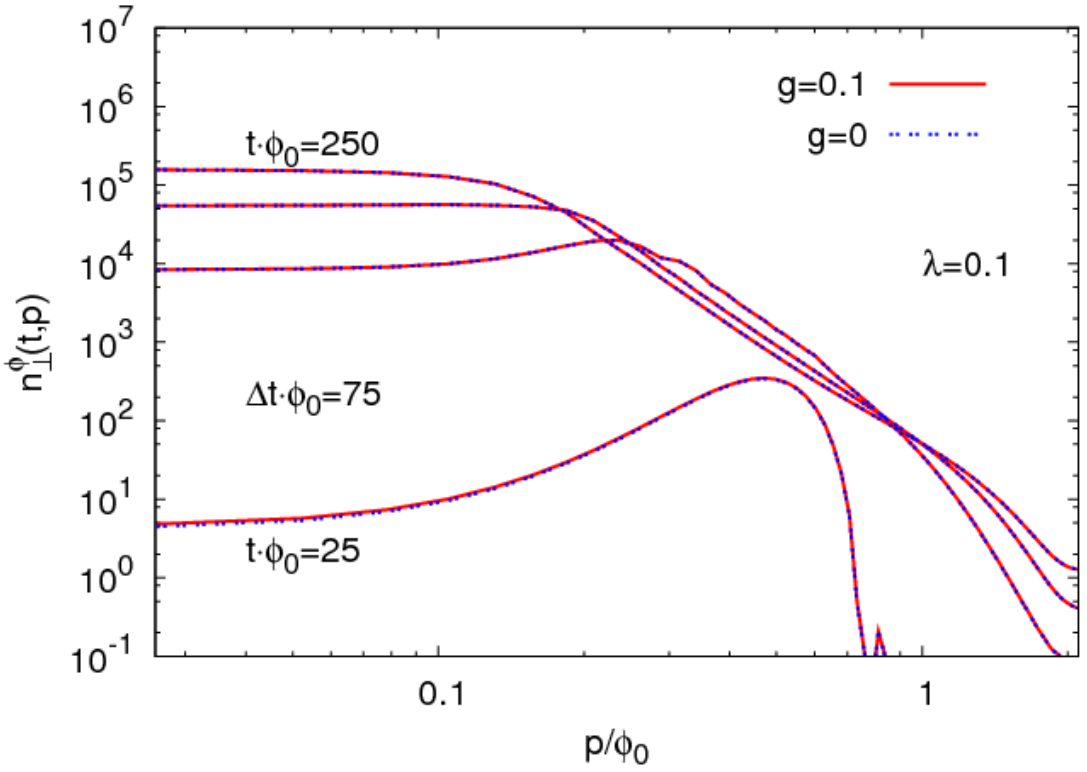
Memory integrals!

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Weak coupling:

- Bosonic dynamics unchanged for weak Yukawa couplings
- High occupation numbers in the IR (classical-statistical regime)
- Strong turbulence at later times

