Transport Theory of Resonant Leptogenesis from Nonequilibrium QFT

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in collaboration with

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Motivation

- Standard approach to leptogenesis calculations is to use Boltzmann equations with vacuum S-matrix elements.

Goal:

First principle description of leptogenesis, including:

- Finite density medium effects
- Flavour coherence effects
- Memory effects, etc.

Questions:

- When Boltzmann description breaks?
- Are finite widths required in the resonant regime: $|M_i - M_j| \sim \Gamma$, even though $M_i \gg \Gamma$?
- Do flavour oscillations in the singlet neutrino sector play any role?

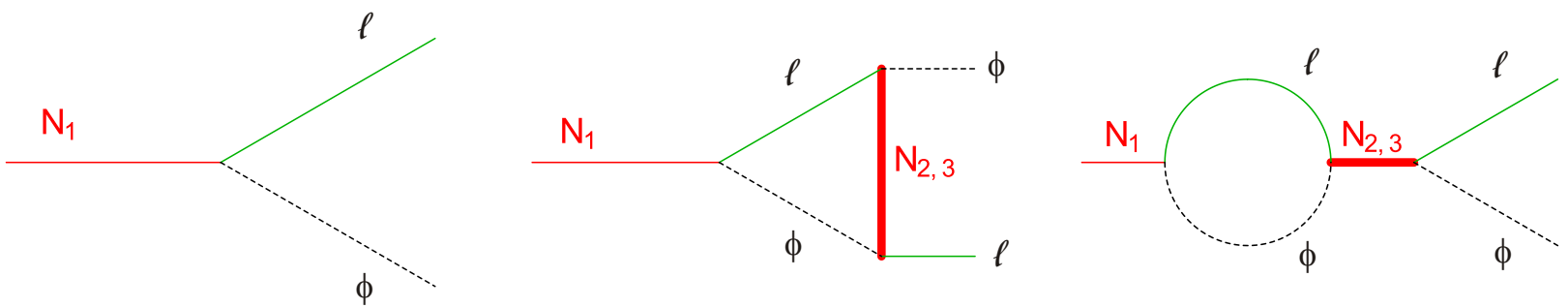
Leptogenesis [Fukugita, Yanagida (1986)]

- The Standard Model is extended by adding heavy right-handed Majorana neutrinos N_i, $i = 1, 2, \ldots$ (eg. see-saw models)

$$\mathcal{L} \supset \frac{1}{2}\bar{\psi}_{Ni}\left(\mathrm{i}\partial\!\!\!/ - M_i\right)\psi_{Ni} - Y^*_{ia}\bar{\psi}_{\ell a}(\epsilon\phi)^\dagger\psi_{Ni} + \mathrm{h.c.}$$

- Lepton asymmetry is generated through out-of-equilibrium L- and CP-violating Yukawa decays: $N_1 \to \ell\phi$

- Resonant enhancement of CP-violation for nearly degenerate neutrino masses: $|M_i - M_j| \sim \Gamma$



Lots of recent progress towards more reliable calculational scheme:

- Finite density medium / thermal effects
  - Buchmüller, Fredenhagen (2000)
  - Asaka, Laine, Shaposhnikov (2006)
  - Garny, Hohenegger, Kartavtsev, Lindner (2009 & 2010)
  - Garny, Hohenegger, Kartavtsev (2010 & 2011)
  - Beneke, Garbrecht, MH, Schwaller (2010)
  - Garbrecht (2010)
  - Anisimov, Besak, Bödeker (2010)
  - Besak, Bödeker (2012)
  - Kiessig, Plumacher, Thoma (2010)
  - Kiessig, Plumacher (2011)
  - Laine, Schroder (2011)
- Flavour effects
  - De Simone, Riotto (2007)
  - Beneke, Garbrecht, Fidler, MH, Schwaller (2010)
- Finite width / memory effects
  - Anisimov, Buchmüller, Drewes, Mendizabal (2010 & 2011)
  - Garny, Hohenegger, Kartavtsev (2011)

