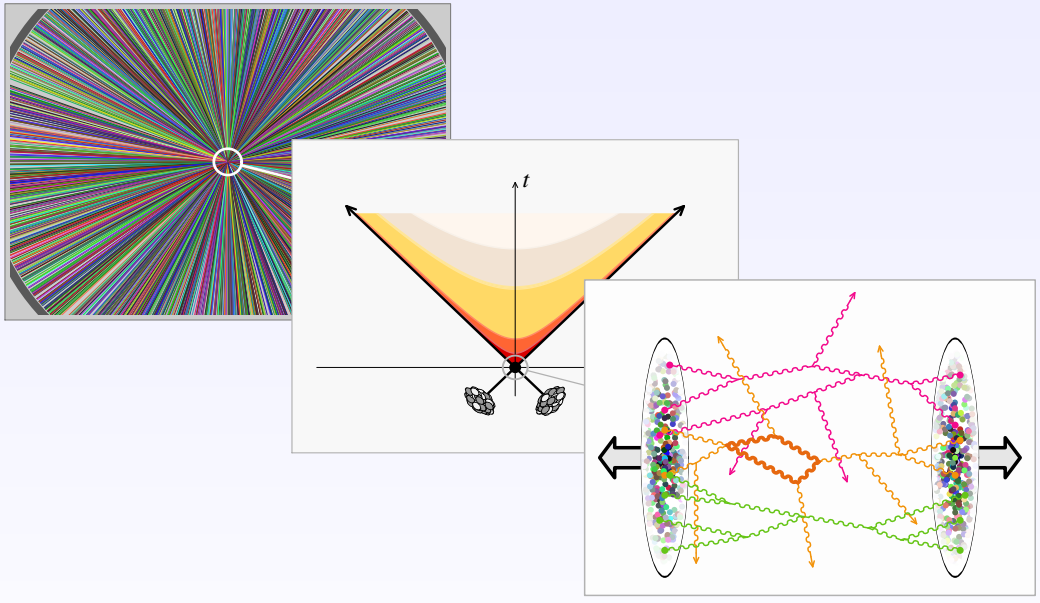
The early stages of Heavy Ion Collisions

SEWM, Swansea University, July 2012



François Gelis
IPhT, Saclay

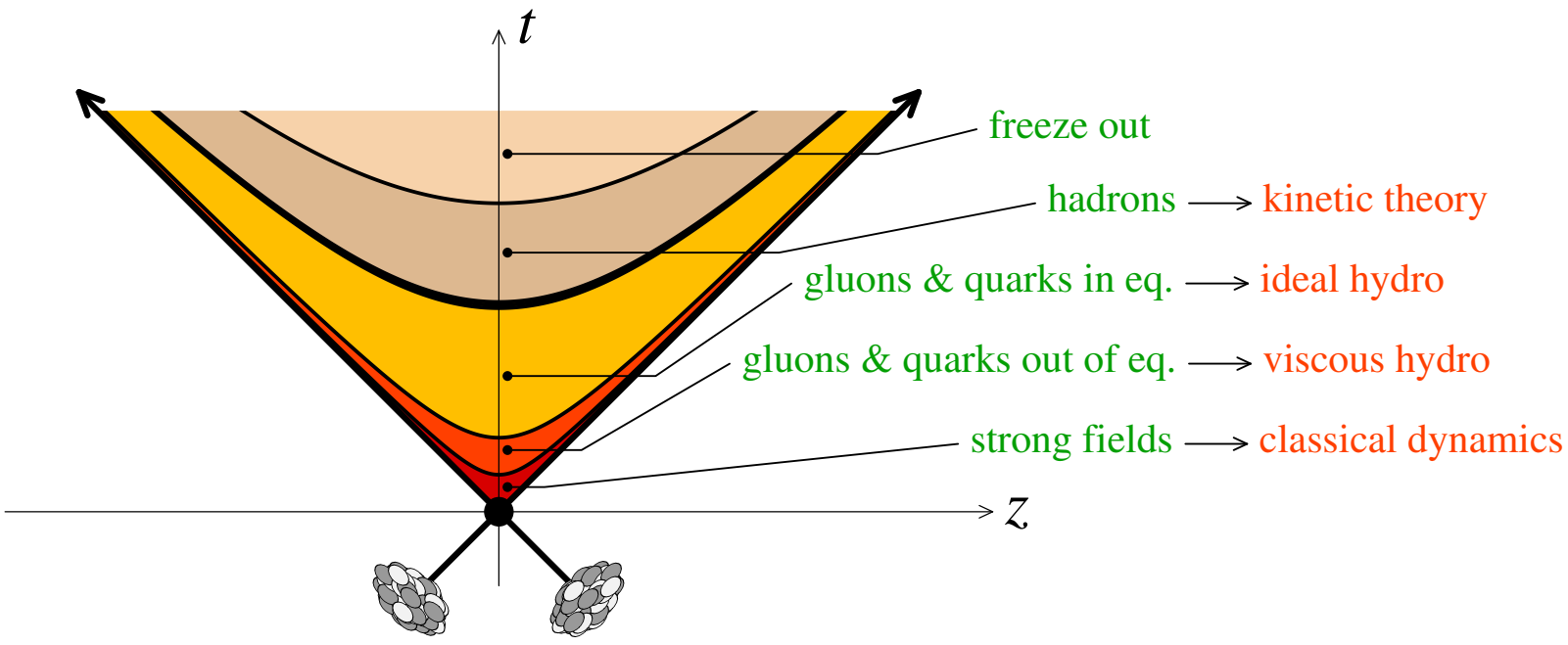
Outline

1. **Initial state factorization**
2. **Final state evolution**

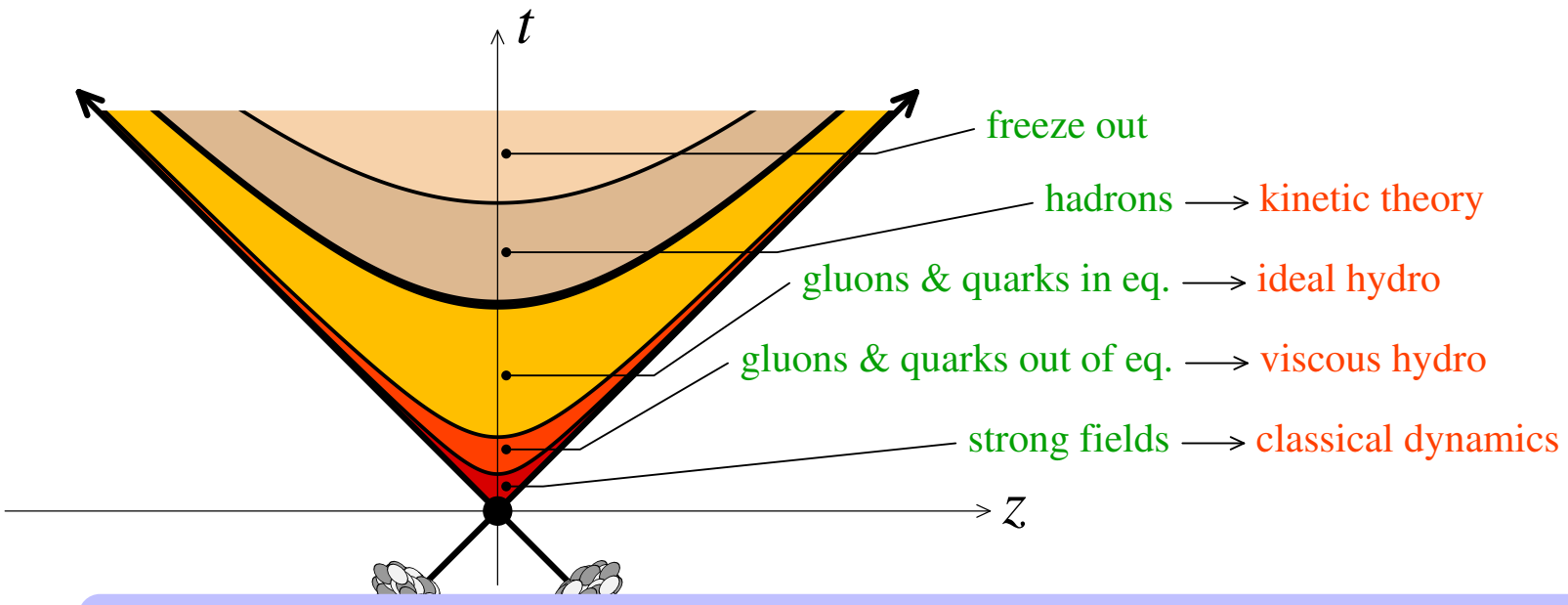
In collaboration with :

K. Dusling	(NCSU)
T. Epelbaum	(IPhT)
T. Lappi	(Jyväskylä U.)
R. Venugopalan	(BNL)

Stages of a nucleus-nucleus collision



Stages of a nucleus-nucleus collision

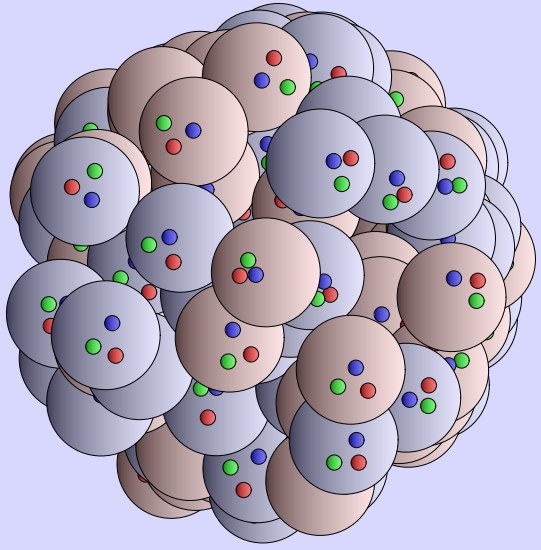


Goals :

- evolution from $\mathrm{t} = -\infty$ to the start of hydrodynamics
- first principles description in QCD
- weakly coupled (but strongly interacting...)

What do we need to know about nuclei?

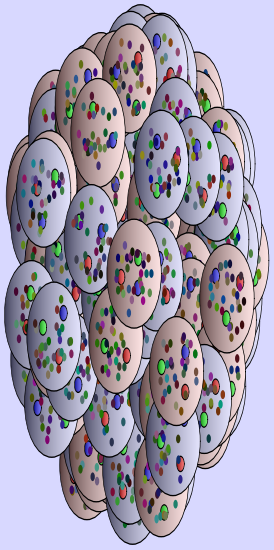
Nucleus at rest



- At low energy : valence quarks

What do we need to know about nuclei?

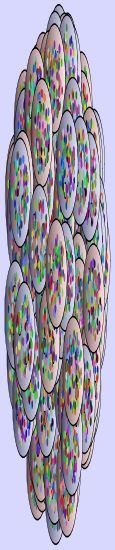
Slightly boosted nucleus



- At low energy : valence quarks
- At higher energy :
  - Lorenz contraction of longitudinal sizes
  - Time dilation $\triangleright$ slowing down of the internal dynamics
  - Gluons start becoming important

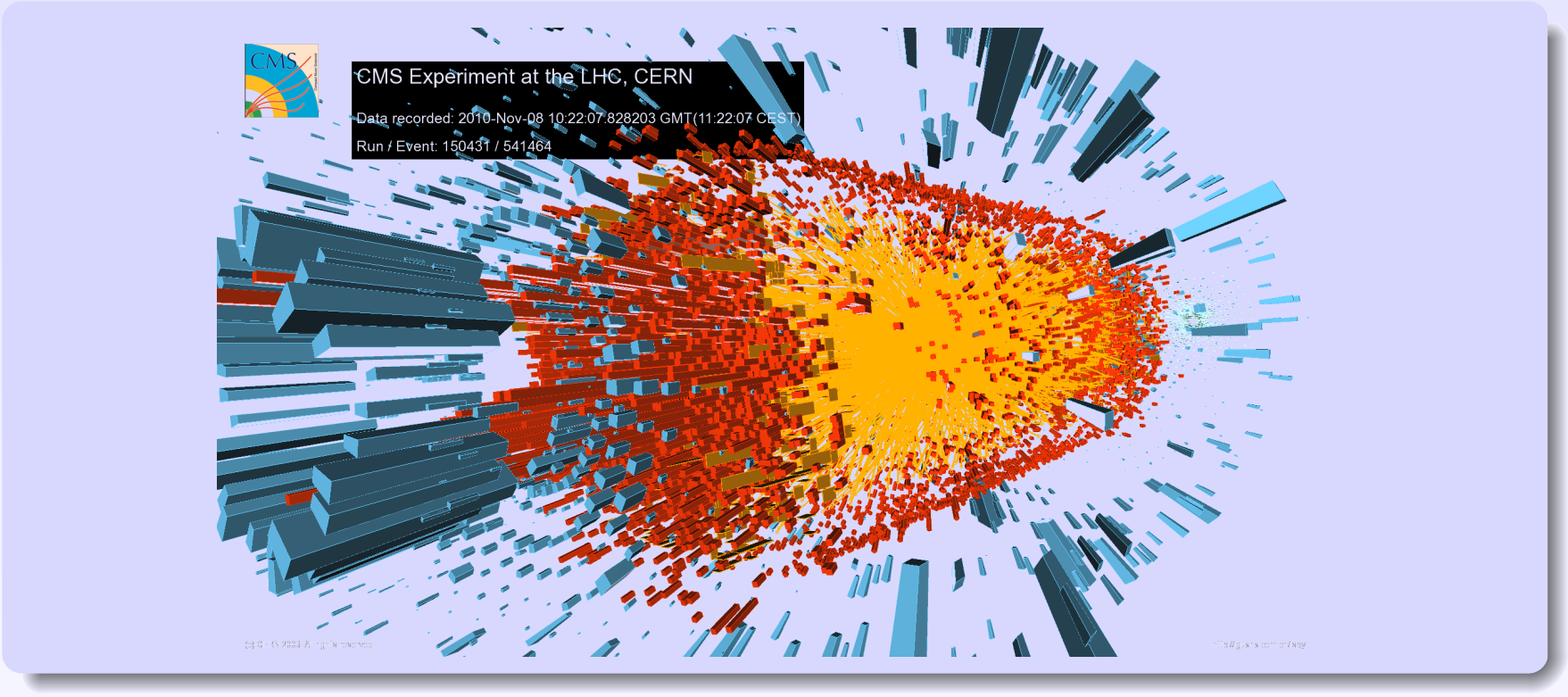
What do we need to know about nuclei?

High energy nucleus



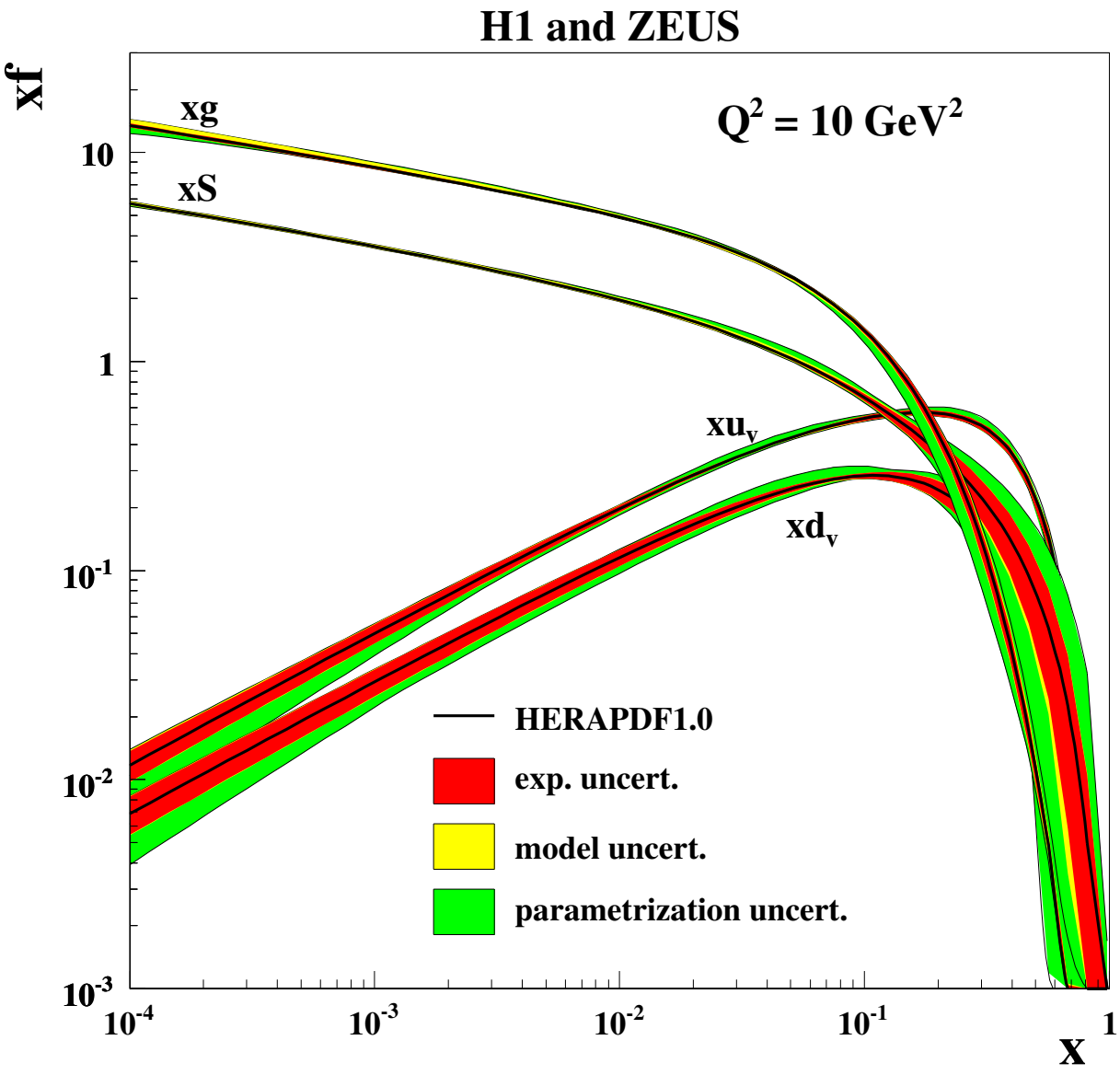
- At low energy : valence quarks
- At higher energy :
  - Lorenz contraction of longitudinal sizes
  - Time dilation $\triangleright$ slowing down of the internal dynamics
  - Gluons start becoming important
- At very high energy : gluons dominate

Kinematics

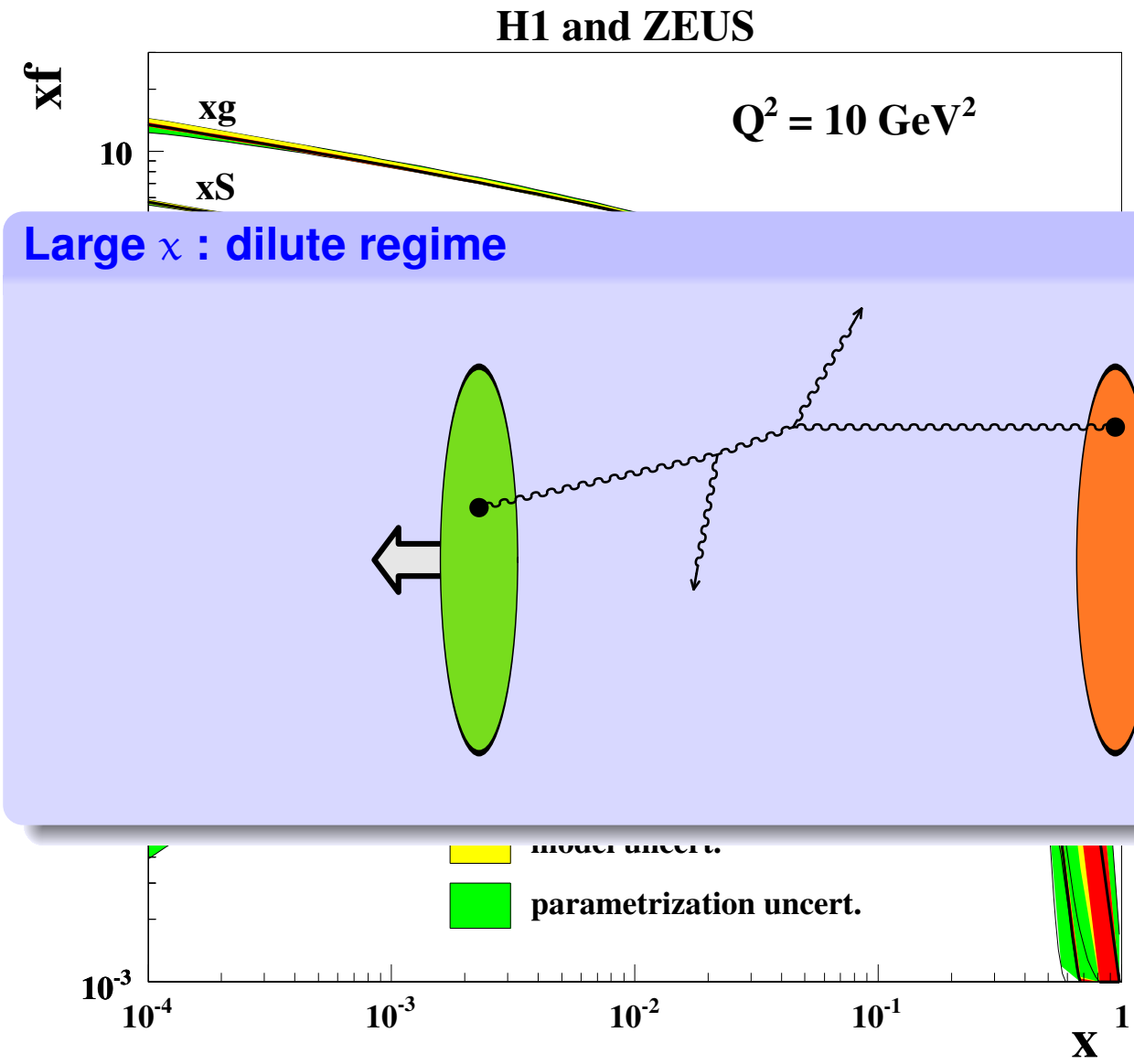


- Low typical final state transverse momentum $p_\perp \lesssim 1$ GeV
- Incoming partons have low momentum fractions $x \sim p_\perp / E$
  - $x \sim 10^{-2}$ at RHIC ($E = 200$ GeV)
  - $x \sim 4.10^{-4}$ at the LHC ($E = 2.76 - 5.5$ TeV)