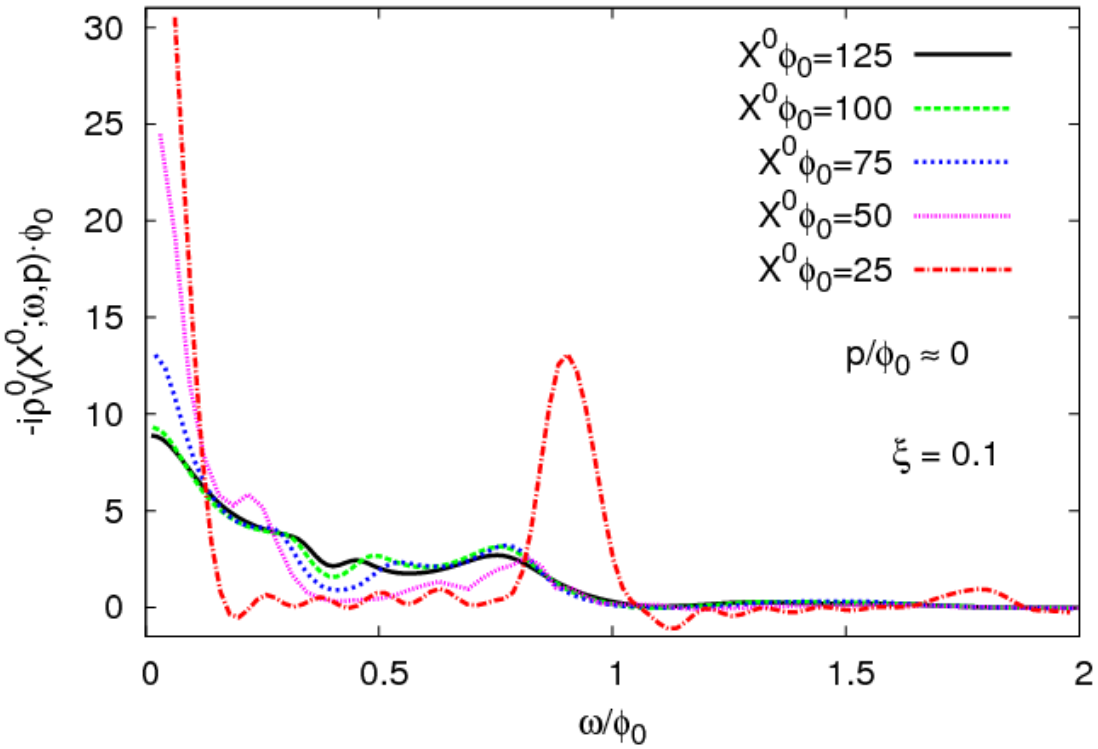


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Weak coupling:

- Wigner transformed dynamical spectral function
- Quasi-particle description valid at early times
- Strongly correlated medium at later times