Closed Time Path* (CTP) Formalism

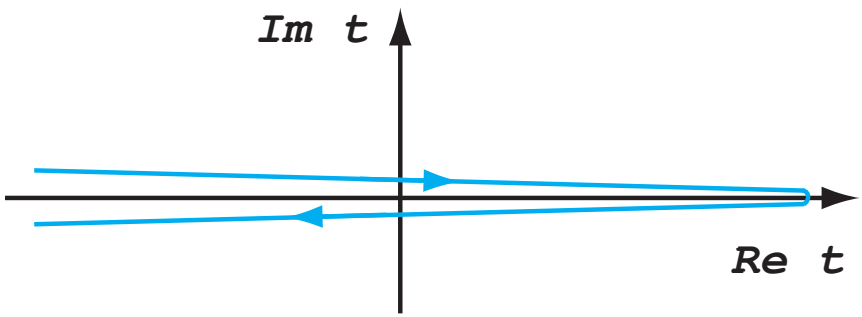
* a.k.a. Schwinger-Keldysh Formalism [Schwinger (1961); Keldysh (1964); Calzetta, Hu (1988)]

- In out-of-equilibrium problems the interesting quantities are expectation values, e.g.

$$j^0(x) = \langle \bar{\psi}(x)\gamma^0\psi(x)\rangle \equiv \mathrm{tr}\left[\hat{\rho}\,\bar{\psi}(x)\gamma^0\psi(x)\right]$$

  - $\hat{\rho}$ is an (unknown) quantum density operator

$\longrightarrow$ In-In generating functional on a Closed Time Path:

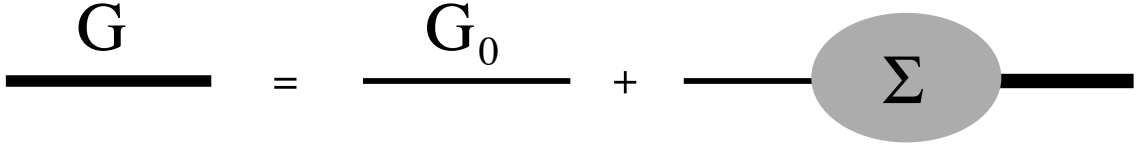


Path-ordered correlators:

$$\mathrm{i}S_{\mathcal{C}}(x,y) = \langle T_{\mathcal{C}}[\psi(x)\bar{\psi}(y)]\rangle$$

Kadanoff-Baym Equations

- Generic Schwinger-Dyson equations for (CTP) propagators:



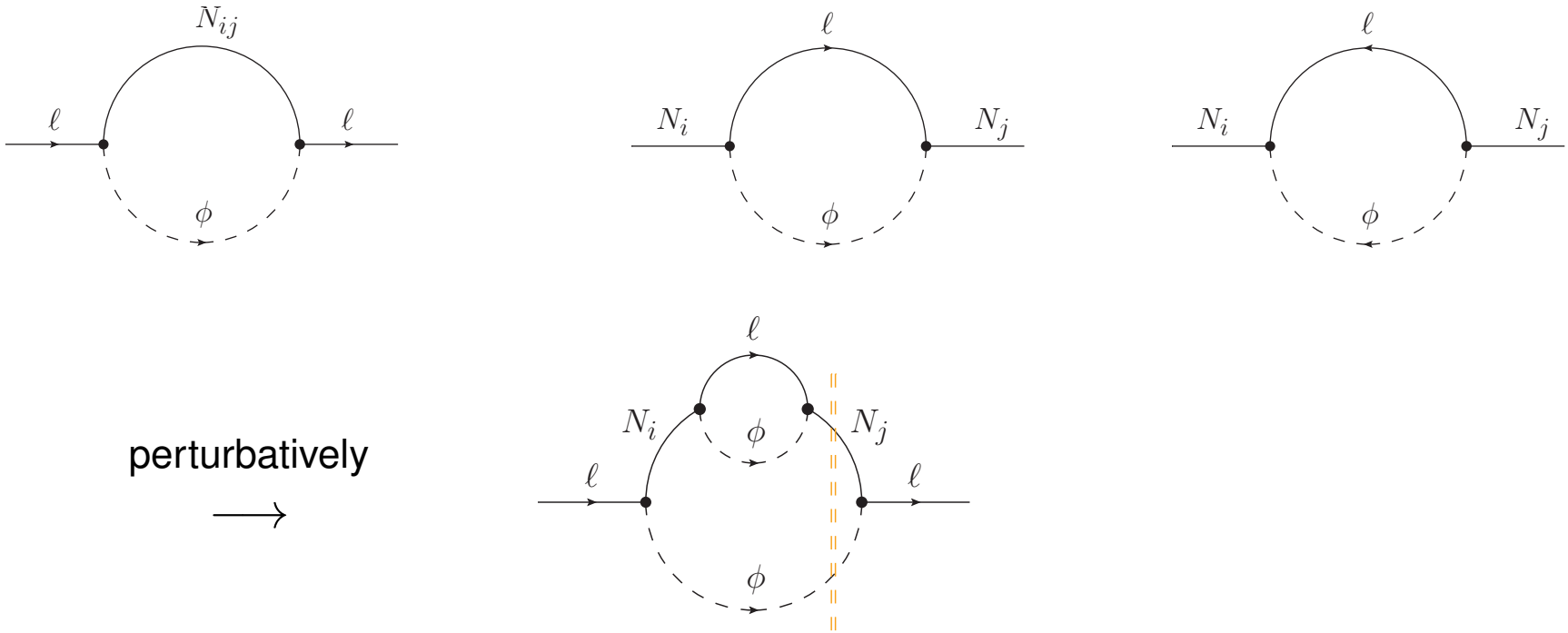
- Kadanoff-Baym equations are the $<,>$ components:

$$(\mathrm{i}\partial\!\!\!/ - m)S^{<,>} - \Sigma^{H} \odot S^{<,>} - \Sigma^{<,>} \odot S^{H} = \frac{1}{2}\left(\Sigma^{>} \odot S^{<} - \Sigma^{<} \odot S^{>}\right)$$

  - Renormalization, thermal corrections (thermal masses etc.)
  - Finite width effects
  - Collision term

Application to resonant leptogenesis

- 2PI-type approach: one-loop self-energies, N_{ij} is independent variable
  [Garbrecht, MH (2011); Garny, Hohenegger, Kartavtsev (2011); MH, Kainulainen, Rahkila (in progress)]



- Wave-function type CP-violation arises from N_i flavour mixing

Approximations

Relevant scales:

- External scale: Hubble expansion rate $H \sim T^2/M_{\rm pl}$
- Internal scale: momentum $k \sim T$

$$\Longrightarrow \quad H \ll k$$

- In addition: For weak coupling the interaction rates:

$$\Gamma \ll k$$

During the leptogenesis at $T > \Lambda_{\rm EW}$:

$$k \gg \Gamma_g \gg \Gamma_Y \sim H$$

- ℓ and ϕ fields remain in kinetic equilibrium due to fast gauge interactions

To separate the scales, we perform Wigner transform ($x \equiv (u+v)/2$):

$$S(k,x) = \int d^4(u-v)\, e^{ik\cdot(u-v)}\, S(u,v)$$

and

Gradient expansion to the lowest order in

- (conformal) time-derivatives: $\partial_t M_i(t)$, $\partial_t \Sigma(k,t)$.
- coupling constants in $\Sigma(k,t)$
- chemical potentials: μ_L and μ_H

Additionally:

- We neglect dispersive self-energy Σ^h corrections

KB-equations $\longrightarrow$ flavour-mixing Constraint and Kinetic Equations for N_i:

$$\text{(CE)} \qquad 2k_0\bar{S}^< = \left\{H, \bar{S}^<\right\} + \left(e^{\frac{i}{2}\partial^{\Sigma}_{k_0}\cdot\partial^{S}_{t}}\bar{\Sigma}^<\bar{S}^h - ie^{\frac{i}{2}\partial^{\Sigma}_{k_0}\cdot\partial^{S}_{t}}\bar{\Sigma}^{\mathcal{A}}(\bar{S}^< - \bar{S}^<_{\rm eq}) + \text{h.c.}\right)$$