Nucleon parton distributions

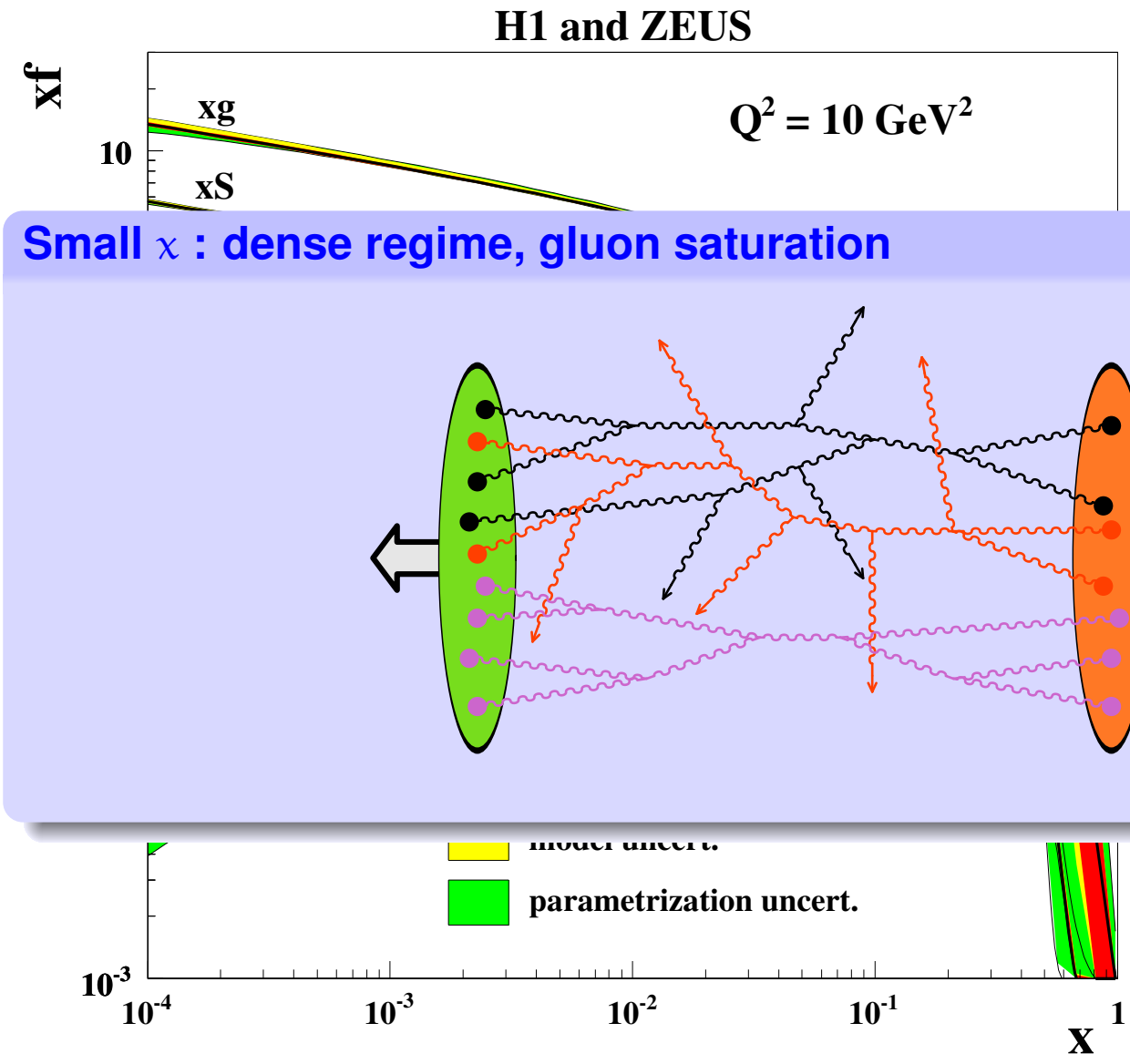


Nucleon parton distributions



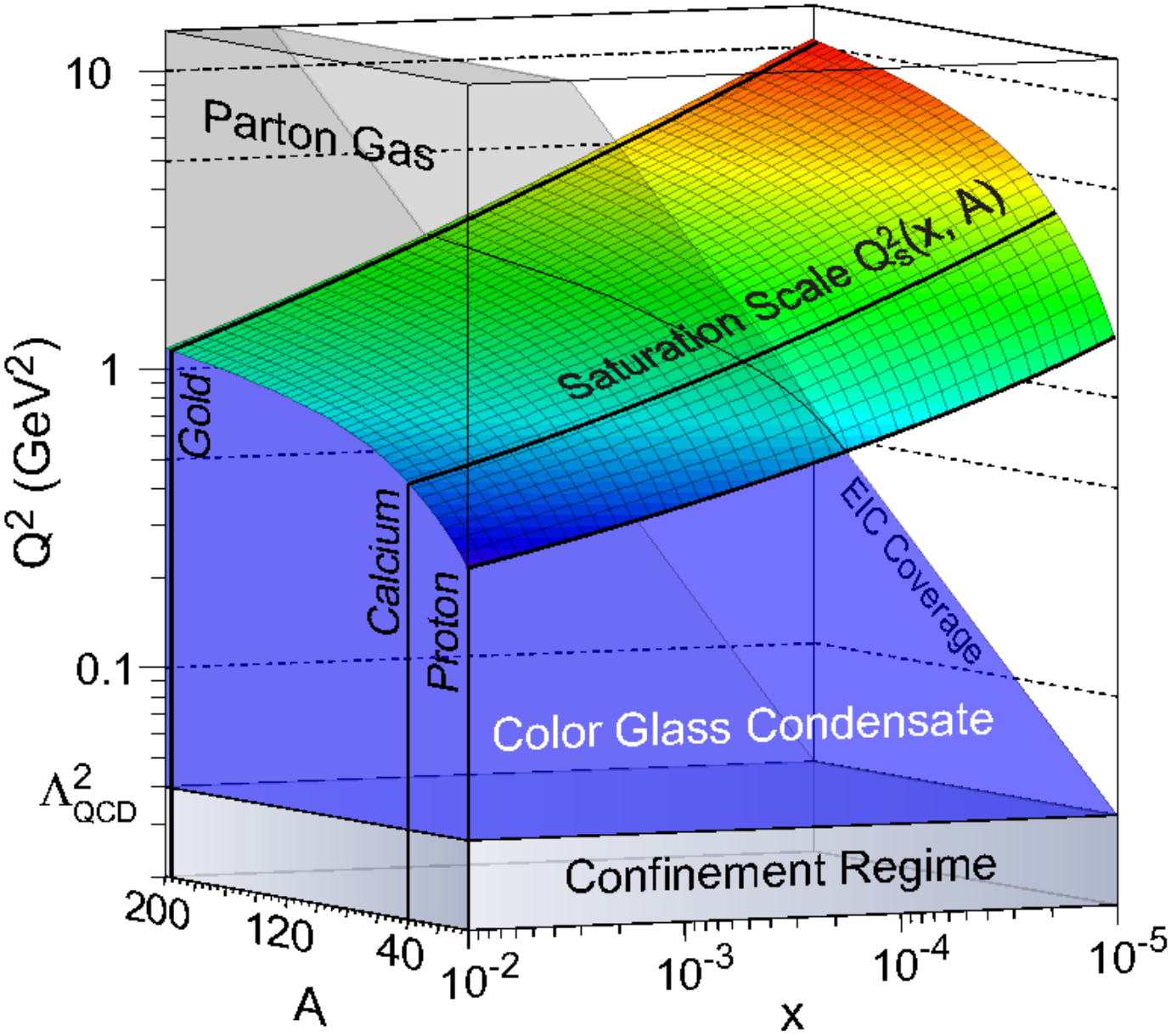
Large χ : dilute regime

Nucleon parton distributions

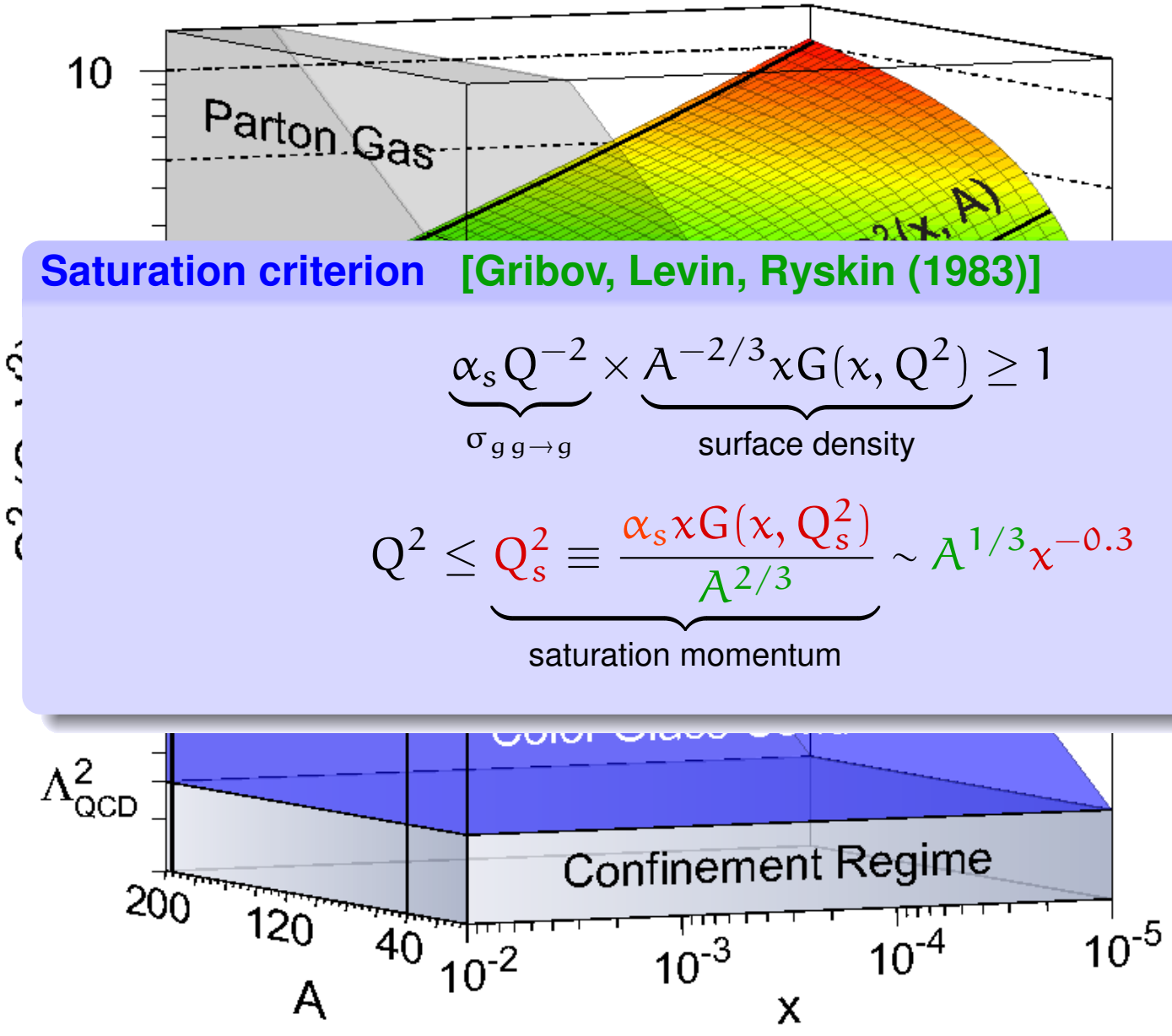


Small χ : dense regime, gluon saturation

Saturation domain



Saturation domain



Saturation criterion [Gribov, Levin, Ryskin (1983)]

$$\underbrace{\alpha_s Q^{-2}}_{\sigma_{g\,g\to g}} \times \underbrace{A^{-2/3} x G(x, Q^2)}_{\text{surface density}} \geq 1$$

$$Q^2 \leq \underbrace{Q_s^2 \equiv \frac{\alpha_s x G(x, Q_s^2)}{A^{2/3}}}_{\text{saturation momentum}} \sim A^{1/3} x^{-0.3}$$

Color Glass Condensate = effective theory of small x gluons

[McLerran, Venugopalan (1994), Jalilian-Marian, Kovner, Leonidov, Weigert (1997), Iancu, Leonidov, McLerran (2001)]

- The fast partons ($k^+ > \Lambda^+$) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

$$J^\mu = \delta^{\mu+}\rho(x^-, \vec{x}_\perp) \qquad (0 < x^- < 1/\Lambda^+)$$

- The color sources ρ are random, and described by a probability distribution $W_\Lambda[\rho]$

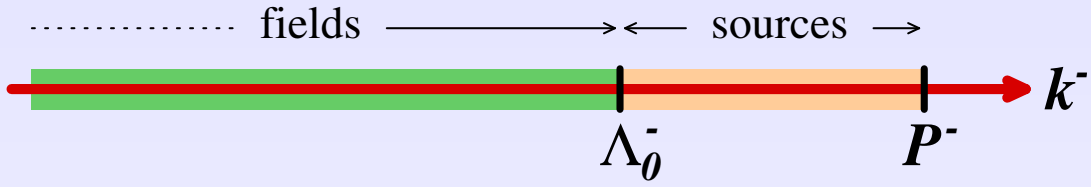
- Slow partons ($k^+ < \Lambda^+$) may evolve during the collision
  ▷ treated as standard gauge fields
  ▷ eikonal coupling to the current J^μ : $J_\mu A^\mu$

$$S = \underbrace{-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}}_{S_{YM}} + \int \underbrace{J^\mu A_\mu}_{\text{fast partons}}$$

Initial state factorization

[FG, Lappi, Venugopalan (2008)]

Renormalization group evolution, JIMWLK equation

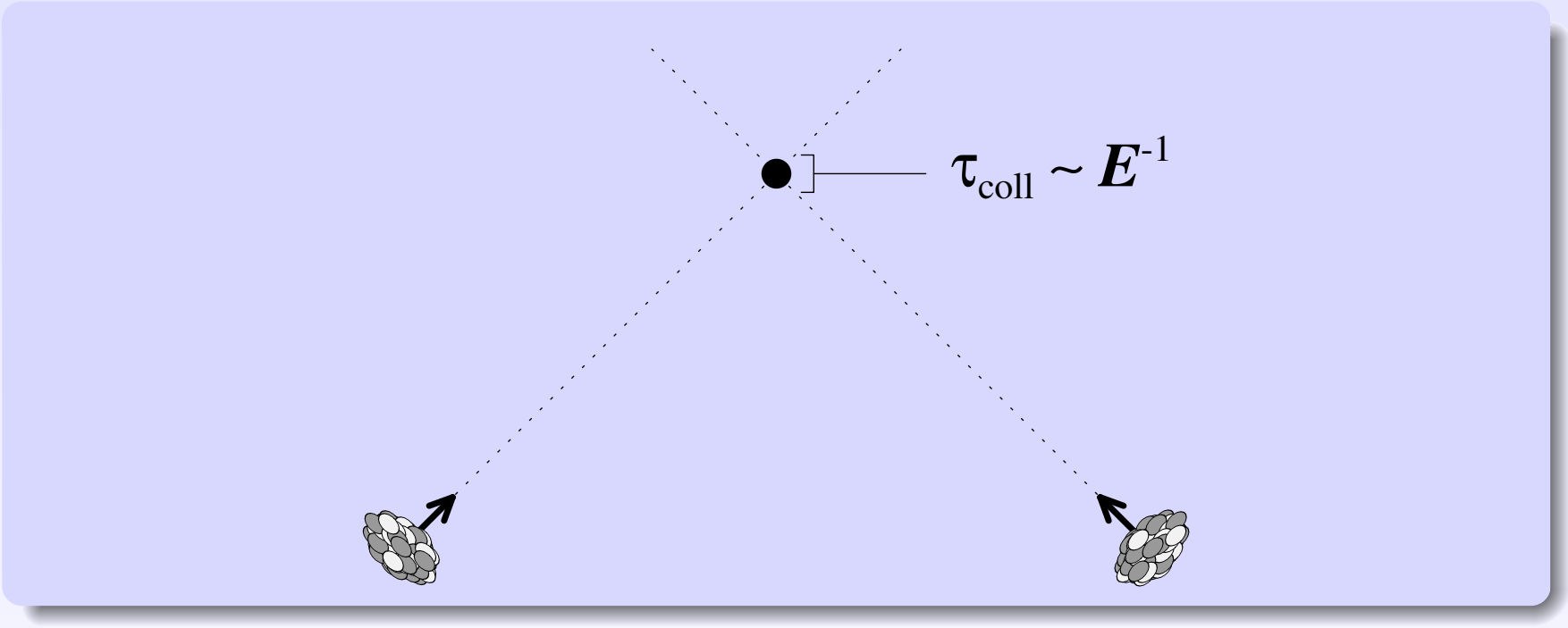


- The cutoff between the sources and the fields is not physical, and should not enter in observables
- Loop corrections contain logs of the cutoff
  (physically, due to soft gluon radiation)
- In inclusive Deep Inelastic Scattering: cancelled by letting the distribution of the sources depend on the cutoff

$$\Lambda \frac{\partial W[\rho]}{\partial \Lambda} = \mathcal{H}\left(\rho, \frac{\delta}{\delta \rho}\right) W[\rho] \qquad \text{(JIMWLK equation)}$$

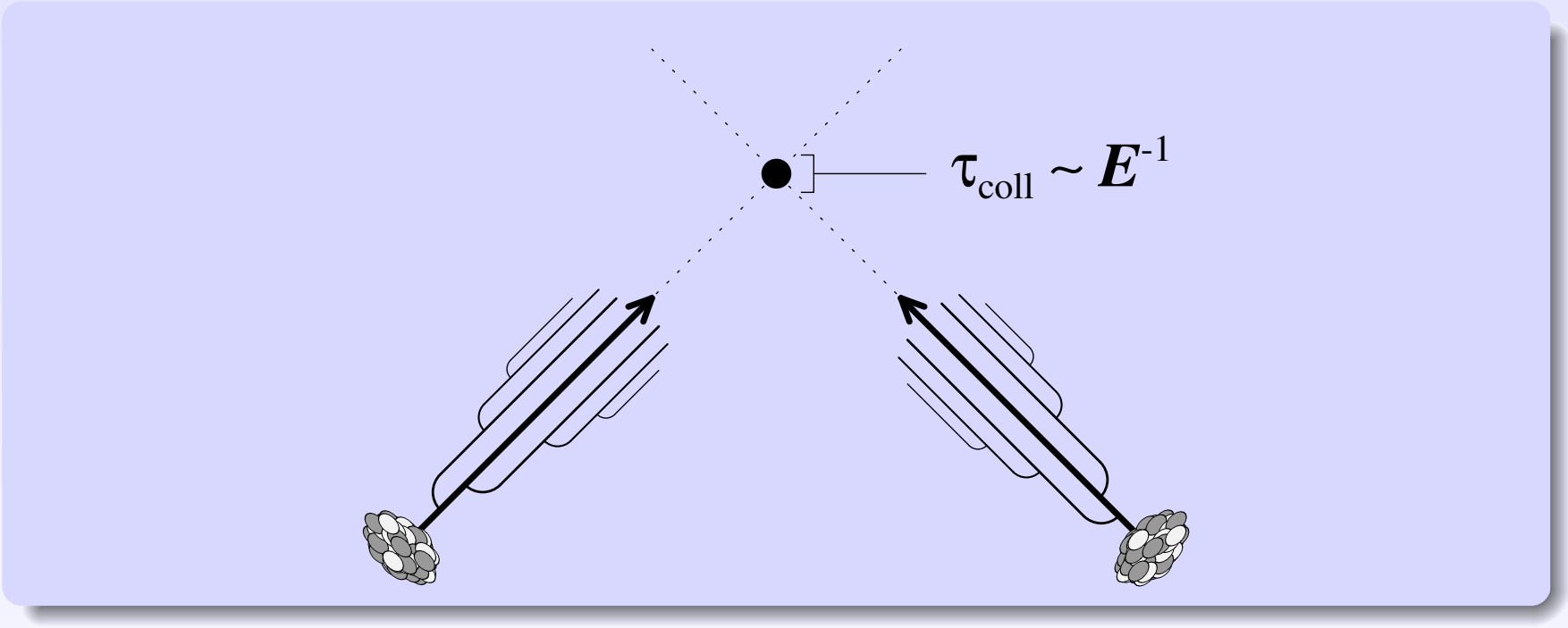
- What about nucleus-nucleus collisions?
  Are the logs still universal?