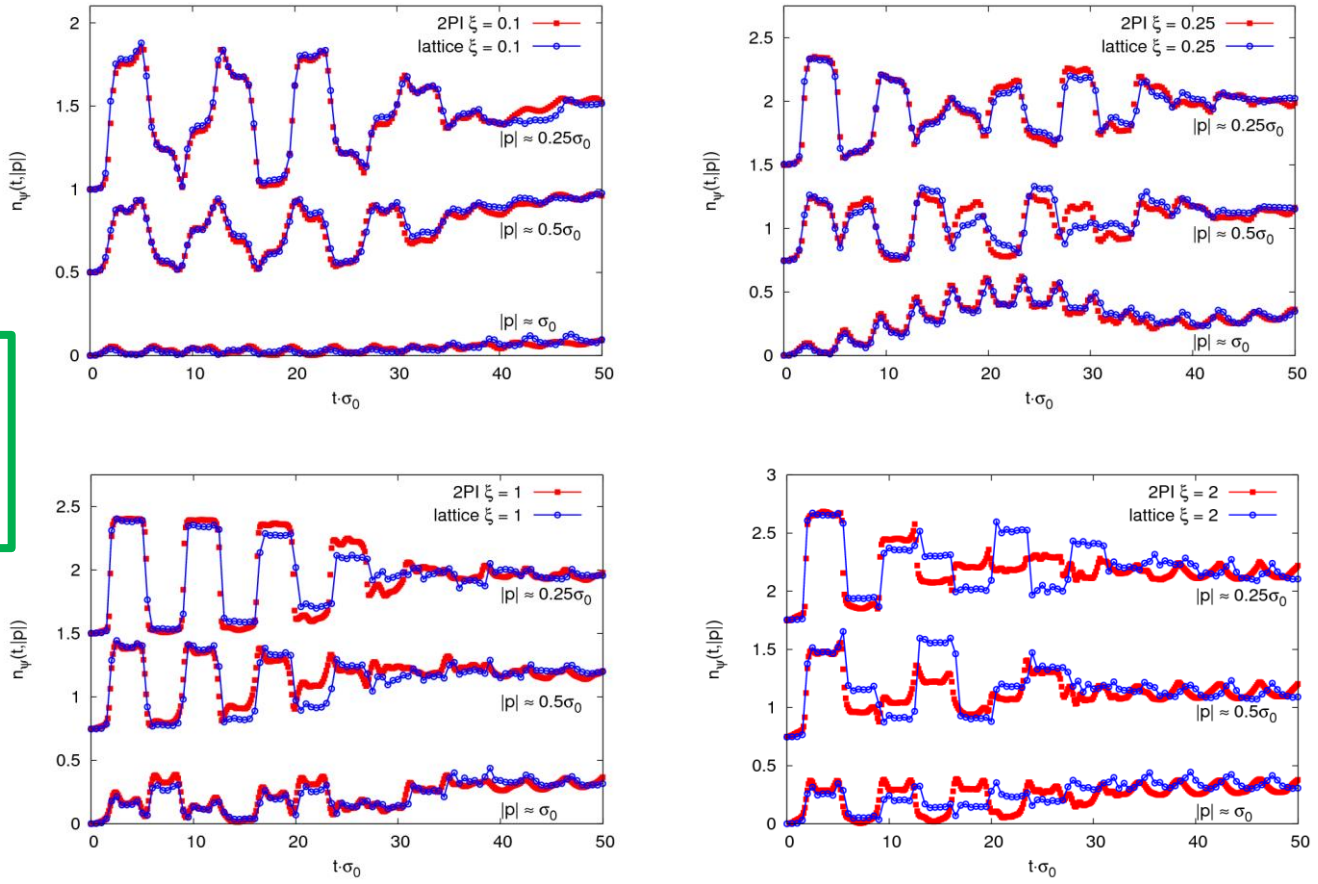


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Increasing coupling:

Loop expansion breaks down at stronger couplings!

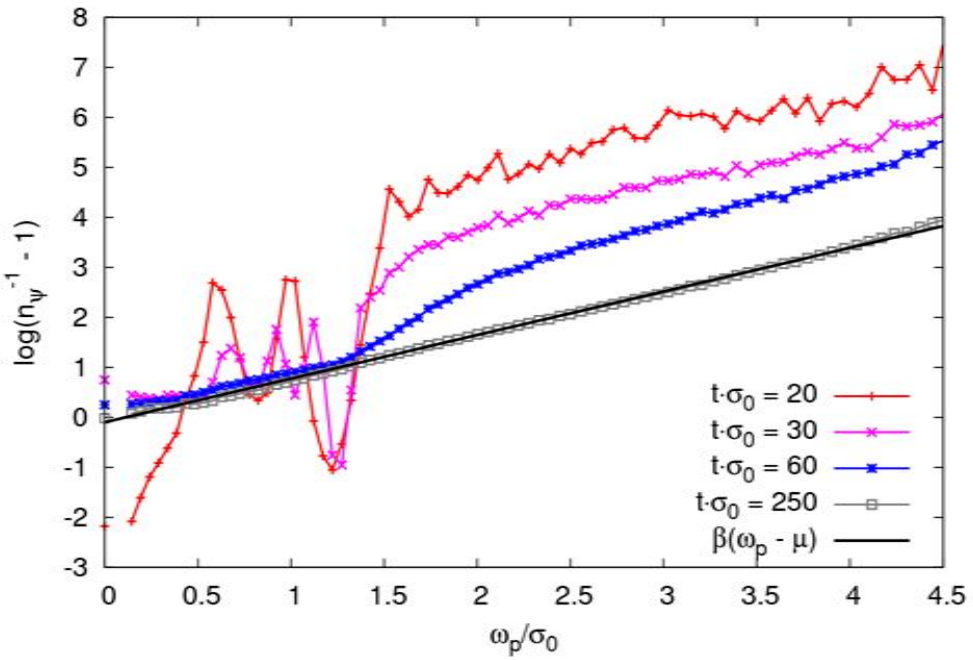


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Strong coupling:

- First scattering smoothens the IR distribution
- Occupancy in the UV is built up much later
- Lattice dispersion relation used