$$\text{(KE)} \qquad i\partial_t\bar{S}^< = \left[H, \bar{S}^<\right] + \left(e^{\frac{i}{2}\partial^{\Sigma}_{k_0}\cdot\partial^{S}_{t}}\bar{\Sigma}^<\bar{S}^h - ie^{\frac{i}{2}\partial^{\Sigma}_{k_0}\cdot\partial^{S}_{t}}\bar{\Sigma}^{\mathcal{A}}(\bar{S}^< - \bar{S}^<_{\rm eq}) - \text{h.c.}\right)$$

where

$$\bar{S}^<(k,t) \equiv iS^<(k,t)\gamma^0\,, \qquad H_{ij} \equiv \left(\mathbf{k}\cdot\alpha + M_i\gamma^0\right)\delta_{ij}$$

Kinetic equation for lepton asymmetry:

$$\partial_t\left(n_\ell - \bar{n}_\ell\right) = -\frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\text{tr}\left[\text{i}\Sigma^>_\ell(k)P_{\rm L}\text{i}S^<_\ell(k) - \text{i}\Sigma^<_\ell(k)P_{\rm L}\text{i}S^>_\ell(k)\right] + h.c.$$

$$\equiv -\mathcal{W}\left(n_\ell - \bar{n}_\ell\right) + \mathcal{S}_{CP}$$

Non-resonant regime: $|M_i - M_j| \gg \Gamma$

- Rapidly oscillating flavour-coherence solutions can be neglected

- Perturbative solution to the constraint equation:

$$i\delta S^{<}_{ii}(k,t) = -2\pi(\not{k} + M_i)\delta f_i(\mathbf{k},t)\delta(k^2 - M_i^2)$$

$$i\delta S^{<}_{ij}(k,t) = \frac{i}{M_i^2 - M_j^2}\left[(\not{k} + M_i)\Sigma^{\mathcal{A}}_{ij}(k) i\delta S^{<}_{jj}(k,t) + i\delta S^{<}_{ii}(k,t)\Sigma^{\mathcal{A}}_{ij}(k)(\not{k} + M_j)\right]$$

- Standard result for *CP*-violating source:

$$\mathcal{S}^{\rm wf}_{CP} = 8\,{\rm Im}[Y_1^2 Y_2^{*2}]\frac{M_1 M_2}{M_1^2 - M_2^2}\sum_{i=1,2}\int \frac{d^3k}{(2\pi)^3\,2\omega_i}\Sigma_\mu(k_i)\,\Sigma^\mu(k_i)\,\delta f_i(\mathbf{k},t)$$

Resonant regime: $|M_i - M_j| \lesssim \Gamma$

- Flavour-coherence important!

- Lowest order solution to the constraint equation:

$$i\delta S^{<}_{ij}(k,t) = -2\pi \sum_{h=\pm} P_h(\not{k} + \bar{M}_{ij})\delta f_{h\,ij}(\mathbf{k},t)\delta(k^2 - \bar{M}^2_{ij})$$

- Neutrino flavour-mixing dynamics from local kinetic equation:

$$\partial_t \delta f_h = -\frac{i}{2\omega}\left[M^2, \delta f_h\right] - \left\{\Gamma_h, \delta f_h\right\} - \partial_t f_{\rm eq}$$

- *CP*-violating source:

$$\mathcal{S}_{CP} = \sum_{i,j}\sum_{h=\pm} Y_i^* Y_j \int \frac{d^3k}{(2\pi)^3\, 2\omega}\left[k^\mu(\delta f_{h\,ij} - \delta f^*_{h\,ij}) + h\tilde{k}^\mu(\delta f_{h\,ij} + \delta f^*_{h\,ij})\right]\Sigma_\mu(k)$$

$$\bar{M}_{ij} \equiv (M_i + M_j)/2\,, \qquad P_h \equiv \tfrac{1}{2}(1 + h\,\hat{\mathbf{k}}\cdot\gamma^0\gamma\gamma^5)\,, \qquad \Gamma_{h\,ij}(\mathbf{k}) \equiv \left(\mathrm{Re}[Y_i Y_j^*]\,\frac{k^\mu}{\omega} + ih\,\mathrm{Im}[Y_i Y_j^*]\,\frac{\tilde{k}^\mu}{\omega}\right)\Sigma_\mu(k)$$