Handwaving argument for factorization



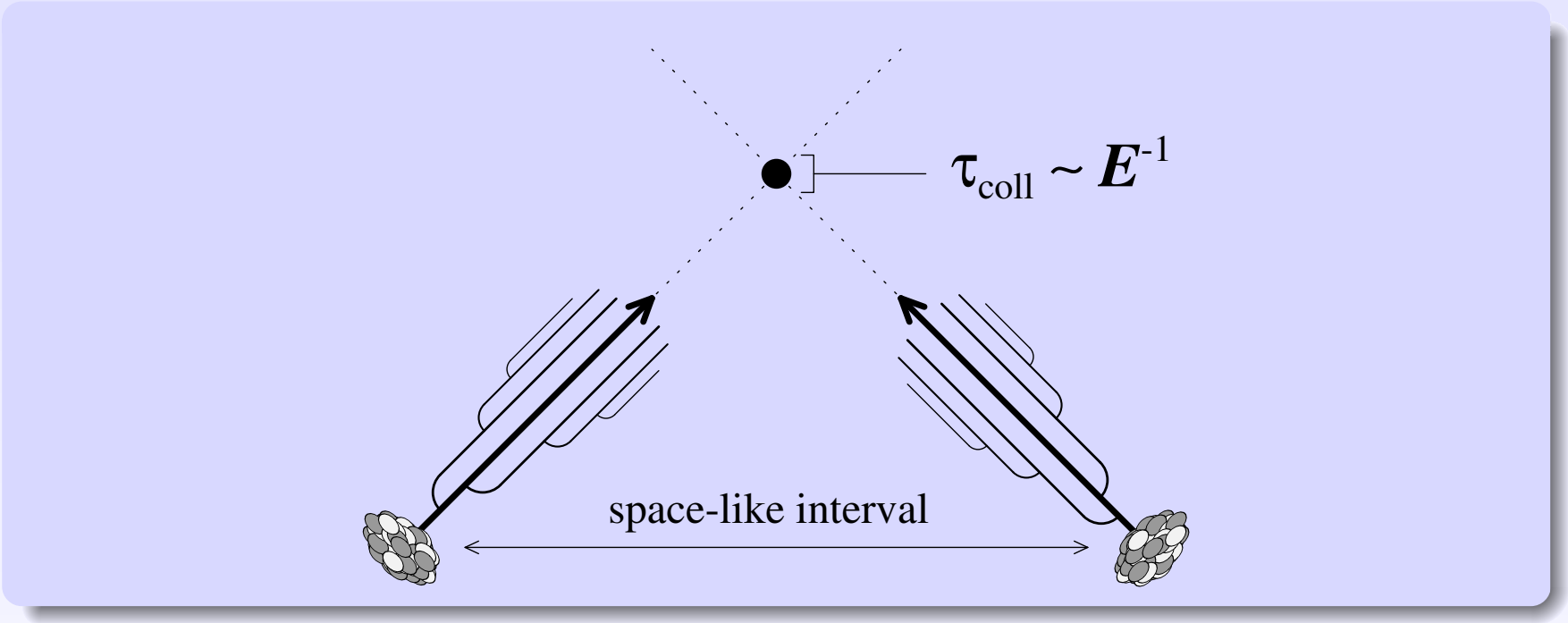
- The duration of the collision is very short: $\tau_{coll} \sim E^{-1}$

Handwaving argument for factorization



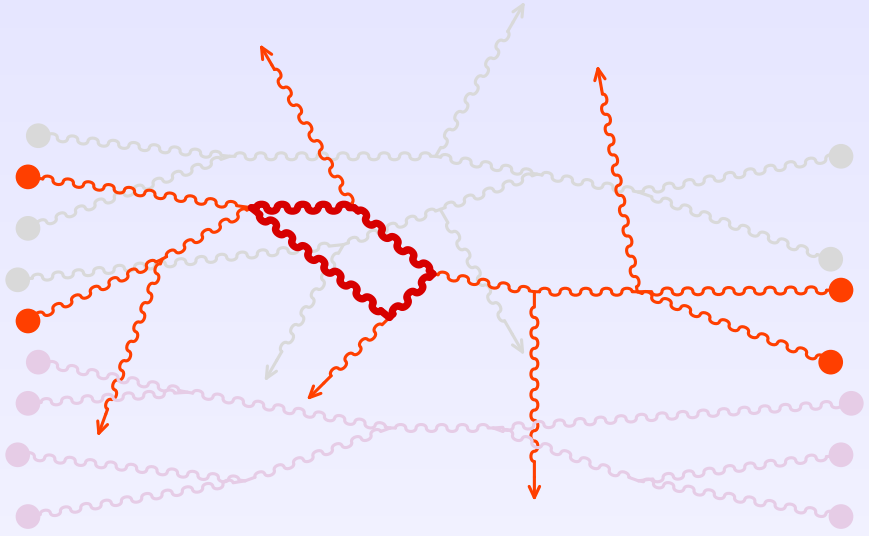
- The duration of the collision is very short: $\tau_{coll} \sim E^{-1}$
- Soft radiation takes a long time
  ▷ it must happen (long) before the collision

Handwaving argument for factorization



- The duration of the collision is very short: $\tau_{coll} \sim E^{-1}$
- Soft radiation takes a long time
  ▷ it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
  ▷ the logarithms are intrinsic properties of the projectiles, independent of the measured observable

Power counting



In the saturated regime: $J^\mu \sim g^{-1}$

$$g^{-2}\ g^{\#\text{ of external legs}}\ g^{2\times(\#\text{ of loops})}$$

- No dependence on the number of sources J^μ
  - ▷ infinite number of graphs at each order

Leading Order

- All observables can be expressed in terms of classical solutions of Yang-Mills equations :

$$[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}] = J_1^\nu + J_2^\nu$$

- Boundary conditions for inclusive observables :
  retarded, with $\mathcal{A} \to 0$ at $x_0 = -\infty$

Example : inclusive spectra at LO

$$\left.\frac{dN_1}{d^3\vec{p}}\right|_{LO} \sim \int d^4x d^4y \; e^{ip\cdot(x-y)} \; \Box_x \Box_y \; \mathcal{A}(x)\,\mathcal{A}(y)$$

$$\left.\frac{dN_n}{d^3\vec{p}_1 \cdots d^3\vec{p}_n}\right|_{LO} = \left.\frac{dN_1}{d^3\vec{p}_1}\right|_{LO} \cdots \left.\frac{dN_1}{d^3\vec{p}_n}\right|_{LO}$$

Next to Leading Order

Relation between LO and NLO

$$\mathcal{O}_{\mathrm{NLO}} = \left[\frac{1}{2} \int_{u,v} \int_{k} \left[a_k \, \mathbb{T} \right]_u \left[a_k^* \, \mathbb{T} \right]_v + \int_u \left[\alpha \, \mathbb{T} \right]_u \right] \mathcal{O}_{\mathrm{LO}}$$

- $\mathbb{T}$ is the generator of the shifts of the initial field :

$$\exp\left[\int_u \left[\alpha \, \mathbb{T} \right]_u \right] \mathcal{O}\Big[\overbrace{\mathcal{A}_\tau(\underbrace{\mathcal{A}_{\mathrm{init}}}_{\text{init. field}})}^{\text{class. field at } \tau} \Big] = \mathcal{O}\Big[\mathcal{A}_\tau(\underbrace{\mathcal{A}_{\mathrm{init}} + \alpha}_{\text{shifted init. field}}) \Big]$$

- a_k, α are calculable analytically
- Valid for all inclusive observables,
  e.g. spectra, the energy-momentum tensor, etc...

Initial state logarithms

- In the CGC, upper cutoff on the loop momentum : $k^{\pm} < \Lambda$, to avoid double counting with the sources $J^{\nu}_{1,2}$
  $\triangleright$ logarithms of the cutoff

Central result for factorization

$$\frac{1}{2}\int_{u,v}\int_{k}\left[\alpha_k\,\mathbb{T}\right]_u\left[\alpha_k^*\,\mathbb{T}\right]_v+\int_u\left[\alpha\,\mathbb{T}\right]_u=$$

$$=\log\left(\Lambda^+\right)\,\mathcal{H}_1+\log\left(\Lambda^-\right)\,\mathcal{H}_2+\text{terms w/o logs}$$

$\mathcal{H}_{1,2}=$ JIMWLK Hamiltonians of the two nuclei

- No mixing between the logs of the two nuclei
- Since the LO $\leftrightarrow$ NLO relationship is the same for all inclusive observables, these logs have a universal structure

Factorization of the logarithms

- The JIMWLK Hamiltonian $\mathcal{H}$ is self-adjoint :

$$\int [D\rho]\, W\, (\mathcal{H}\, \mathcal{O}) = \int [D\rho]\, (\mathcal{H}\, W)\, \mathcal{O}$$

Inclusive observables at Leading Log accuracy

$$\langle \mathcal{O} \rangle_{\text{Leading Log}} = \int [D\rho_1\, D\rho_2]\, W_1[\rho_1]\, W_2[\rho_2]\, \underbrace{\mathcal{O}_{\text{LO}}[\rho_1, \rho_2]}_{\text{fixed } \rho_{1,2}}$$

- Logs absorbed into the scale evolution of $W_{1,2}$

$$\Lambda \frac{\partial W}{\partial \Lambda} = \mathcal{H}\, W \qquad \text{(JIMWLK equation)}$$

- Universality : the same W's for all inclusive observables

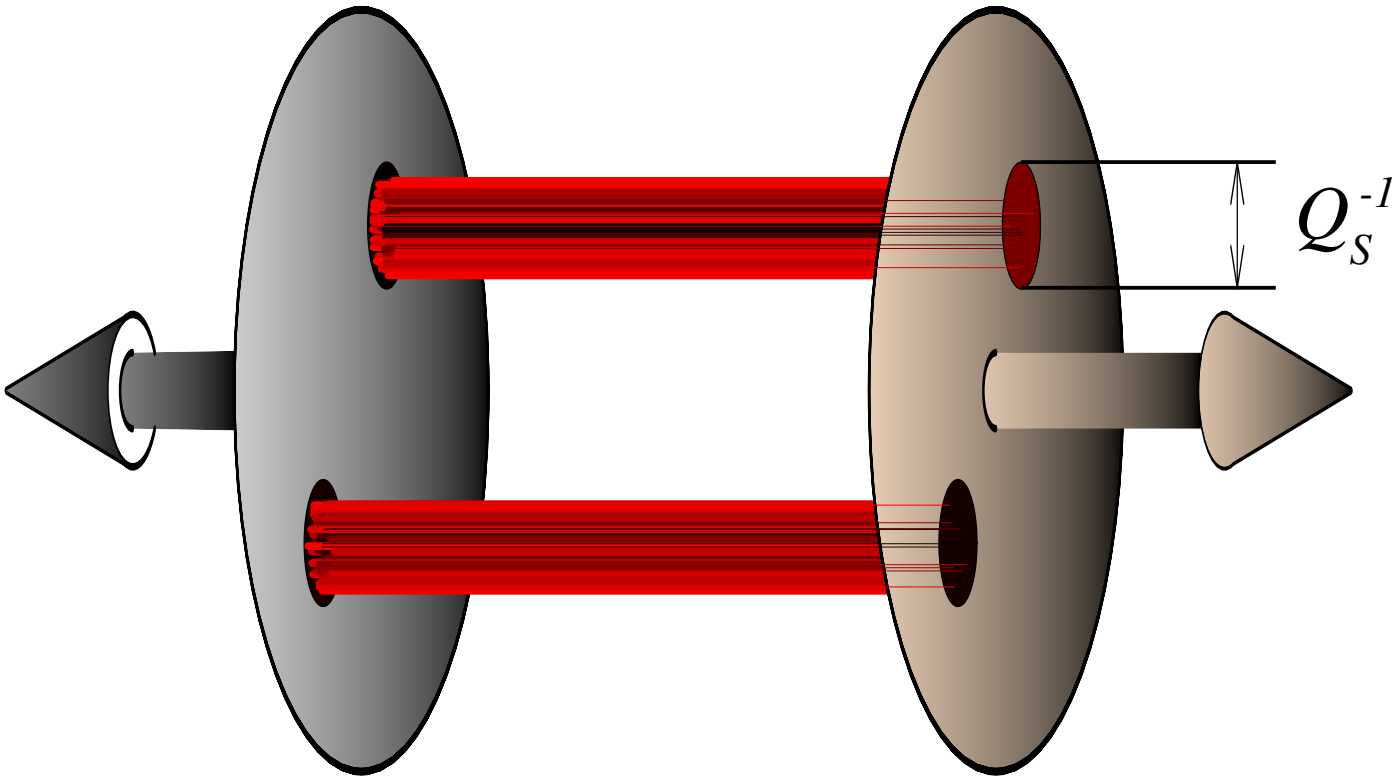
Final state evolution

[Dusling, Epelbaum, FG, Venugopalan (2010)]

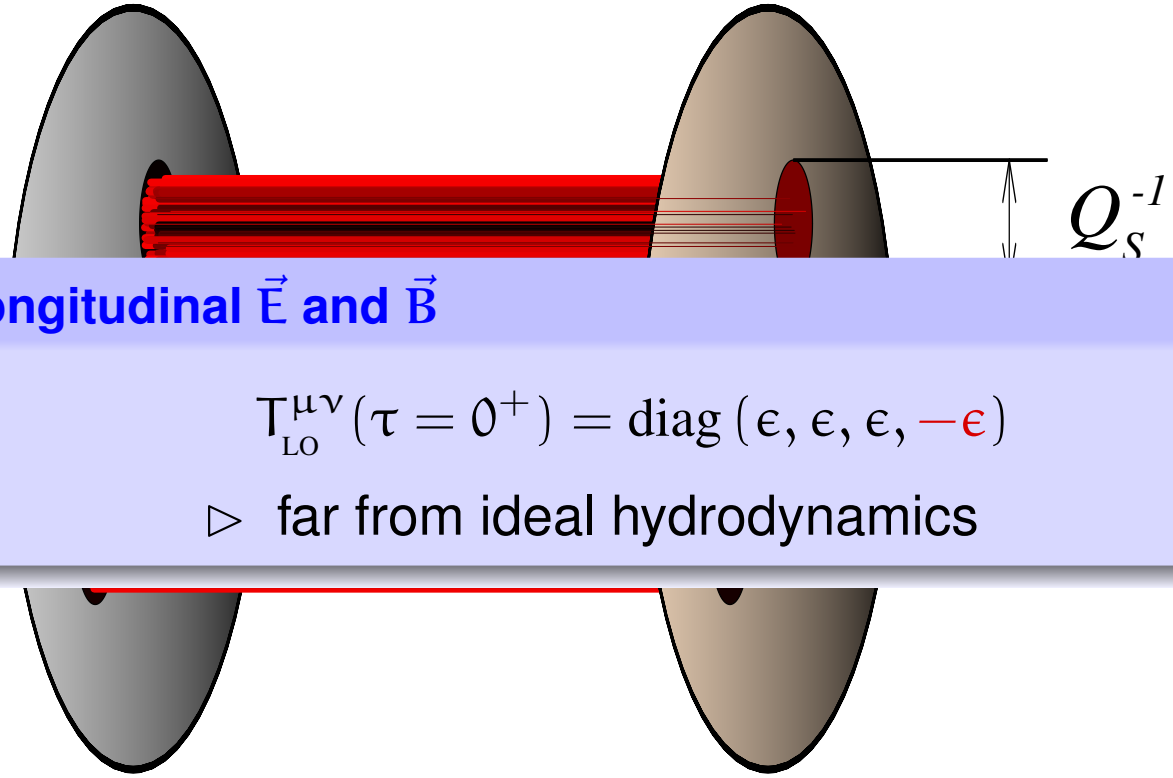
[Dusling, FG, Venugopalan (2011)]

[Epelbaum, FG (2011)]

Energy momentum tensor at LO



Energy momentum tensor at LO



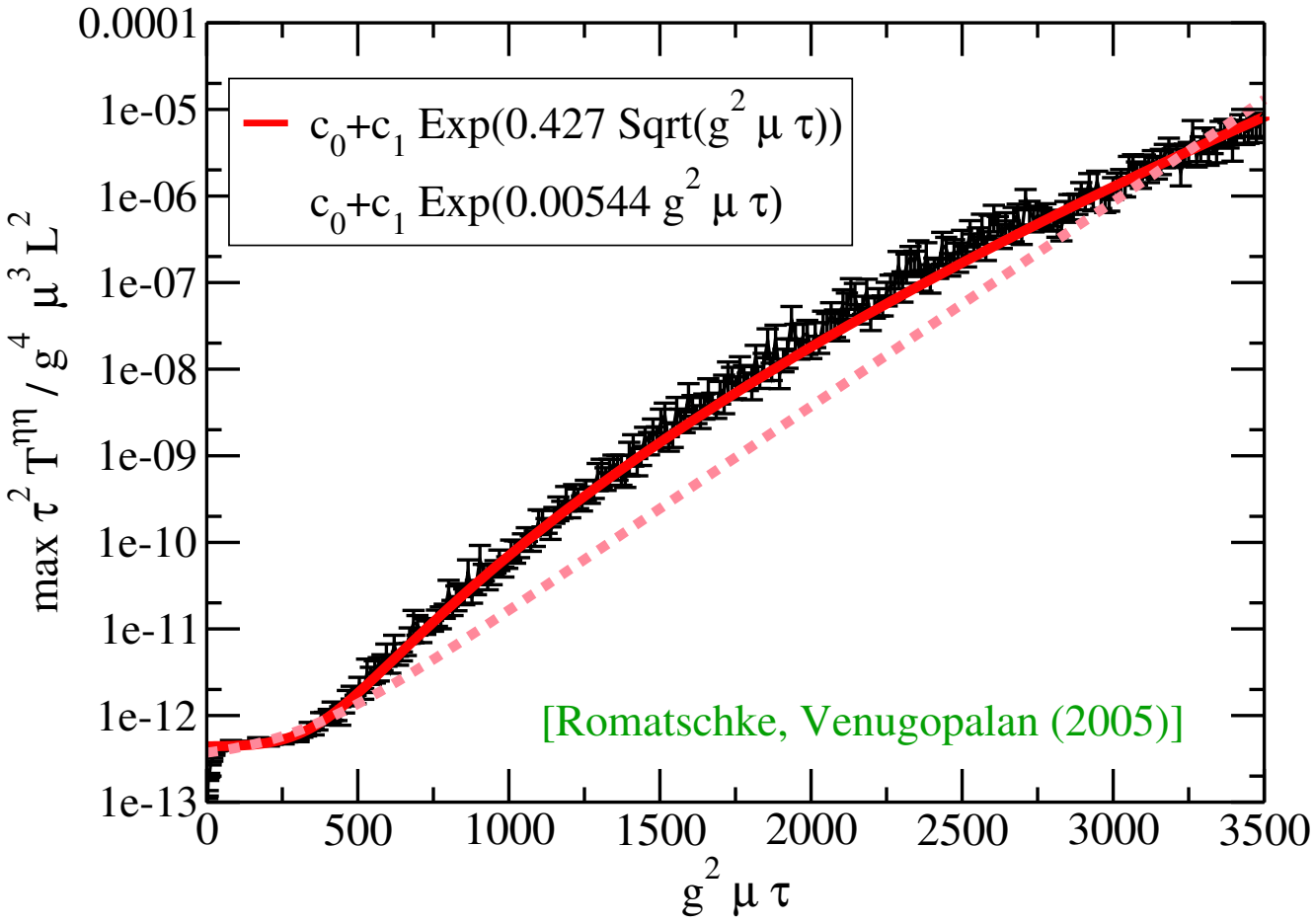
$T^{\mu\nu}$ for longitudinal $\vec{E}$ and $\vec{B}$

$$T^{\mu\nu}_{\rm LO}(\tau = 0^+) = \mathrm{diag}\,(\epsilon, \epsilon, \epsilon, -\epsilon)$$

