News on electroweak baryogenesis

Stephan Huber, University of Sussex

SEWM 2012

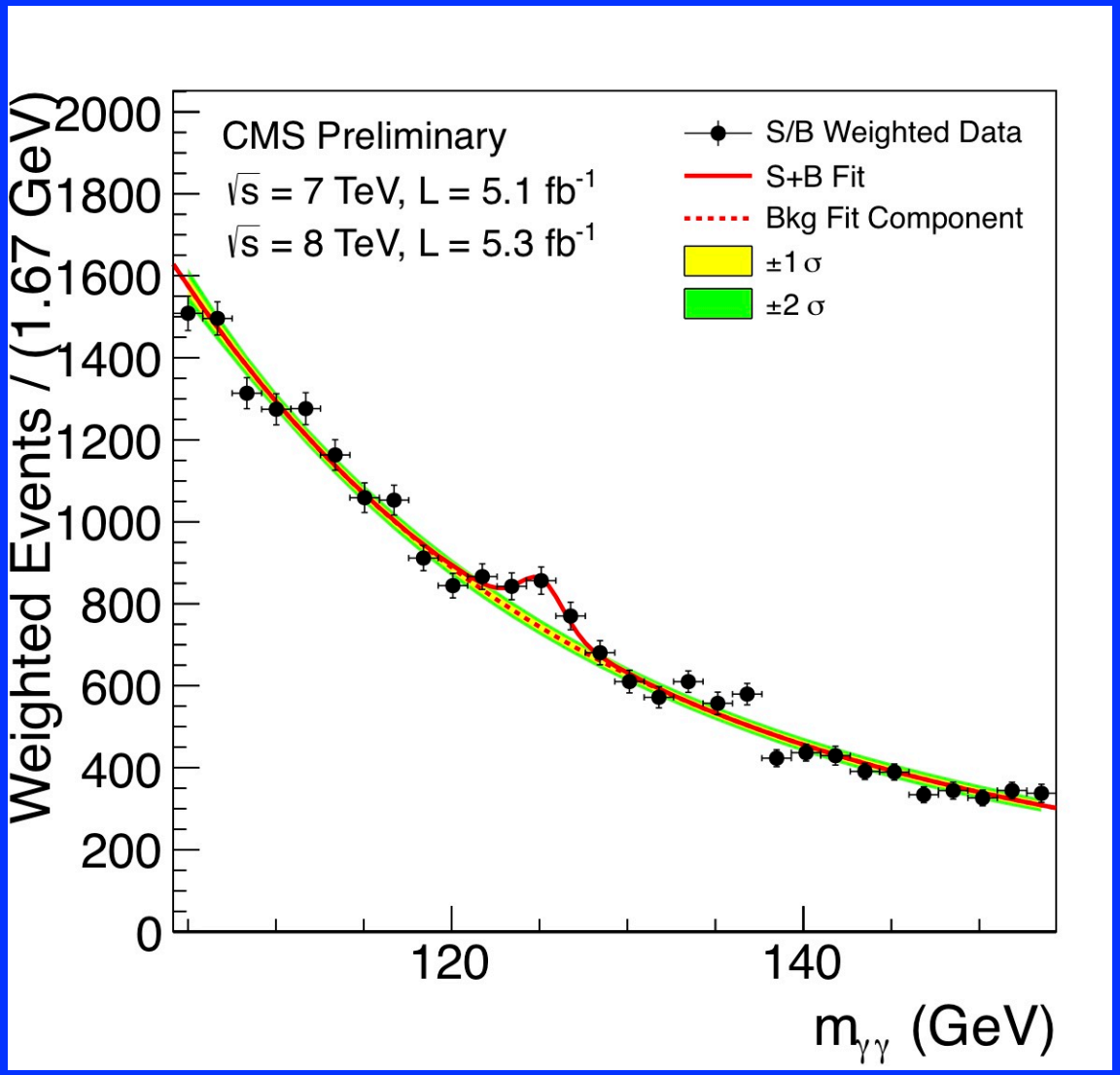
Swansea, July 2012

Outline

- Electroweak phase transition
- Electroweak baryogenesis
- Models: Two Higgs doublet model
  SM with higher-dimensional operators
  MSSM +extensions
- wall velocity
- Gravitational waves from the electroweak phase transition
- Summary & outlook