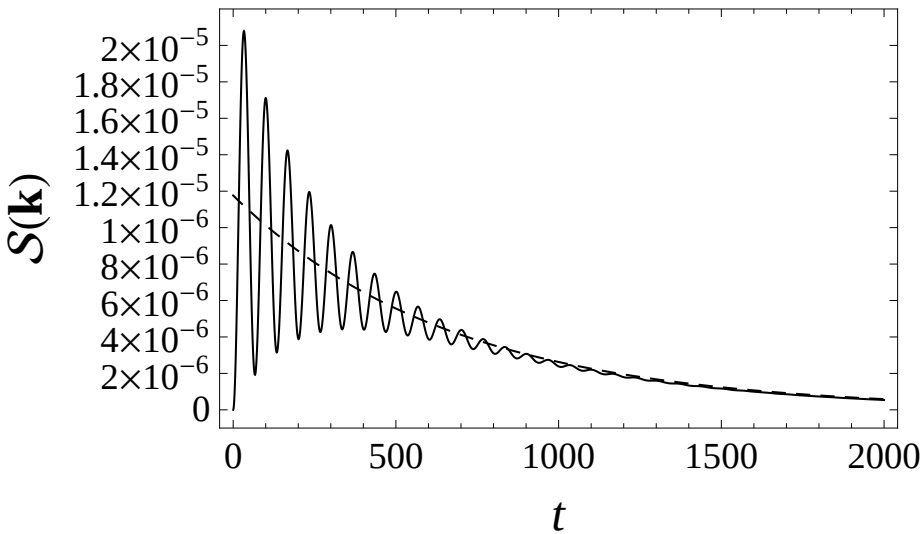
Toy model: constant background

- Initial conditions (N_1 vacuum, N_2 thermal):

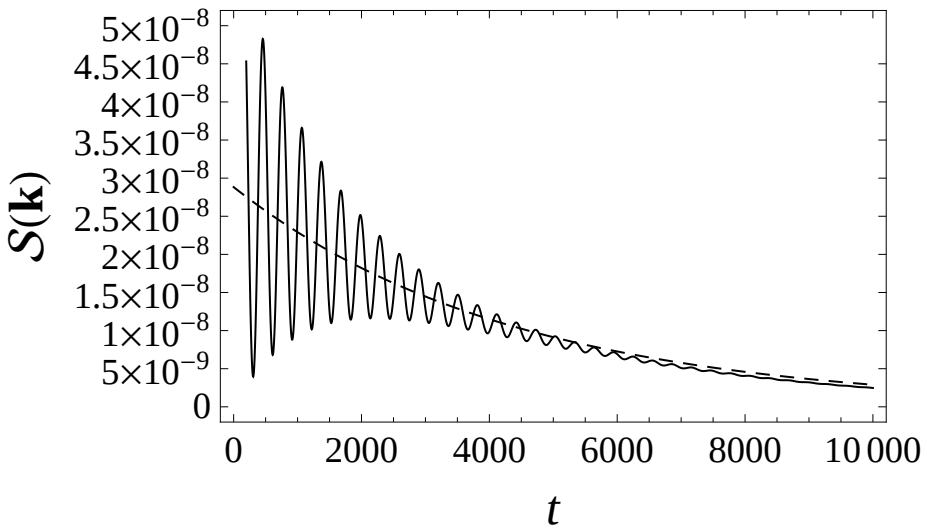
$$\delta f_{h\,11}(\mathbf{k},0) = -f_{\rm eq}(\mathbf{k})\,, \qquad \delta f_{h\,22}(\mathbf{k},0) = \delta f_{h\,12}(\mathbf{k},0) = \delta f_{h\,21}(\mathbf{k},0) = 0$$

- Non-resonant regime: $|M_2 - M_1| \gg \Gamma$

$$M_1 = T\,, \qquad M_2 = 1.1\,T\,, \qquad Y_1 = 0.1\,, \qquad Y_2 = 0.2 + 0.1\mathrm{i}$$