$\triangleright$ far from ideal hydrodynamics

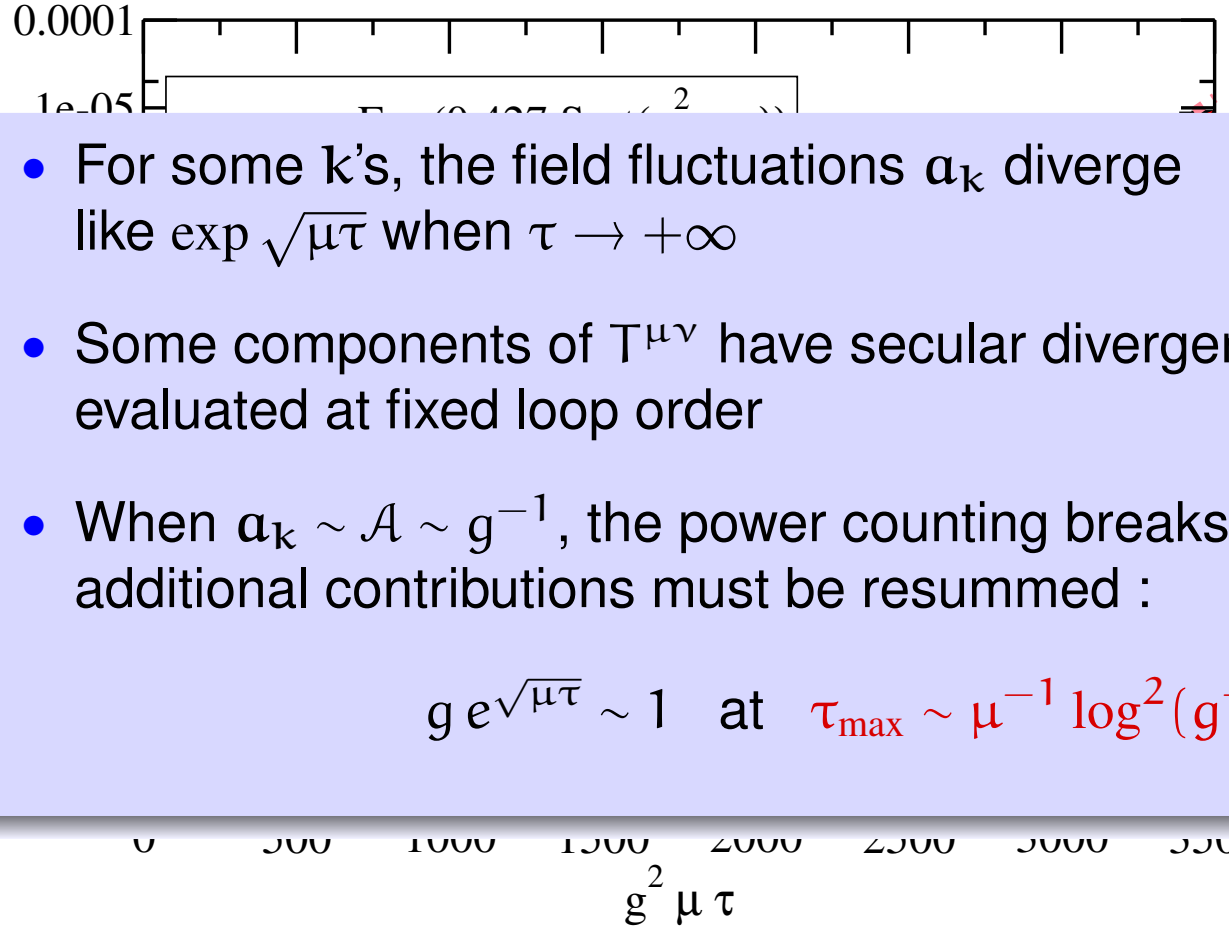
Weibel instabilities for small perturbations

[Mrowczynski (1988), Romatschke, Strickland (2003), Arnold, Lenaghan, Moore (2003), Rebhan, Romatschke, Strickland (2005), Arnold, Lenaghan, Moore, Yaffe (2005), Romatschke, Rebhan (2006), Bodeker, Rummukainen (2007),...]



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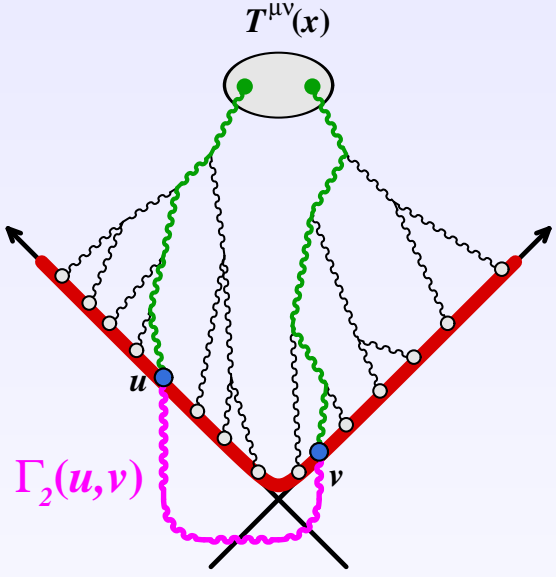


- For some $\mathbf{k}$'s, the field fluctuations $\alpha_{\mathbf{k}}$ diverge like $\exp\sqrt{\mu\tau}$ when $\tau \to +\infty$
- Some components of $T^{\mu\nu}$ have secular divergences when evaluated at fixed loop order
- When $\alpha_{\mathbf{k}} \sim \mathcal{A} \sim g^{-1}$, the power counting breaks down and additional contributions must be resummed :

$$g\, e^{\sqrt{\mu\tau}} \sim 1 \quad \text{at} \quad \tau_{\max} \sim \mu^{-1} \log^2(g^{-1})$$

Improved power counting

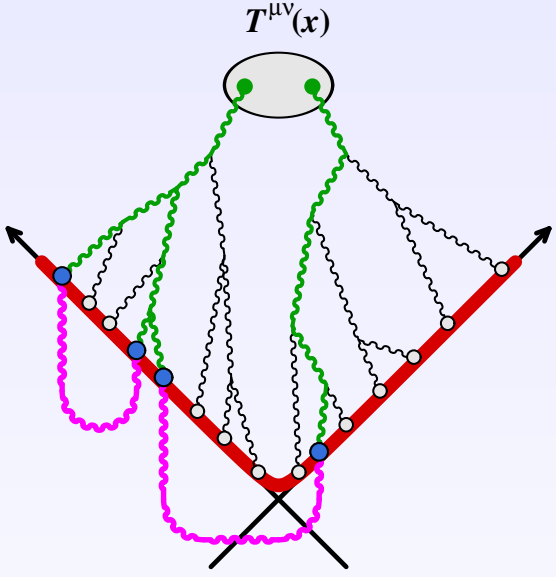
$$\text{Loop} \sim g^2 \quad , \quad \mathbb{T} \sim e^{\sqrt{\mu\tau}}$$



- 1 loop : $(ge^{\sqrt{\mu\tau}})^2$

Improved power counting

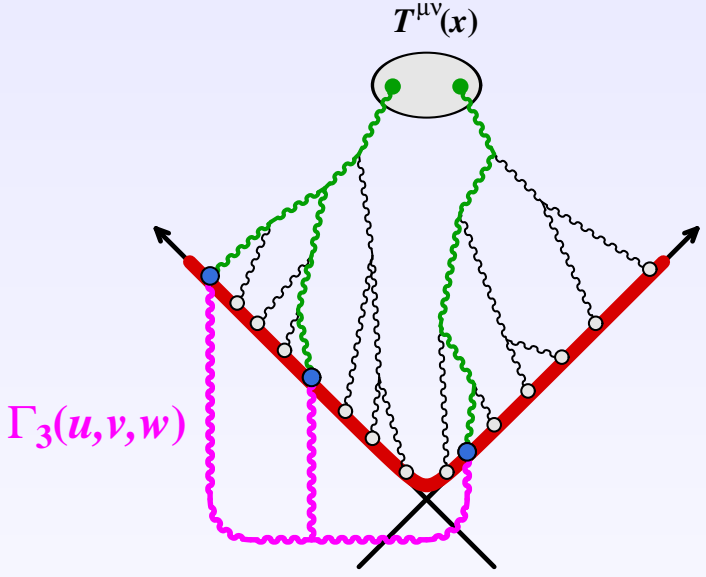
$$\text{Loop} \sim g^2 \quad , \quad \mathbb{T} \sim e^{\sqrt{\mu\tau}}$$



- 1 loop : $(ge^{\sqrt{\mu\tau}})^2$
- 2 disconnected loops : $(ge^{\sqrt{\mu\tau}})^4$

Improved power counting

$$\text{Loop} \sim g^2 \quad , \quad \mathbb{T} \sim e^{\sqrt{\mu\tau}}$$



- 1 loop : $(ge^{\sqrt{\mu\tau}})^2$
- 2 disconnected loops : $(ge^{\sqrt{\mu\tau}})^4$
- 2 nested loops : $g(ge^{\sqrt{\mu\tau}})^3$
  - ▷ subleading

Leading terms at $\tau_{\max}$

- All disconnected loops to all orders
  - ▷ exponentiation of the 1-loop result

Resummation of the leading secular terms

$$T^{\mu\nu}_{\text{resummed}} = \exp\left[\frac{1}{2}\int_{u,v}\underbrace{\int_k [\alpha_k \mathbb{T}]_u [\alpha_k^* \mathbb{T}]_v}_{\mathcal{G}(u,v)} + \int_u [\alpha \mathbb{T}]_u\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}}]$$

$$= \underbrace{T^{\mu\nu}_{\text{LO}} + T^{\mu\nu}_{\text{NLO}}}_{\text{in full}} + \underbrace{T^{\mu\nu}_{\text{NNLO}} + \cdots}_{\text{partially}}$$

Resummation of the leading secular terms

$$T^{\mu\nu}_{\text{resummed}} = \exp\left[\frac{1}{2}\int_{u,v}\underbrace{\int_k [a_k \mathbb{T}]_u [a_k^* \mathbb{T}]_v}_{\mathcal{G}(u,v)} + \int_u [\alpha \mathbb{T}]_u\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}}]$$

$$= \int [D\chi]\, \exp\left[-\frac{1}{2}\int_{u,v}\chi(u)\mathcal{G}^{-1}(u,v)\chi(v)\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}} + \chi + \alpha]$$

- The evolution remains classical, but we must average over a Gaussian ensemble of initial conditions
- The 2-point correlation $\mathcal{G}(u,v)$ of the fluctuations is known
- Fluctuations $\Leftrightarrow$ 1/2 quantum per mode

Analogous scalar toy model

ϕ^4 field theory coupled to a source

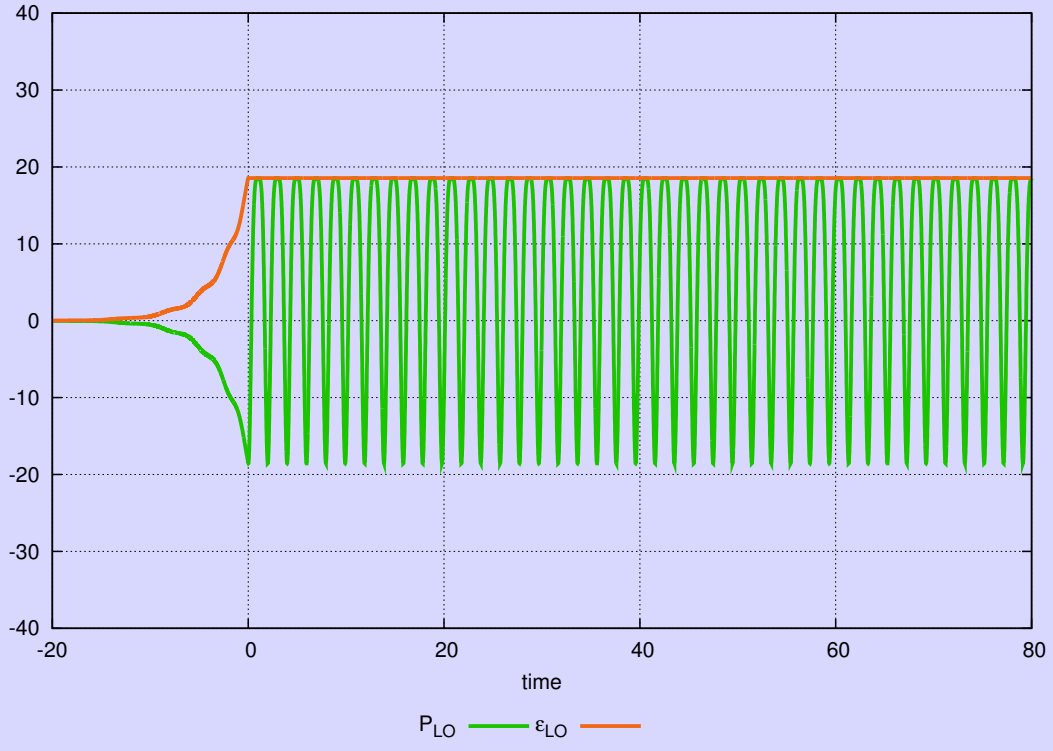
$$\mathcal{L} = \frac{1}{2}(\partial_\alpha \phi)^2 - \frac{g^2}{4!}\phi^4 + J\phi$$

Strong external source: $J \propto \frac{Q^3}{g}$

- In 3+1-dim, g is dimensionless, and the only scale in the problem is Q
- This theory has unstable modes (parametric resonance)
- Two setups have been studied :
  - Fixed volume system
  - Longitudinally expanding system

Pathologies in fixed order calculations

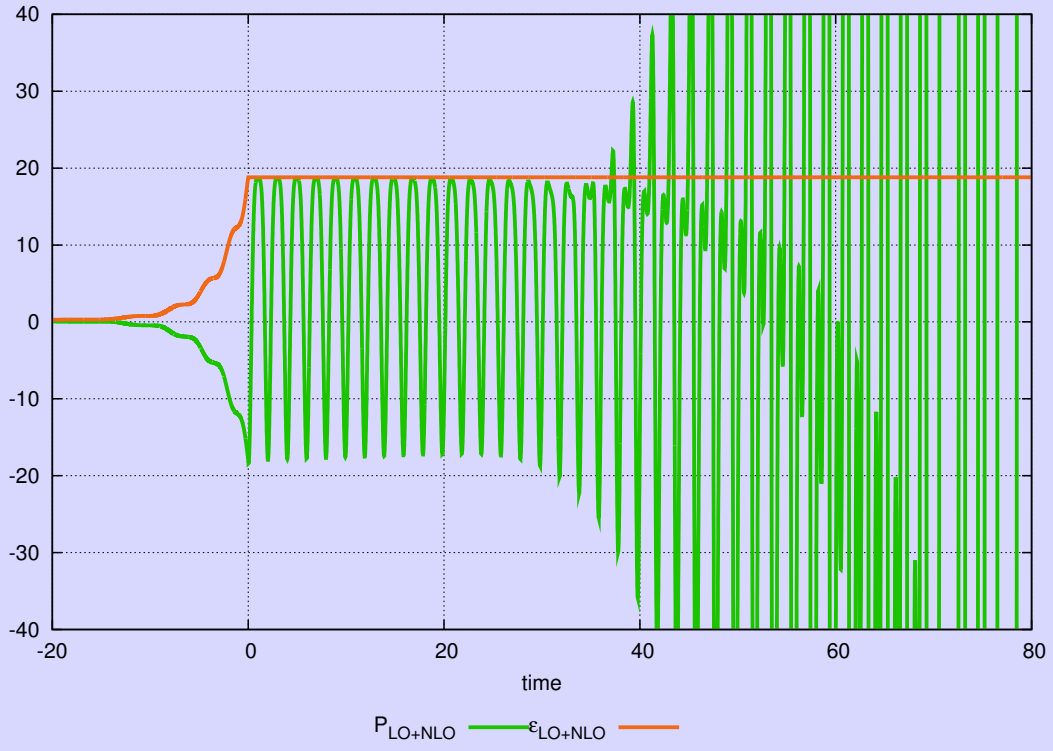
Tree



- Oscillating pressure at LO : no equation of state

Pathologies in fixed order calculations

Tree + 1-loop



- Oscillating pressure at LO : no equation of state
- Small NLO correction to the energy density (protected by energy conservation)
- Secular divergence in the NLO correction to the pressure

Spectrum of fluctuations

Initial conditions

$$\phi(t_0, \mathbf{x}) = \varphi(t_0, \mathbf{x}) + \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k} \left[c_{\mathbf{k}}\, e^{i\lambda_{\mathbf{k}} t_0}\, V_{\mathbf{k}}(\mathbf{x}) + \text{c.c.} \right]$$

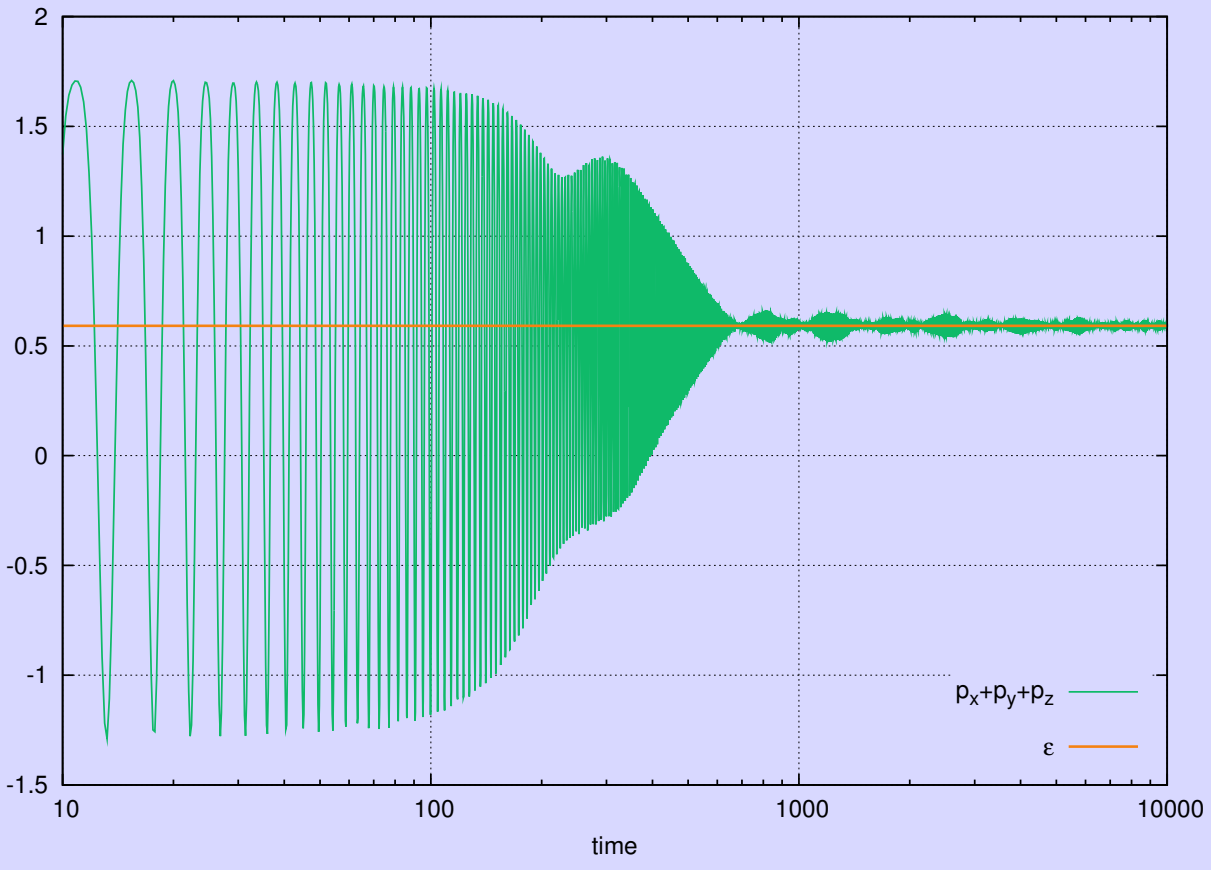
- $\varphi(t_0, \mathbf{x})$ = classical background field
- $c_{\mathbf{k}}$ = complex random Gaussian number:

$$\langle c_{\mathbf{k}} \rangle = 0\,, \qquad \langle c_{\mathbf{k}} c_{\mathbf{l}}^* \rangle = (2\pi)^3\, k\, \delta(\mathbf{k} - \mathbf{l})$$

- $\lambda_{\mathbf{k}}, V_{\mathbf{k}}$ determined by:

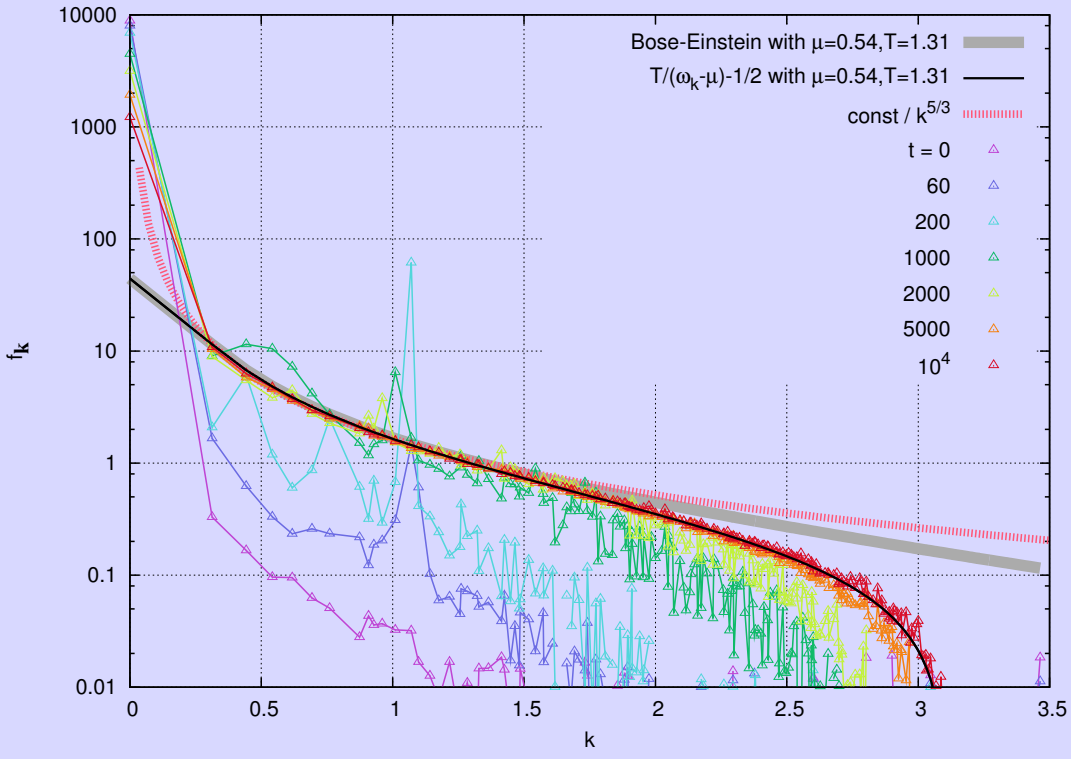
$$\left[-\boldsymbol{\nabla}^2 + \frac{1}{2} g^2 \varphi^2(t_0, \mathbf{x}) \right] V_{\mathbf{k}}(\mathbf{x}) = \lambda_{\mathbf{k}}^2 V_{\mathbf{k}}(\mathbf{x})$$

Resummed energy momentum tensor



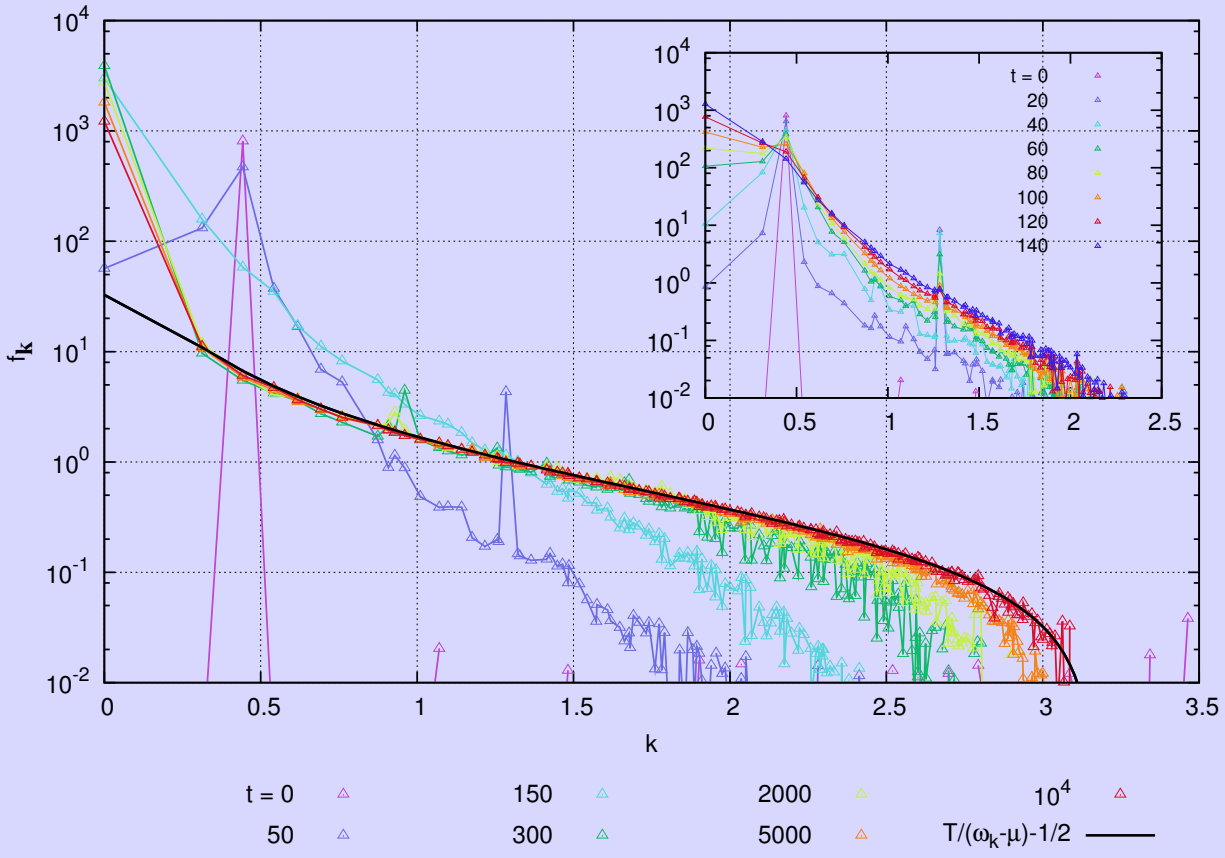
- No secular divergence in the resummed pressure
- The pressure relaxes to the equilibrium equation of state

Time evolution of the occupation number



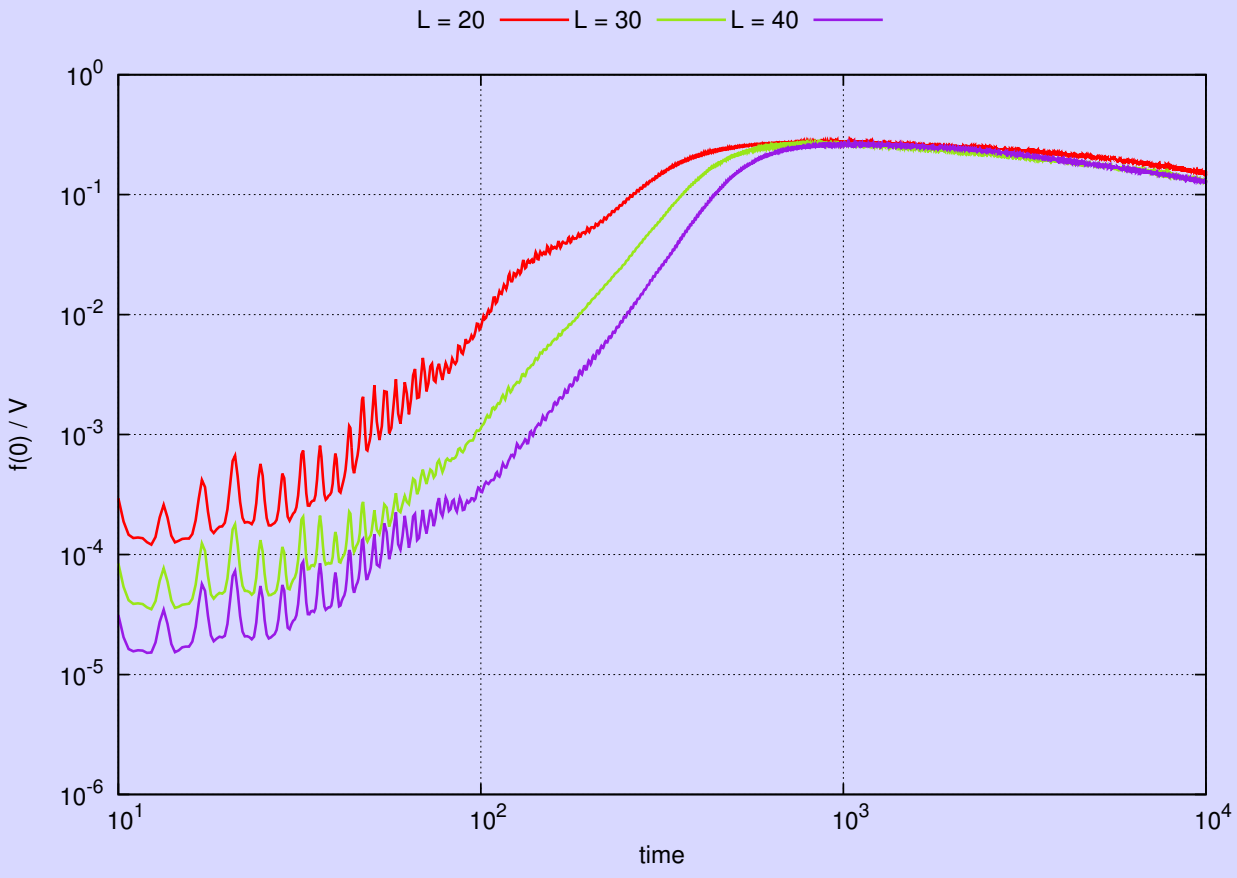
- Resonant peak at early times
- Late times : classical equilibrium with a chemical potential
- $\mu \approx m$ + excess at $k = 0$: Bose-Einstein condensation?

Bose-Einstein condensation



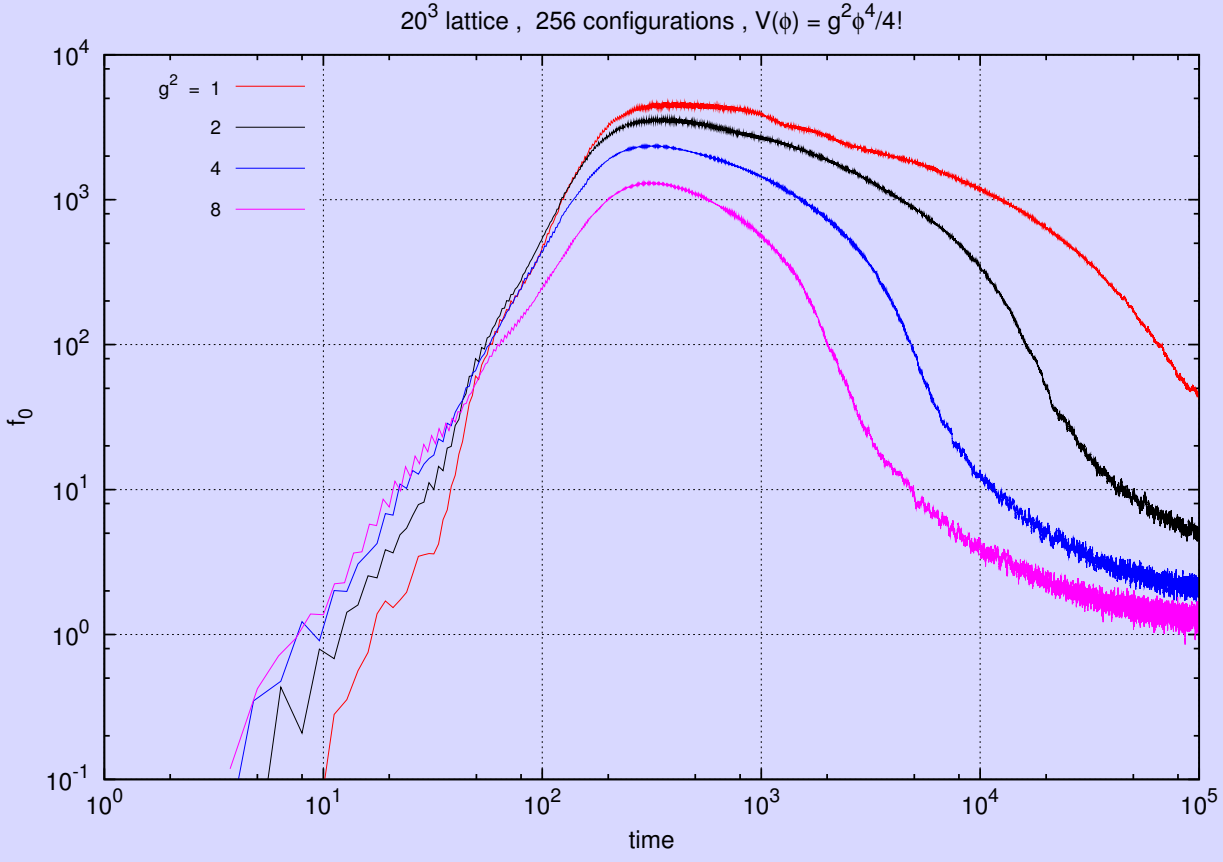
- Start with the same energy density, but an empty zero mode
- Very quickly, the zero mode becomes highly occupied
- Same distribution as before at late times

Volume dependence



$$f(k) = \frac{1}{e^{\beta(\omega_k - \mu)} - 1} + n_0 \delta(k) \quad \Longrightarrow f(0) \propto V = L^3$$

Evolution of the condensate



- Formation time almost independent of the coupling
- Condensate lifetime much longer than its formation time
- Smaller amplitude and faster decay at large coupling

Longitudinal expansion: spectrum of fluctuations

Initial conditions

$$\phi(\tau_0,\eta,\mathbf{x}_\perp) = \varphi(\tau_0,\mathbf{x}_\perp) + \int \frac{d^2\mathbf{k}_\perp d\nu}{(2\pi)^3}\left[c_{\nu\mathbf{k}}\, H^{(2)}_{i\nu}(\lambda_k\tau_0)\, V_k(\mathbf{x}_\perp)\, e^{-i\nu\eta} + \text{c.c.}\right]$$

- $\varphi(\tau_0,\mathbf{x}_\perp)$ = classical background field (boost invariant)
- $c_{\nu\mathbf{k}}$ = complex random Gaussian number:

$$\left\langle c_{\nu\mathbf{k}}\right\rangle = 0\,, \qquad \left\langle c_{\nu\mathbf{k}} c^*_{\sigma\mathbf{l}}\right\rangle = (2\pi)^3\,\delta(\mathbf{k}_\perp-\mathbf{l}_\perp)\delta(\nu-\sigma)$$

- λ_k, V_k determined by:

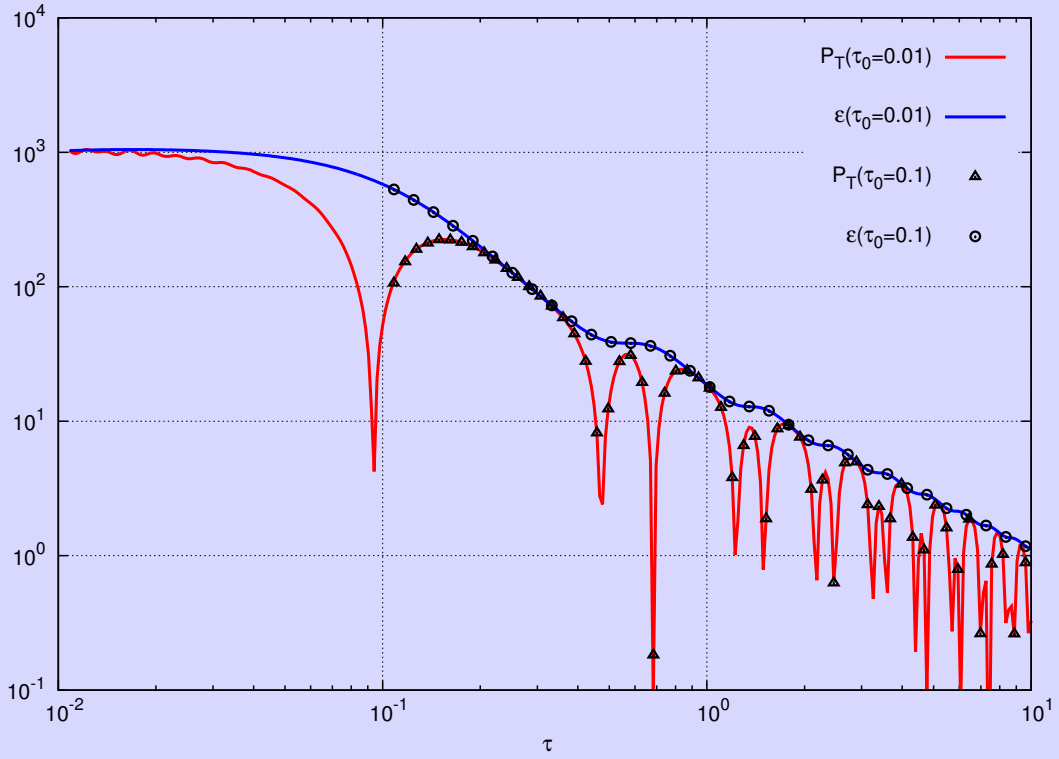
$$\left[-\boldsymbol{\nabla}^2_\perp + \frac{1}{2}g^2\varphi^2(\tau_0,\mathbf{x}_\perp)\right] V_k(\mathbf{x}_\perp) = \lambda^2_k V_k(\mathbf{x}_\perp)$$

Longitudinal expansion: initial time

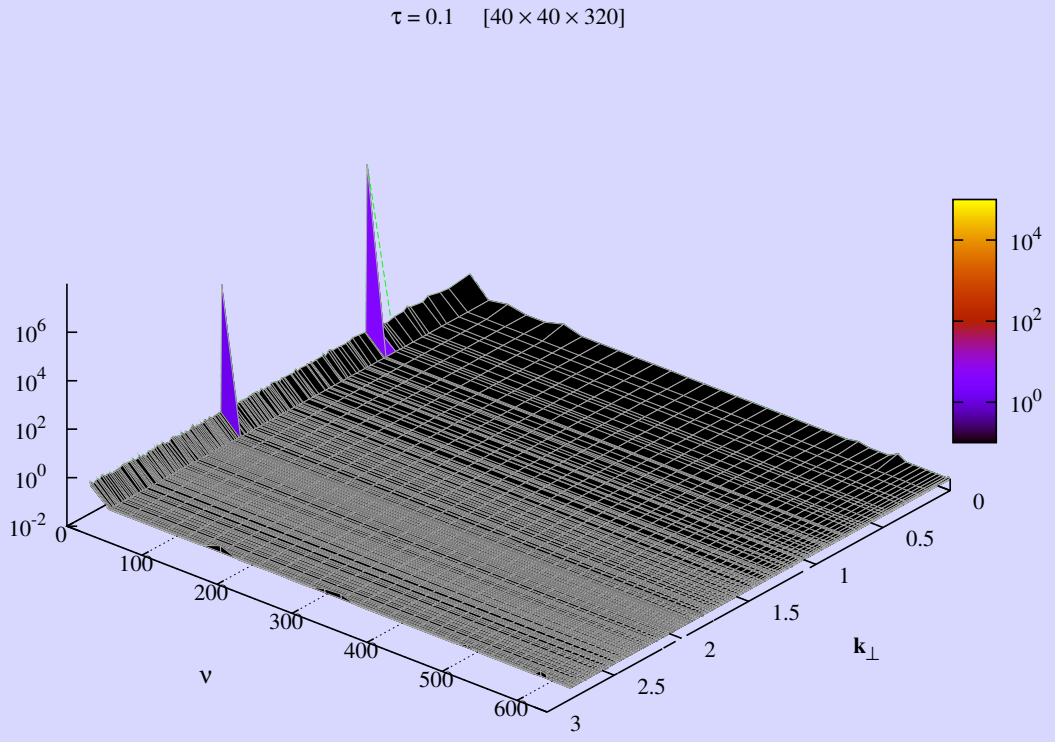
- The EoM is singular when $\tau \to 0$: one must start at $\tau_0 \neq 0$

$$\partial_\tau^2 \phi + \frac{1}{\tau}\partial_\tau \phi - \frac{1}{\tau^2}\partial_\eta^2 \phi - \nabla_\perp^2 \phi + \frac{g^2}{6}\phi^3 = 0$$

- The spectrum of fluctuations of the initial field also depends on τ_0, in such a way that the end result is independent of τ_0