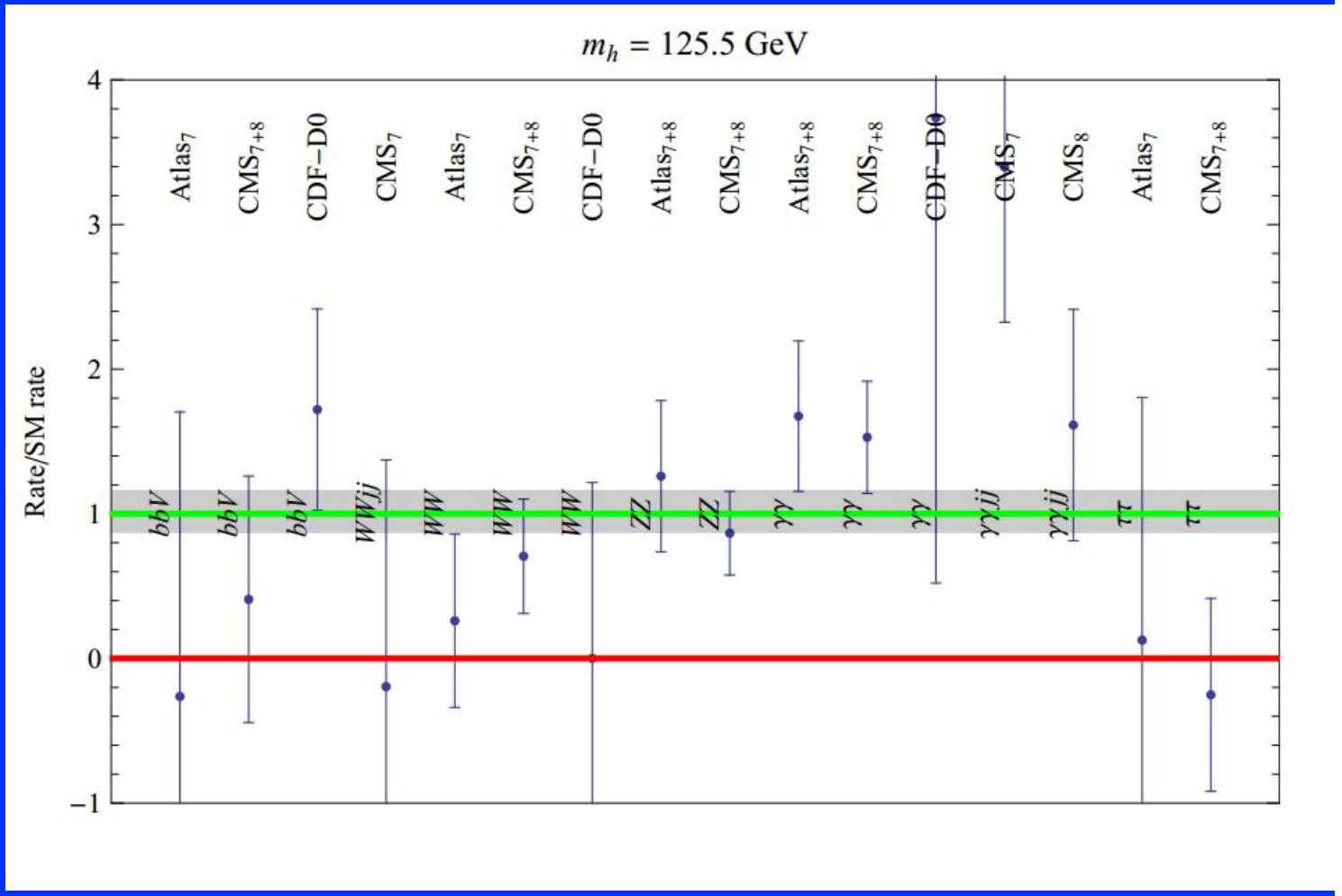
Here it is:

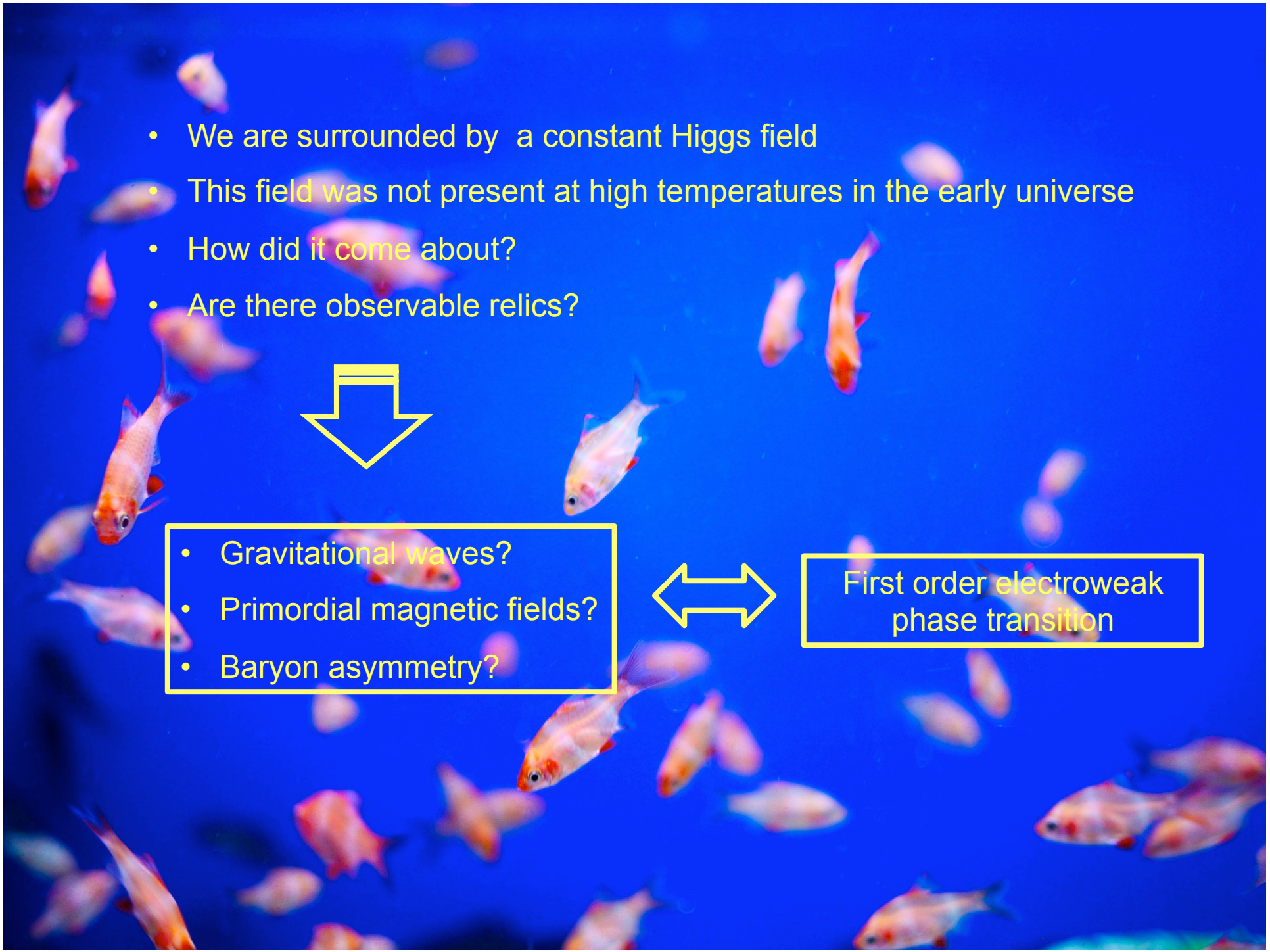
The Higgs boson



m_H=125.3 ± 0.6 GeV

Looking closer….