Non-relativitic case: $|\mathbf{k}| = 0.5\,T$



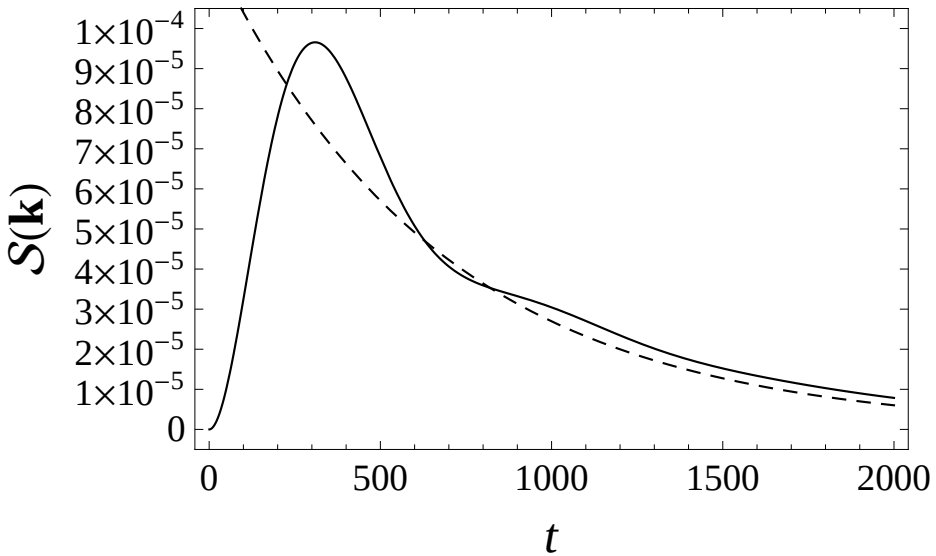
Relativitic case: $|\mathbf{k}| = 5\,T$

—— full kinetic equations

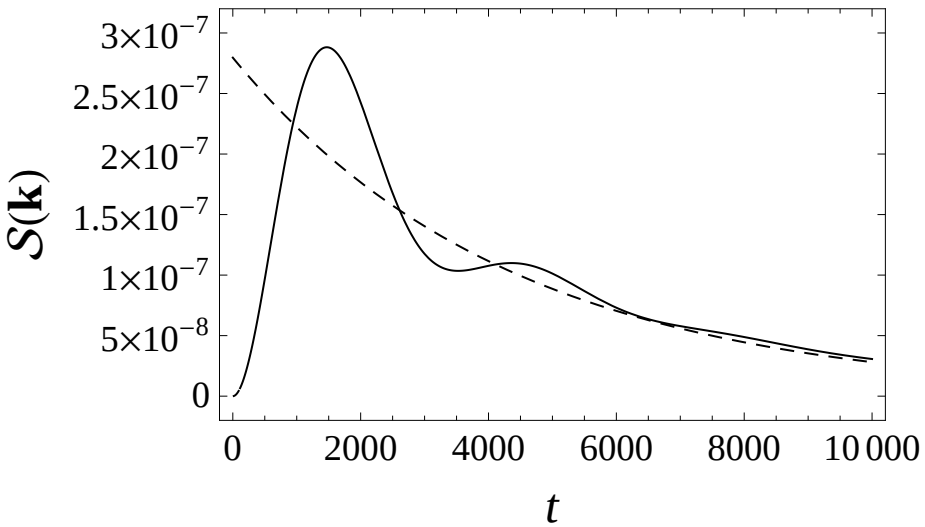
--- peturbative result

- Resonant regime: $|M_2 - M_1| \sim \Gamma$

$$M_1 = T\,, \qquad M_2 = 1.01\,T\,, \qquad Y_1 = 0.1\,, \qquad Y_2 = 0.2 + 0.1\mathrm{i}$$