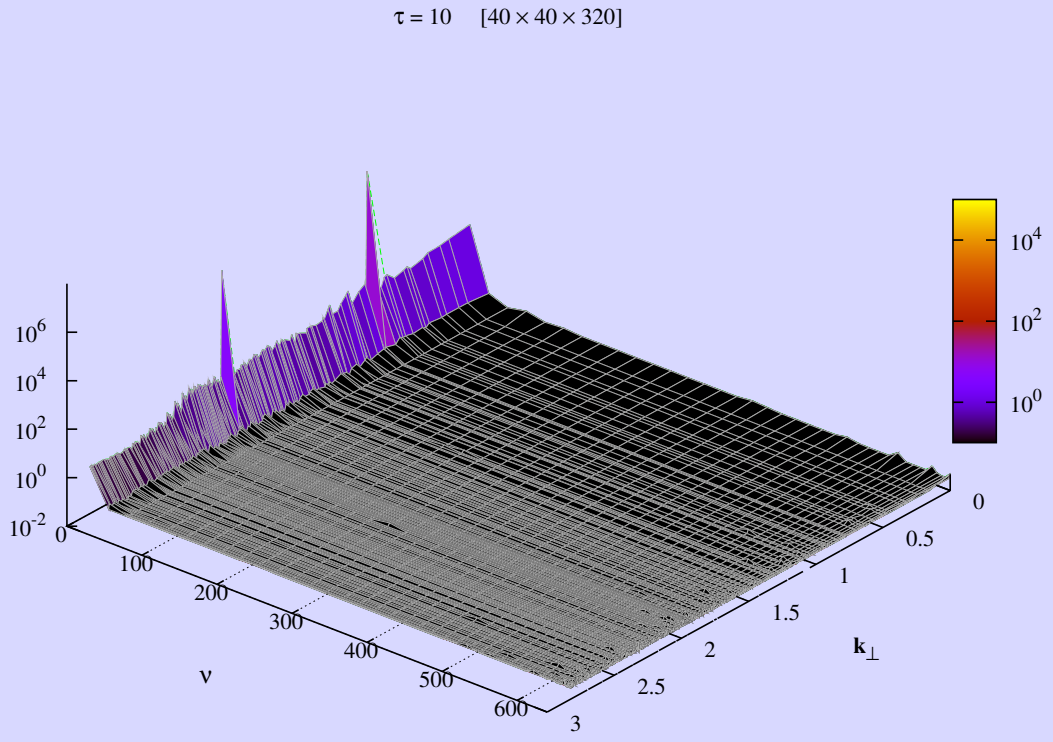


Longitudinal expansion : occupation number



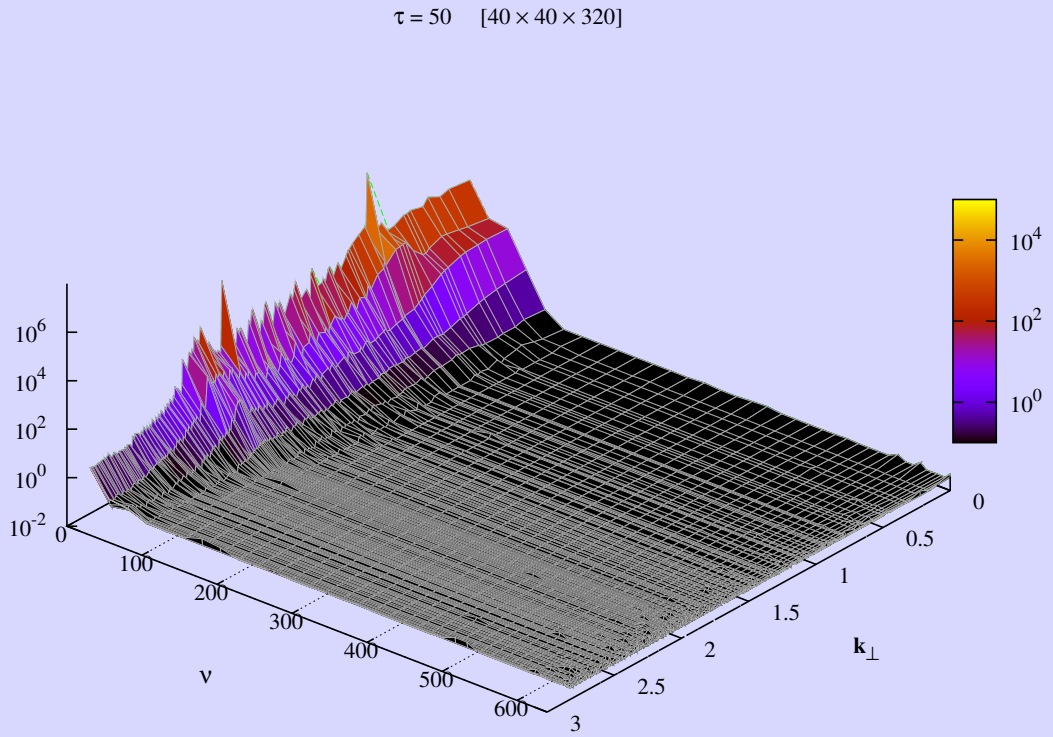
- Note : ν is the Fourier conjugate of the rapidity η
- Initially, only the $\nu = 0$ modes are occupied

Longitudinal expansion : occupation number



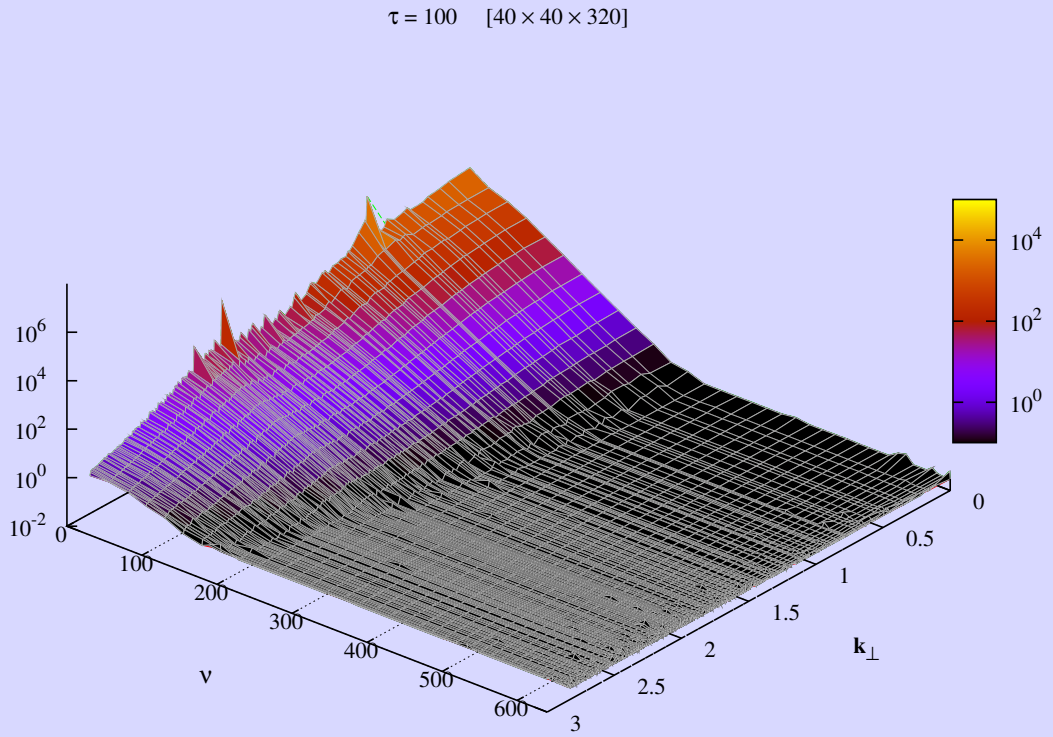
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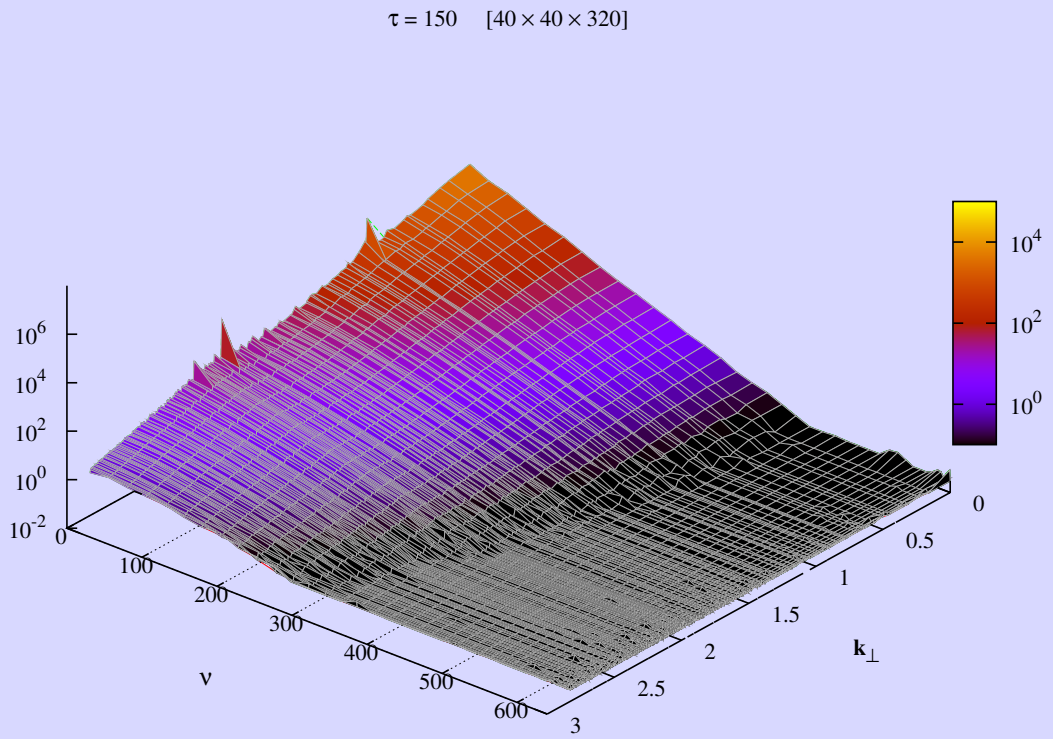
- Note : ν is the Fourier conjugate of the rapidity η
- Initially, only the $\nu = 0$ modes are occupied
- Rapid expansion of the distribution in ν ($\nu_{\max} \sim \tau^{2/3}$ for a thermalized system)

Longitudinal expansion : occupation number



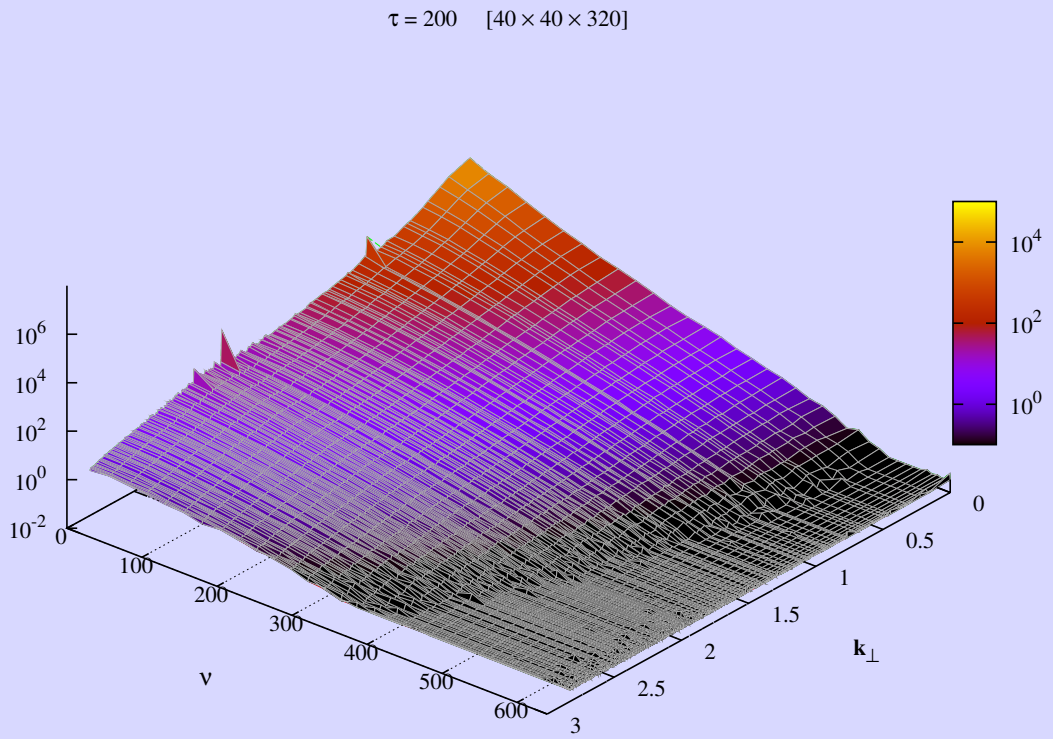
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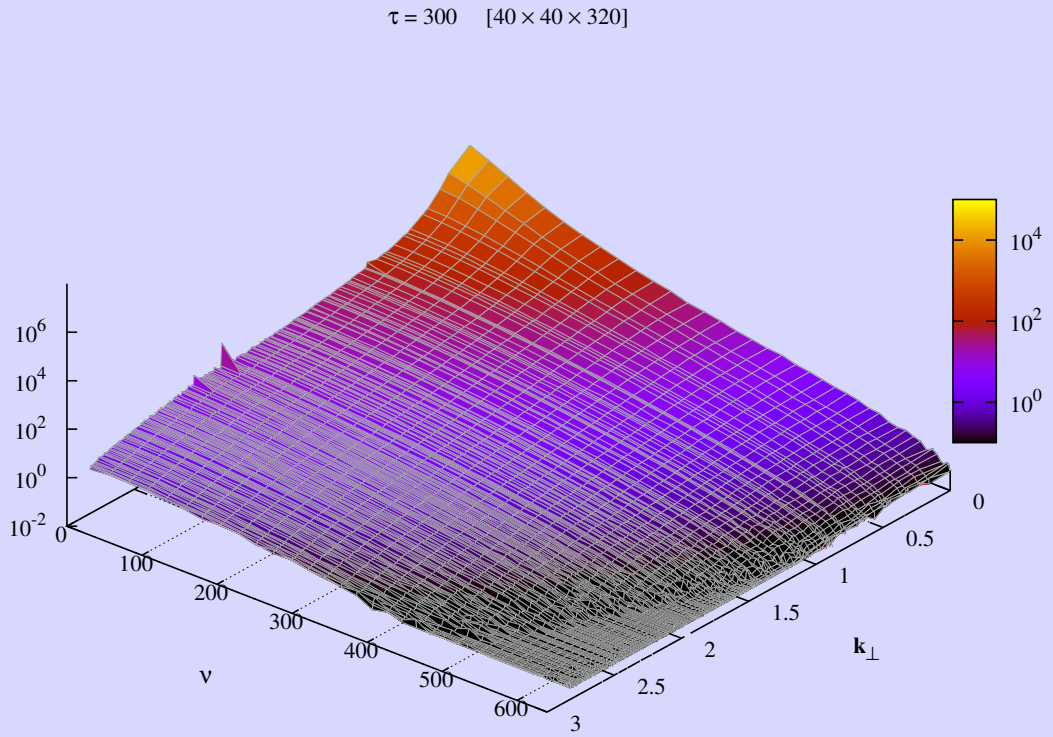
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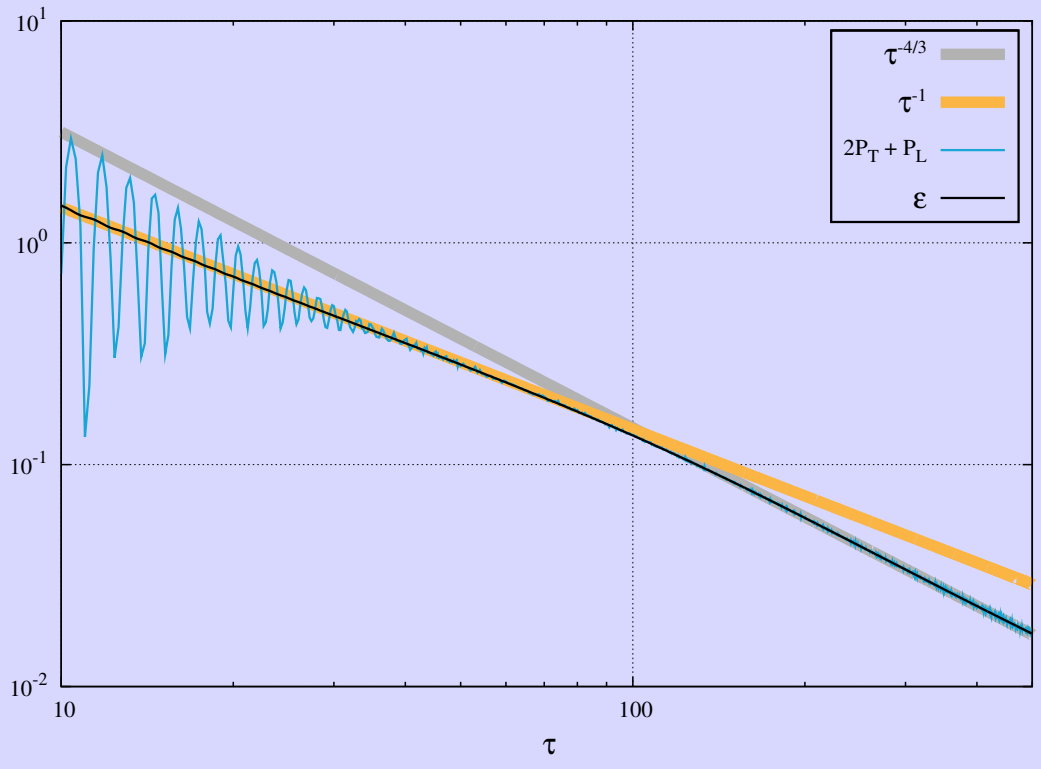
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Longitudinal expansion : occupation number



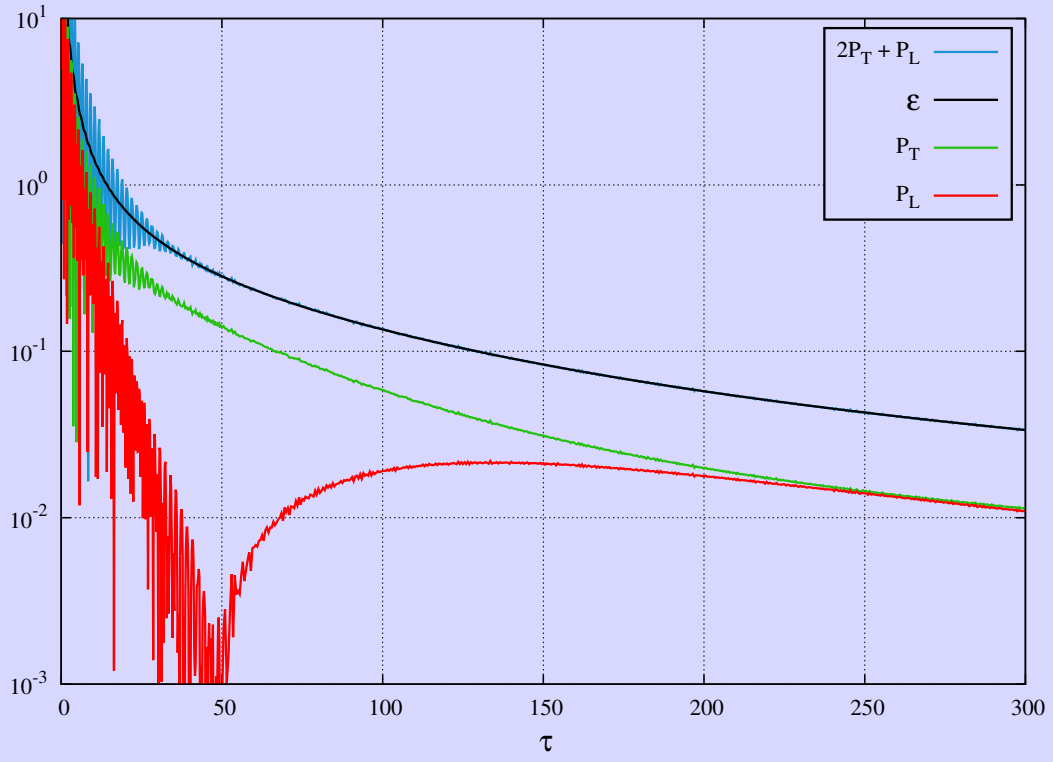
- Note : ν is the Fourier conjugate of the rapidity η
- Initially, only the $\nu = 0$ modes are occupied
- Rapid expansion of the distribution in ν ($\nu_{\max} \sim \tau^{2/3}$ for a thermalized system)

Longitudinal expansion : equation of state



- After a short time, one has $2P_T + P_L \approx \epsilon$
- Change of behavior of the energy density: $\tau^{-1} \rightarrow \tau^{-4/3}$. Since one has $\partial_\tau \epsilon + (\epsilon + P_L)/\tau = 0$, this suggests that P_L gets close to $\epsilon/3$

Longitudinal expansion : isotropization



- At early times, P_L drops much faster than P_T (redshifting of the longitudinal momenta due to the expansion)
- Drastic change of behavior when the expansion rate becomes smaller than the growth rate of the unstability
- Eventually, isotropic pressure tensor : $P_L \approx P_T$

Summary and Outlook

Summary

- Factorization of high energy logarithms in AA collisions
  - universal distributions, also applicable in pA or DIS
  - controls the rapidity dependence of correlations
  - limited to inclusive observables
- Resummation of secular terms in the final state evolution
  - stabilizes the NLO calculation
  - leads to the equilibrium equation of state
  - isotropization even with longitudinal expansion
  - full thermalization on longer time-scales
  - Bose-Einstein condensation if overoccupied initial state