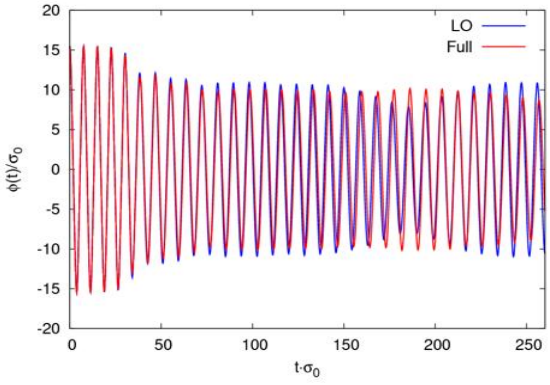
$$\xi = 1.0$$

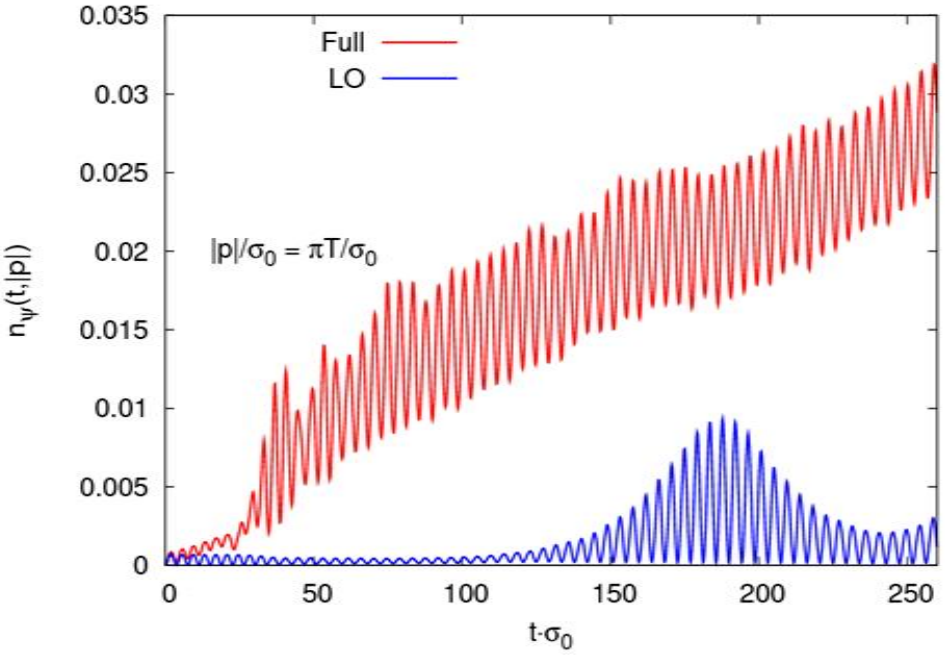
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Strong coupling:

- Difference between LO and full dynamics matters most for high momentum modes





$$\xi = 1.0$$

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Further issues:

- Lattice discetization causes so-called fermion doublers
  - Spatial doublers addressed with pseudoscalar Wilson term

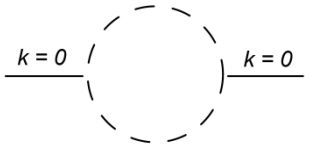
$$W_{PS}\psi_g(x) = i\gamma_5 \frac{r a_s}{2} \triangle_x \psi_g(x) \longrightarrow \omega(\mathbf{p}) = \sqrt{m_\psi^2 + \bar{p}_i \bar{p}^i + \frac{r^2 a_s^2}{4}(\mathbf{p})^4_{lat}}$$

- Quadratically divergent scalar mass has to be renormalized

$$m^2_{0,\sigma/\vec{\pi}} + \Sigma_{\sigma/\vec{\pi}}(m_0^2, k = 0) = m_R^2$$

  - Perturbative one-loop renormalization:





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Lattice formulas and definitions:

Scalar (standard) Wilson term:

$$W_S \psi_g(x) = \frac{r a_s}{2} \triangle_x \psi_g(x)$$

$$\omega(\mathbf{p}) = \sqrt{m_\psi^2 + \bar{p}_i \bar{p}^i + r a_s m_\psi (\mathbf{p})_{lat}^2 + \frac{r^2 a_s^2}{4} (\mathbf{p})_{lat}^4}$$

Free statistical propagator:

$$D(t=0, \mathbf{p}) = \frac{m_\psi - \gamma^i p_i - i\gamma_5 \frac{r a_s}{2} (\mathbf{p})^2}{2\omega(\mathbf{p})} \left(1 - 2n_\psi(\mathbf{p})\right)$$

Particle number definitions:

$$n_\psi(t, \mathbf{p}) = \frac{1}{2} - \frac{\left[\bar{p}_i D_V^i(t, \mathbf{p}) + m_\psi(t) D_S(t, \mathbf{p}) + i \frac{r a_s}{2} (\mathbf{p})_{lat}^2 D_{PS}(t, \mathbf{p})\right]}{\sqrt{\bar{p}_i \bar{p}^i + m_\psi^2(t, \mathbf{p}) + \frac{r^2 a_s^2}{4} (\mathbf{p})_{lat}^4}}$$

$$\epsilon_a(t, \mathbf{p}) = \sqrt{\frac{\partial_t \partial_{t'} F_a(t, t', \mathbf{p}) \mid_{t=t'}}{F_a(t, t', \mathbf{p}) \mid_{t=t'}}}$$

$$n_\phi^a(t, \mathbf{p}) = F_a(t, t, \mathbf{p}) \epsilon_a(t, \mathbf{p}) - \frac{1}{2}$$

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