Non-relativitic case: $|\mathbf{k}| = 0.5\,T$



Relativitic case: $|\mathbf{k}| = 5\,T$

- In resonant regime the correction to perturbative result can be sizeble

- In the limit $|\Gamma_{12}| \ll |\Gamma_{11} - \Gamma_{22}|$ and $|\Gamma_{12}| \lesssim |M_1 - M_2|$, we get:

$$\mathcal{S}_{CP} = -\,8\,\mathrm{Im}[Y_1^2 Y_2^{*2}] \int \frac{d^3k}{(2\pi)^3\,2\omega}\, \frac{M_1 M_2 (M_1^2 - M_2^2)}{(M_1^2 - M_2^2)^2 + \omega^2(\Gamma_{11} - \Gamma_{22})^2} \Sigma_\mu(k)\Sigma^\mu(k)$$
$$\times \left[e^{-\Gamma_{11} t} - \left(\cos(\Delta\omega\, t) + \frac{\Gamma_{11} - \Gamma_{22}}{2\Delta\omega} \sin(\Delta\omega\, t) \right) e^{-(\Gamma_{11}+\Gamma_{22})t/2} \right] f_{\mathrm{eq}}(\mathbf{k})$$

where $\Delta\omega(\mathbf{k}) \equiv |M_1^2 - M_2^2|/(2\omega(\mathbf{k}))$

- The 'resonant' structure of the denominator is in agreement with previous works [Buchmuller, Plumacher (1997); Anisimov, Broncano, Plumacher (2005); Garny, Hohenegger, Kartavtsev (2011)]

- The new terms arise due to singlet neutrino flavour oscillations

  $\longrightarrow$ Important in the resonant regime: $\Delta\omega \sim \Gamma$

Conclusions and outlook

Conclusions:

- Nonequilibrium QFT approach to resonant leptogenesis:
  - LO dynamics can be described by local transport equations within quasiparticle picture(?)
  - Resonant enhancement is obtained through N_i flavour-mixing dynamics (observed for mixing scalar fields in [Liu, Segre (1993); Covi, Roulet (1996)])
- In non-resonant regime: $|M_i - M_j| \gg \Gamma$, the standard perturbative result is recovered
- In resonant regime: $|M_i - M_j| \lesssim \Gamma$, neutrino N_i flavour oscillations can be important

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Outlook:

- Systematic inclusion of thermal effects
- Spectator processes

One-loop self-energies

- Neutrino N_i self-energy:

$$\mathrm{i}\Sigma_{ij}^{<,>}(k) = g_w \int \frac{d^4k'}{(2\pi)^4}\frac{d^4k''}{(2\pi)^4}(2\pi)^4\delta^{(4)}(k-k'-k'') \times \left\{ Y_i Y_j^* P_\mathrm{L}\mathrm{i}S_\ell^{<,>}(k')P_\mathrm{R}\mathrm{i}\Delta_\phi^{ab}(k'') - Y_i^* Y_j P_\mathrm{R} C \mathrm{i}S_\ell^{>,<T}(-k')C^\dagger P_\mathrm{L}\mathrm{i}\Delta_\phi^{>,<}(-k'') \right\}$$

$\longrightarrow$

$$\Sigma_{ij}^{\mathcal{A}}(k) = \frac{1}{2}(\mathrm{i}\Sigma_{ij}^{>}(k) - \mathrm{i}\Sigma_{ij}^{<}(k)) \equiv (Y_j^* Y_i P_\mathrm{L} + Y_i^* Y_j P_\mathrm{R})\gamma^\mu \Sigma_\mu(k)$$

$$\Sigma^\mu(k) = \int \frac{d^3p}{(2\pi)^3 2|\mathbf{p}|}\frac{d^3q}{(2\pi)^3 2|\mathbf{q}|}(2\pi)^4\delta^4(k-p-q)\,p^\mu \left(1 - f_\ell^{\mathrm{eq}}(\mathbf{p}) + f_\phi^{\mathrm{eq}}(\mathbf{q})\right)$$

- Charged lepton ℓ self-energy (lowest order in gradients):

$$\mathrm{i}\Sigma_\ell^{<,>}(k,t) = \int \frac{d^4k'}{(2\pi)^4}\frac{d^4k''}{(2\pi)^4}(2\pi)^4\delta^4(k-k'-k'')Y_i^* Y_j P_\mathrm{R}\mathrm{i}S_{ij}^{<,>}(k',t)P_\mathrm{L}\mathrm{i}\Delta_\phi^{>,<}(-k'')$$