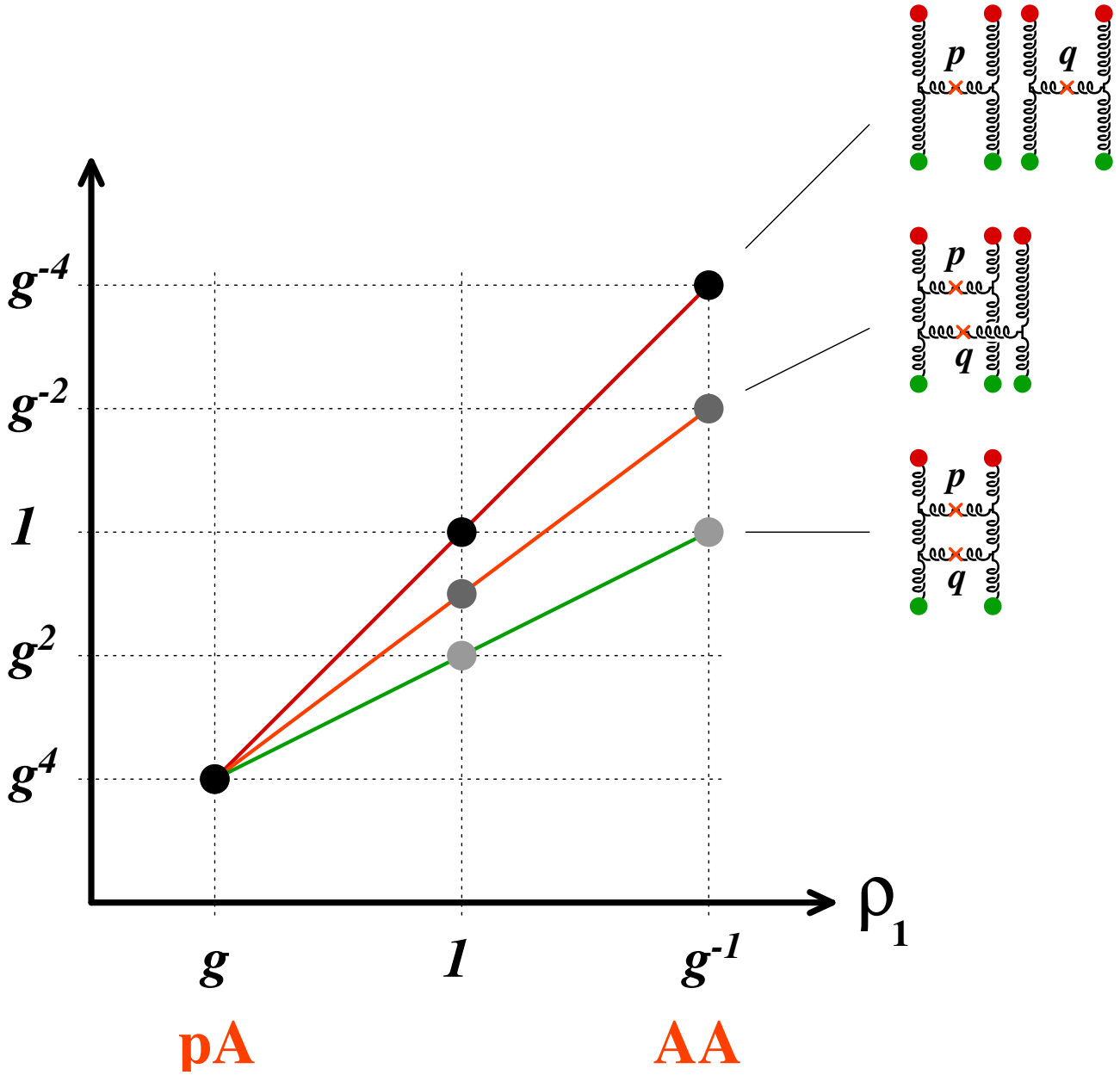
(so far, all numerical studies done for a toy scalar model)

What's next? **QCD**

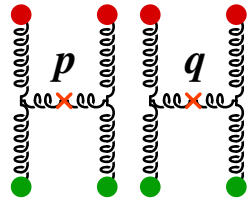
- generalizable to Yang-Mills theory
- gauge invariant
- seamless integration with the CGC description of AA collisions
- computationally expensive ($\sim$ [scalar case] $\times 3 \times (N_c^2 - 1)$)

Extra

Dense-dilute collisions



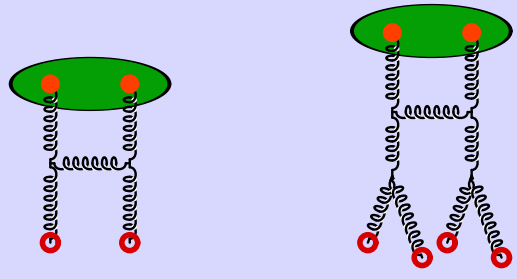
Dense-dilute collisions



Expected complications

- More diagrams to consider even at Leading Order
- More terms in the evolution Hamiltonian if $\rho \sim g$:

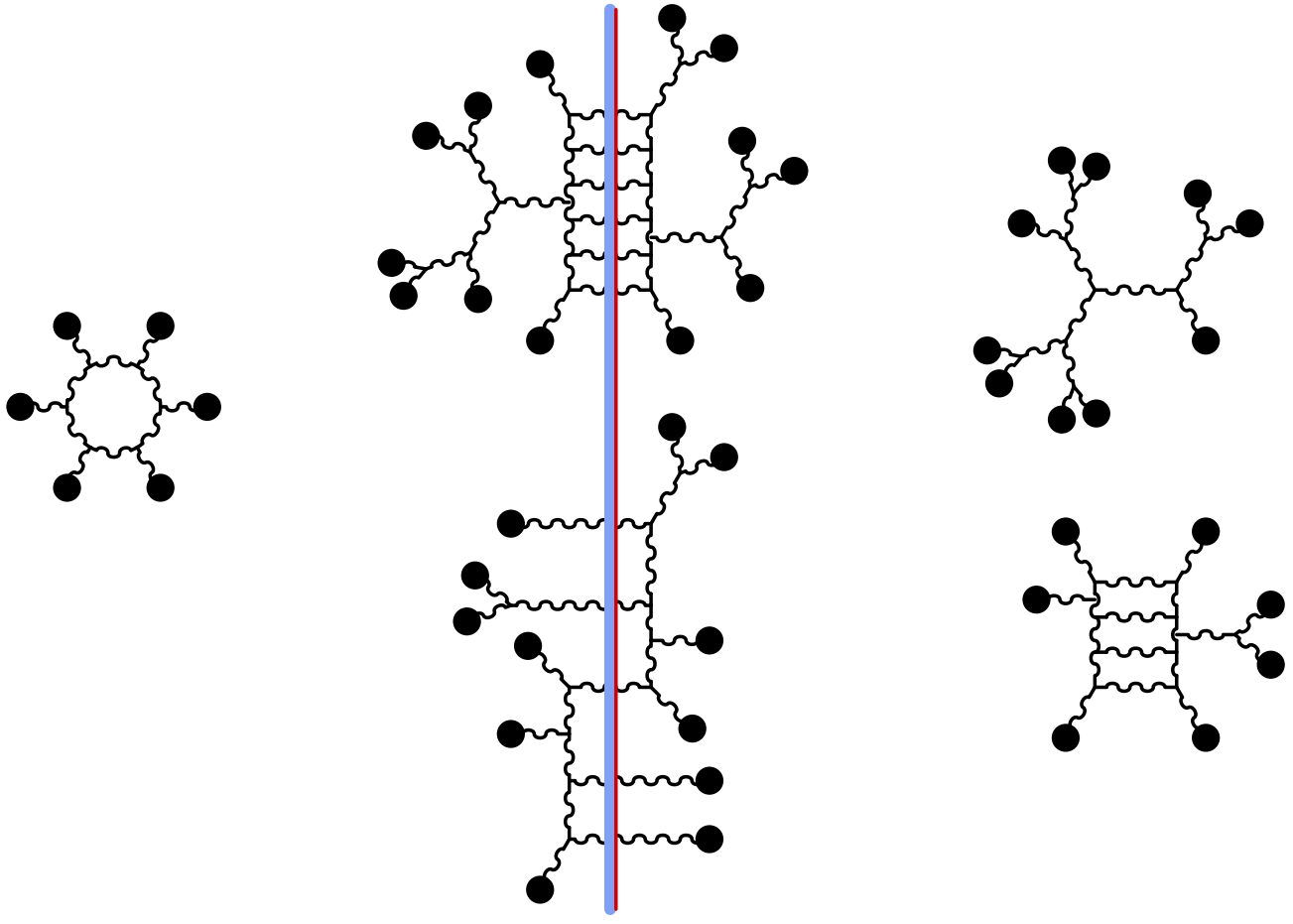
$$g^2\rho^2\left(\frac{\partial}{\partial\rho}\right)^2 \sim g^4\rho^2\left(\frac{\partial}{\partial\rho}\right)^4$$



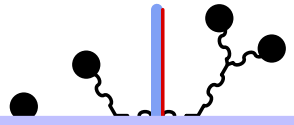
g $\quad$ 1 $\quad$ g^{-1}

pA $\quad$ AA

Exclusive processes



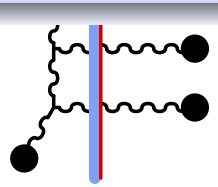
Exclusive processes



Example : differential probability to produce 1 particle at LO

$$\left.\frac{dP_1}{d^3\vec{p}}\right|_{LO} = F[0] \times \int d^4x d^4y \; e^{ip\cdot(x-y)} \, \Box_x \Box_y \, \mathcal{A}_+(x)\mathcal{A}_-(y)\Big|_{z=0}$$

- The vacuum-vacuum graphs do not cancel in exclusive quantities : $F[0] \neq 1$ (in fact, $F[0] = \exp(-c/g^2) \ll 1$)
- $\mathcal{A}_+$ and $\mathcal{A}_-$ are classical solutions of the Yang-Mills equations, but with non-retarded boundary conditions

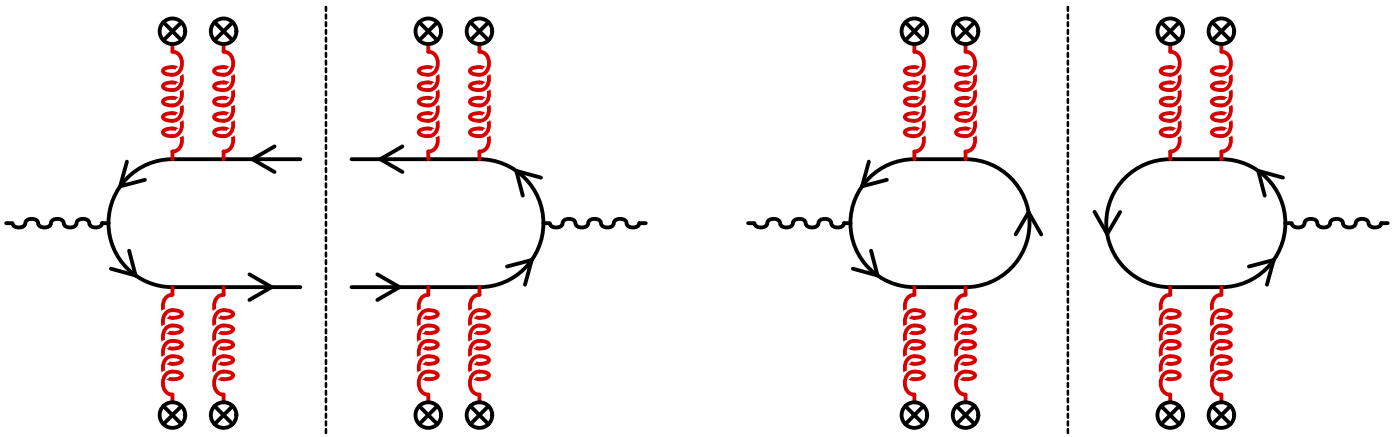


Thermalization in Yang-Mills theory

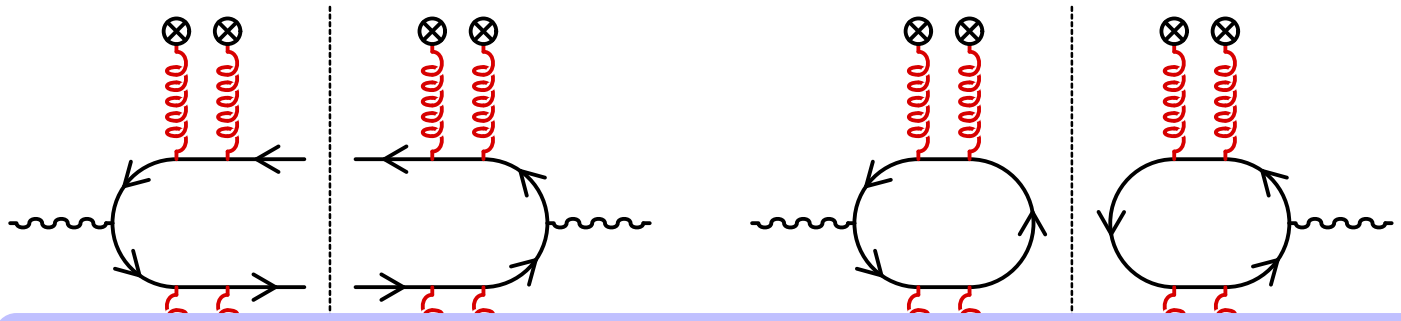
- Recent analytical work : Kurkela, Moore (2011)

- Going from scalars to gauge fields :
  - More fields per site (3 Lorentz components $\times$ 8 colors)
  - More complicated spectrum of initial conditions
  - Expansion : UV overflow on a fixed grid in η

BEC and dilepton production



BEC and dilepton production



Two topologies for virtual photons at LO

Connected	$\omega \sim M_{\rm inv} \sim Q_s$	$k_\perp \sim Q_s$
Disconnected	$\omega \sim M_{\rm inv} \sim Q_s$	$k_\perp \ll Q_s$

▷ excess of dileptons with $k_\perp \ll M_{\rm inv}$

Links to Quantum Chaos

- Quantum Chaos : how does the chaos at the classical level manifests itself in quantum mechanics?

- Berry's conjecture [M.V. Berry (1977)]

  High lying eigenstates of such systems have nearly random wavefunctions. The corresponding Wigner distribution is almost uniform on the energy surface

- Srednicki's eigenstate thermalization hypothesis

  [M. Srednicki (1994)]

  For sufficiently inclusive measurements, these high lying eigenstates look thermal. If the system starts in a coherent state, decoherence is the main mechanism to thermalization

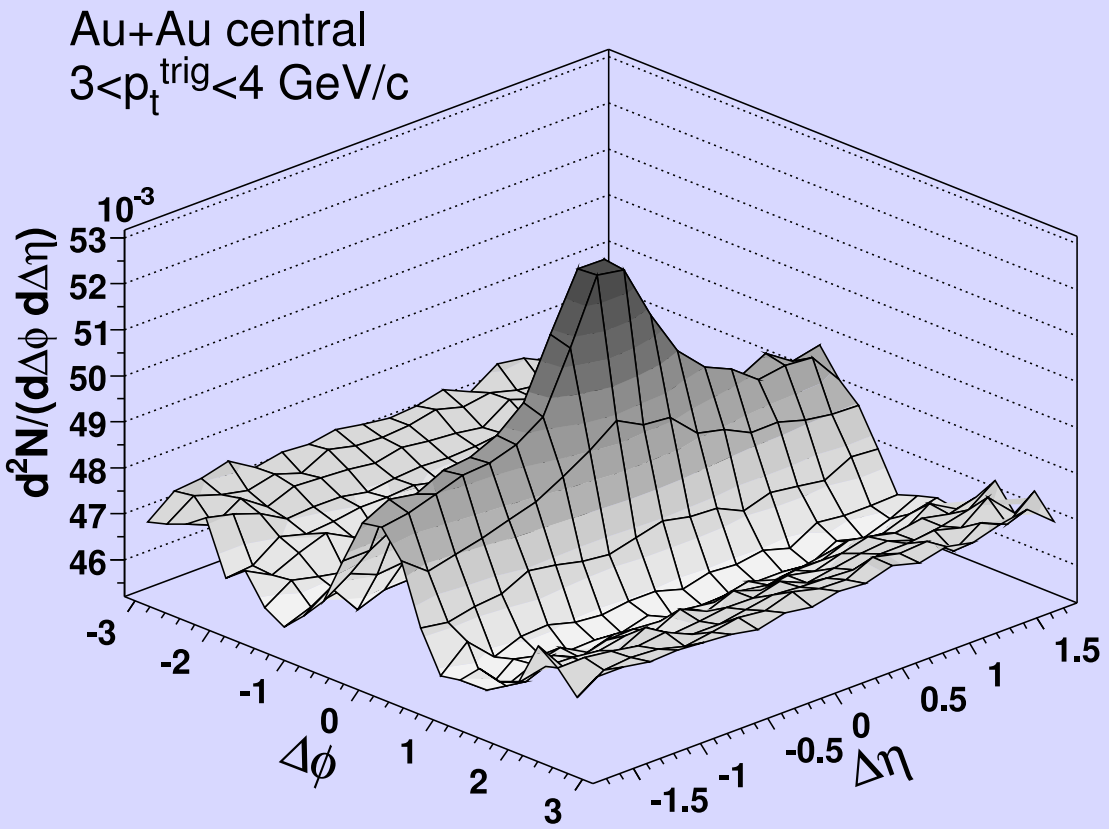
Phenomenology

[Dumitru, FG, McLerran, Venugopalan (2008)]

[Dusling, FG, Lappi, Venugopalan (2010)]

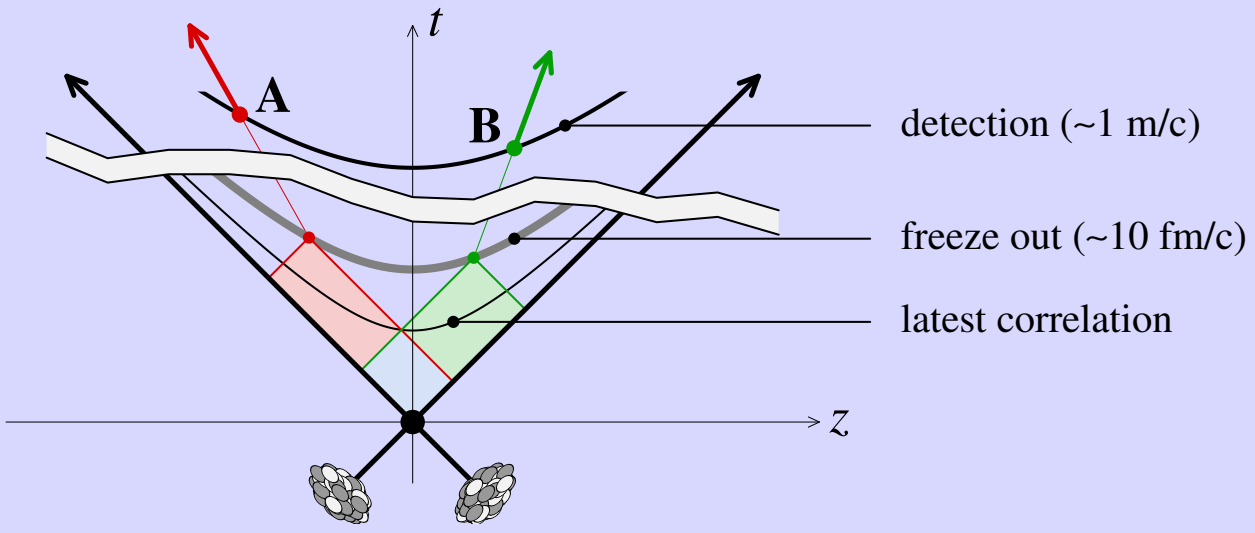
2-particle correlations in AA collisions

[STAR Collaboration, RHIC]



- Long range rapidity correlation
- Narrow correlation in azimuthal angle
- Narrow jet-like correlation near $\Delta y = \Delta\varphi = 0$

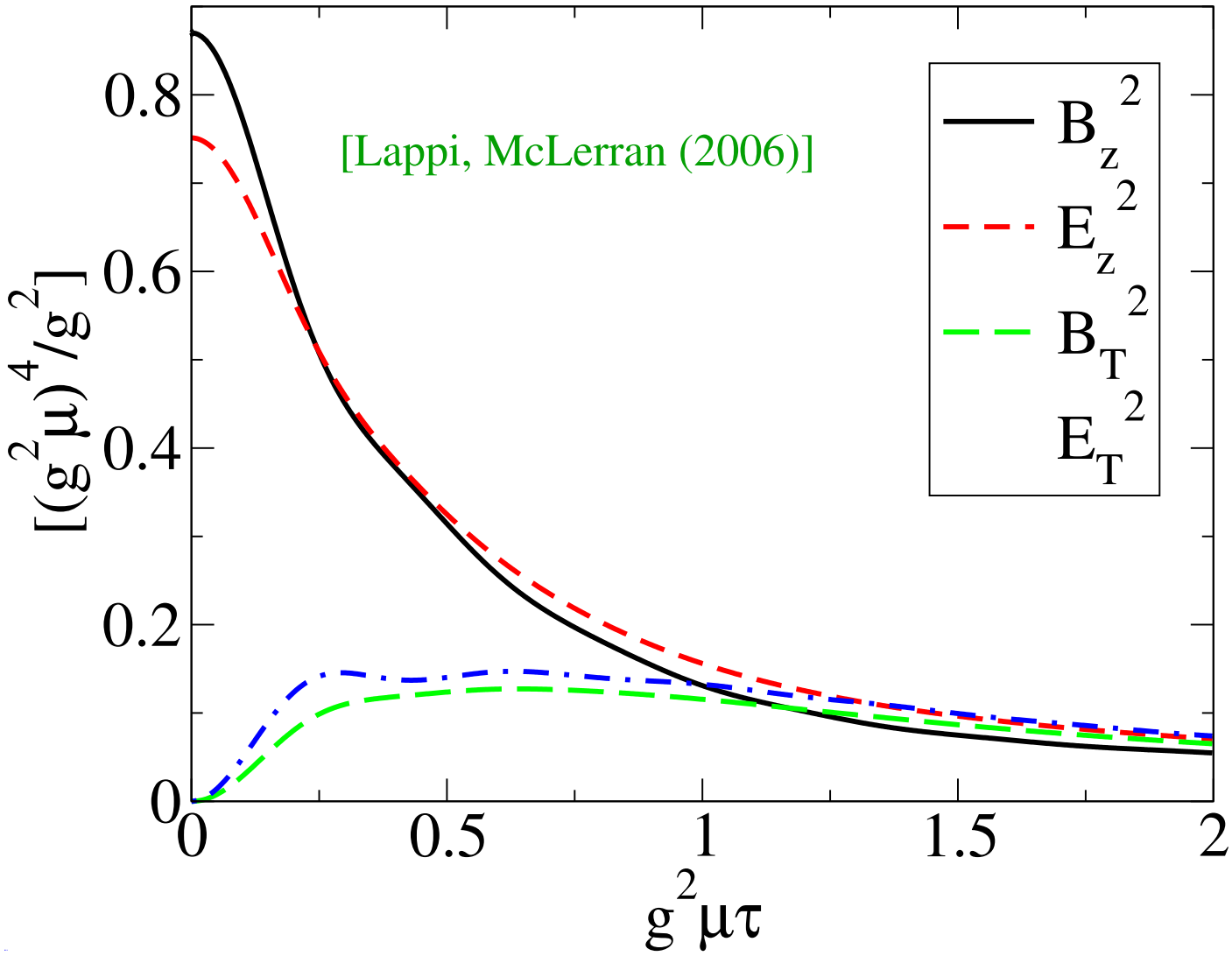
Probing early times with rapidity correlations



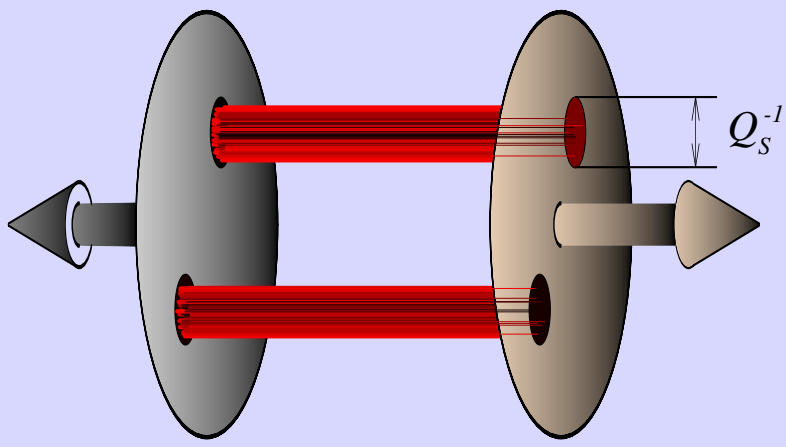
- By causality, long range rapidity correlations are sensitive to the dynamics of the system at early times :

$$\tau_{\text{correlation}} \leq \tau_{\text{freeze out}} \; e^{-|\Delta y|/2}$$

Color field at early time



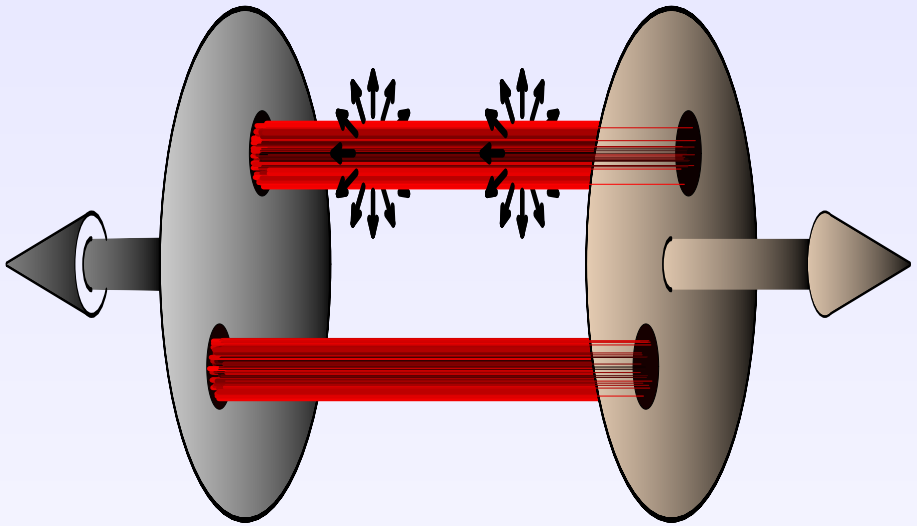
Color field at early time



- The field lines form tubes of transverse size $\sim Q_s^{-1}$
- Rapidity correlation length : $\Delta\eta \sim \alpha_s^{-1}$

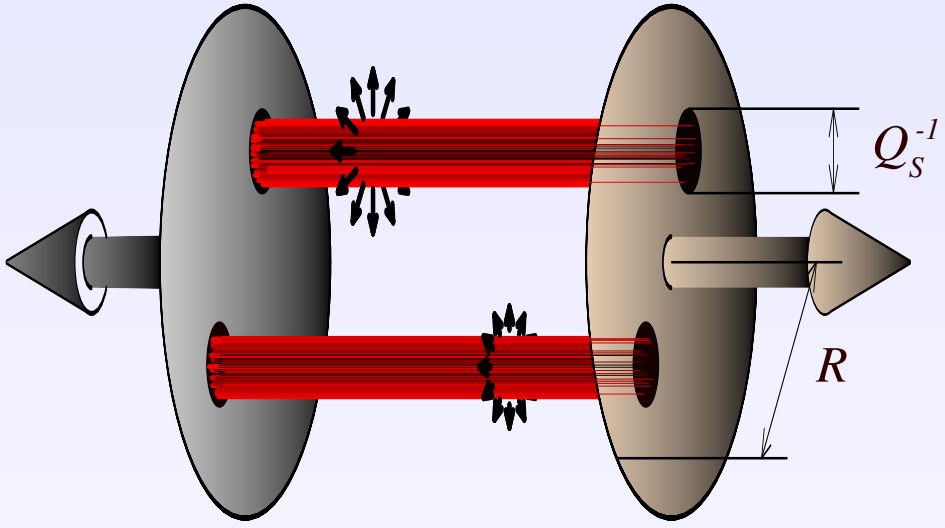
2-hadron correlations from color flux tubes

- η-independent fields lead to long range correlations :



2-hadron correlations from color flux tubes

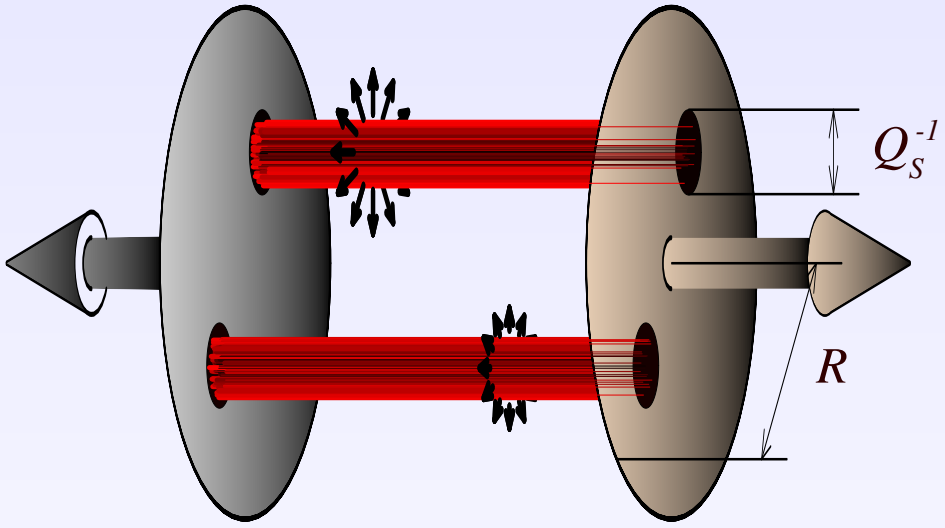
- η-independent fields lead to long range correlations :



- Particles emitted by different flux tubes are not correlated
  - ▷ $(RQ_s)^{-2}$ sets the strength of the correlation

2-hadron correlations from color flux tubes

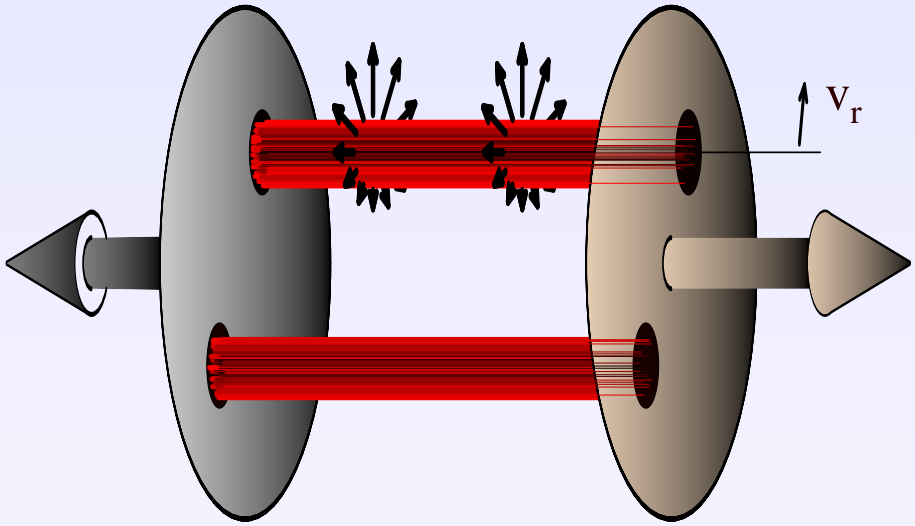
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2-hadron correlations from color flux tubes

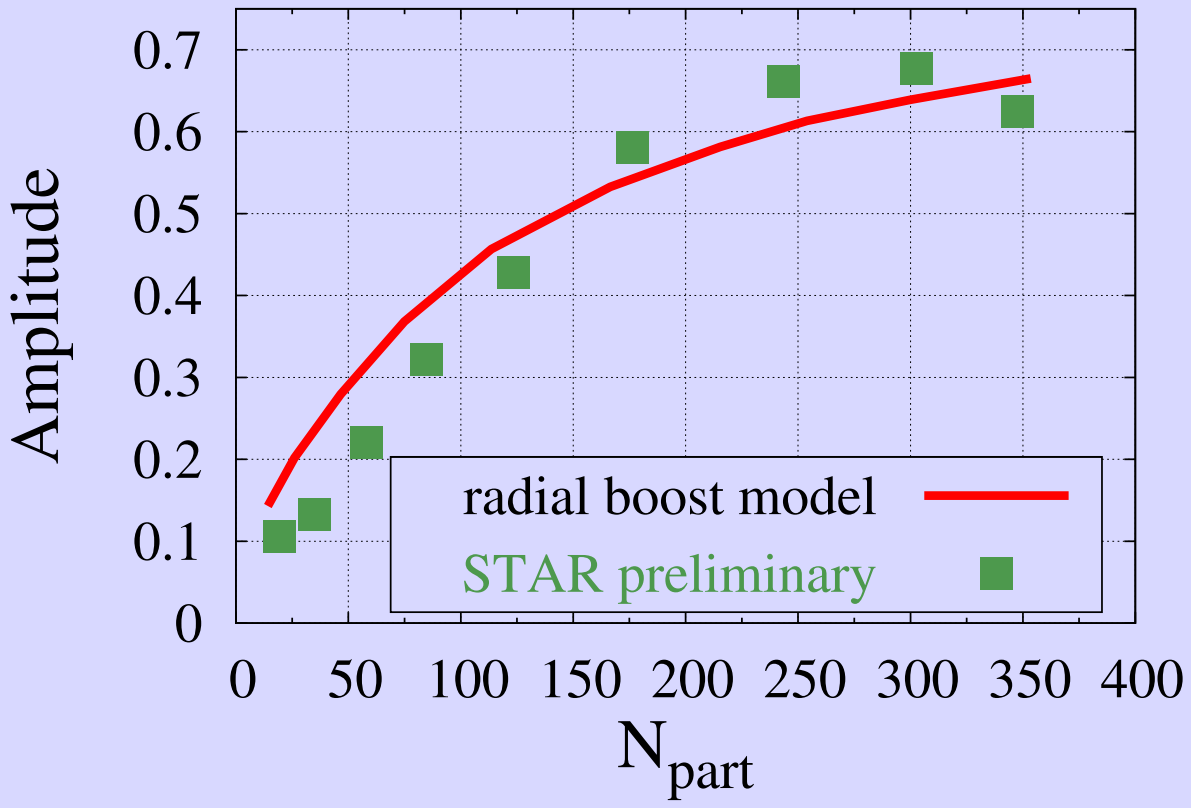
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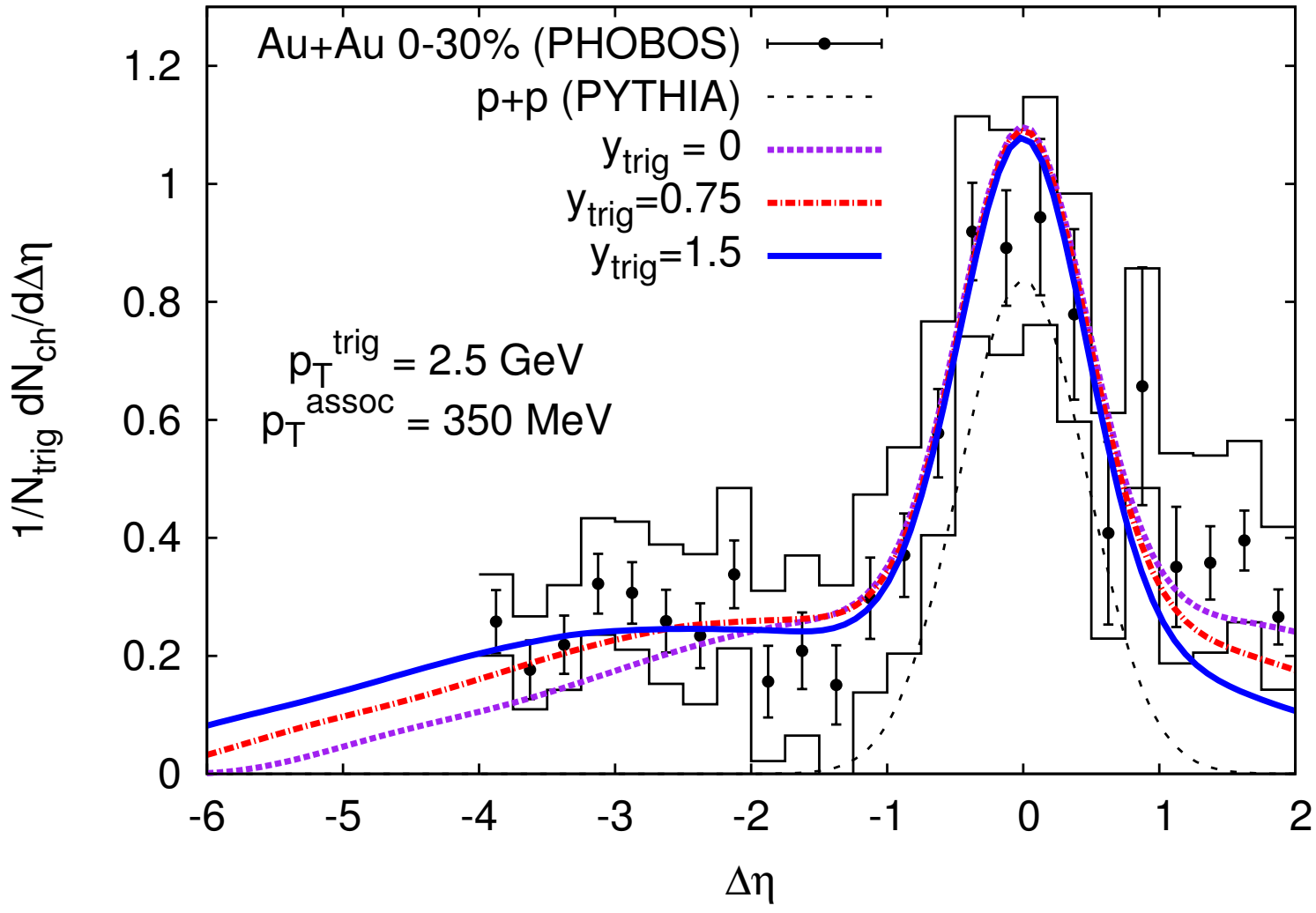
  The collimation in $\Delta\varphi$ is produced later by radial flow

Centrality dependence



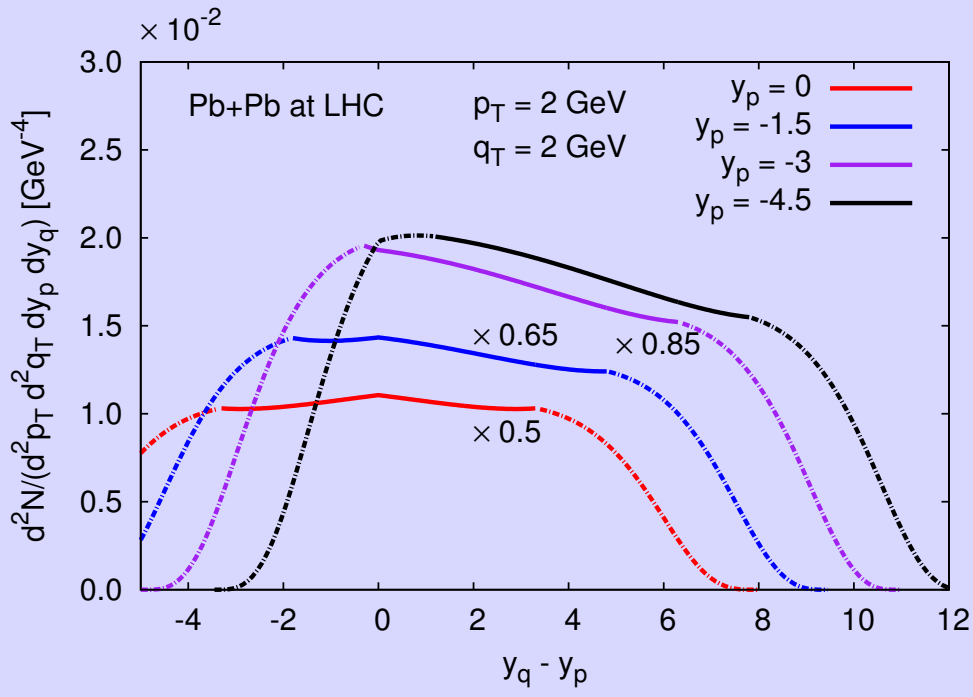
- Main effect : increase of the radial flow velocity with the centrality of the collision

Rapidity dependence



Rapidity dependence

Prediction at LHC energy



Au+Au 0-30% (PHOBOS)

-6 -5 -4 -3 -2 -1 0 1 2

Δη

Backup

Boundary conditions

$$\mathcal{A}_+(x) = \int_y G^0_{++}(x,y)\left[J(y) - V'(\mathcal{A}_+(y))\right] - G^0_{+-}(x,y)\left[J(y) - V'(\mathcal{A}_-(y))\right]$$

$$\mathcal{A}_-(x) = \int_y G^0_{-+}(x,y)\left[J(y) - V'(\mathcal{A}_+(y))\right] - G^0_{--}(x,y)\left[J(y) - V'(\mathcal{A}_-(y))\right]$$

$$\widetilde{G}^0_{+-}(p) = 2\pi z(\mathbf{p})\,\theta(-p_0)\delta(p^2)$$

- $\mathcal{A}_+$ and $\mathcal{A}_-$ are solutions of the classical EoM
- Decompose the fields in Fourier modes :

$$\mathcal{A}_\epsilon(x) \equiv \int \frac{d^3\vec{\mathbf{p}}}{(2\pi)^3 2E_{\mathbf{p}}}\left[a^{(+)}_\epsilon(x_0,\vec{\mathbf{p}})\,e^{-ip\cdot x} + a^{(-)}_\epsilon(x_0,\vec{\mathbf{p}})\,e^{+ip\cdot x}\right]$$

Boundary conditions

$$x^0 = -\infty : \quad a^{(+)}_+(\vec{\mathbf{p}}) = a^{(-)}_-(\vec{\mathbf{p}}) = 0$$

$$x^0 = +\infty : \quad a^{(+)}_-(\vec{\mathbf{p}}) = z(\mathbf{p})\,a^{(+)}_+(\vec{\mathbf{p}})\,, \quad a^{(-)}_+(\vec{\mathbf{p}}) = z(\mathbf{p})\,a^{(-)}_-(\vec{\mathbf{p}})$$

Moyal bracket

- Wigner transform of the commutator
- Explicit expression :

$$\{\{A,B\}\} = \frac{2}{\hbar} A(Q,P)\ \sin\left(\frac{\hbar}{2}(\overleftarrow{\nabla}_Q \overrightarrow{\nabla}_P - \overleftarrow{\nabla}_P \overrightarrow{\nabla}_Q)\right)\ B(Q,P)$$

- Quantum deformation of the Poisson bracket :

$$\{\{A,B\}\} = \{A,B\} + \mathcal{O}(\hbar^2)$$