• We are surrounded by a constant Higgs field
• This field was not present at high temperatures in the early universe
• How did it come about?
• Are there observable relics?
• Gravitational waves?
• Primordial magnetic fields?
• Baryon asymmetry?
First order electroweak phase transition

Electroweak Phase Transition

Bubbles

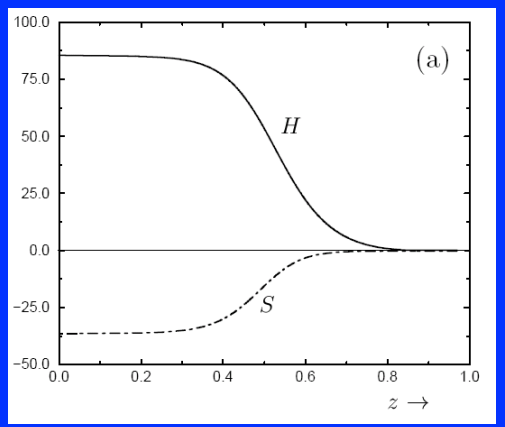
broken phase

symmetric phase

H(z)

v_w

- temperature T_c
- vev in the broken phase v_c
- wall width L_w
- wall velocity v_w



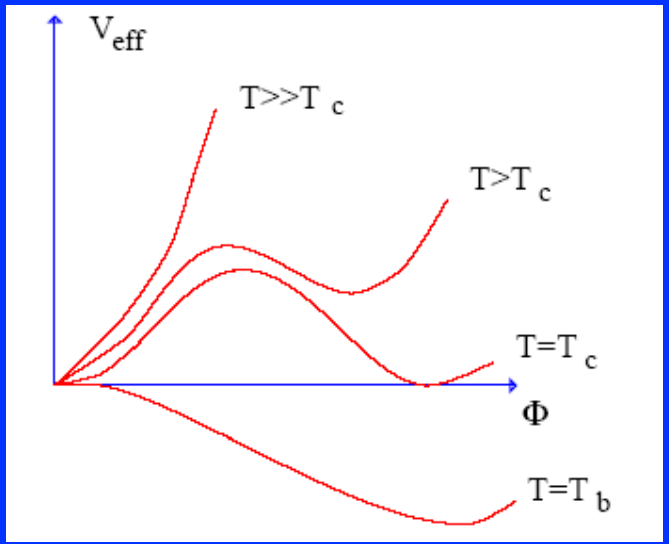
The strength of the PT

Thermal potential:

$$V(H,T) = m^2(T)H^2 - E(T)H^3 + \lambda(T)H^4$$

- Boson loops:

  SM: gauge bosons



Lattice: crossover for m_h>80 GeV → the SM fails

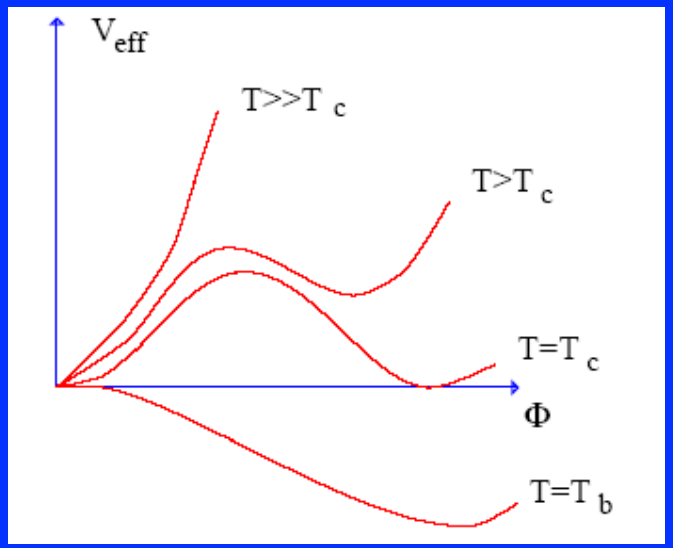
Kajantie, Laine, Rummukainen, Shaposhnikov 1996

Csikor, Fodor, Heitger 1998

The strength of the PT

Thermal potential:

$$V(H,T) = m^2(T)H^2 - E(T)H^3 + \lambda(T)H^4$$



- Boson loops:

  SM: gauge bosons

  SUSY: light stops

  2HDM: extra Higgses

- tree-level: extra singlets: λSH^2, NMSSM, etc.
- replace H^4 by H^6, etc.

Electroweak Baryogenesis

The baryon asymmetry

$$\eta_B = \frac{n_B}{s} = (8.7 \pm 0.3) \times 10^{-11}$$

[WMAP, SDSS]

Reasonable agreement between CMBR+LSS and primordial nucleosynthesis

→ we understand the universe up to T~MeV

Can we repeat this success for the baryon asym.?

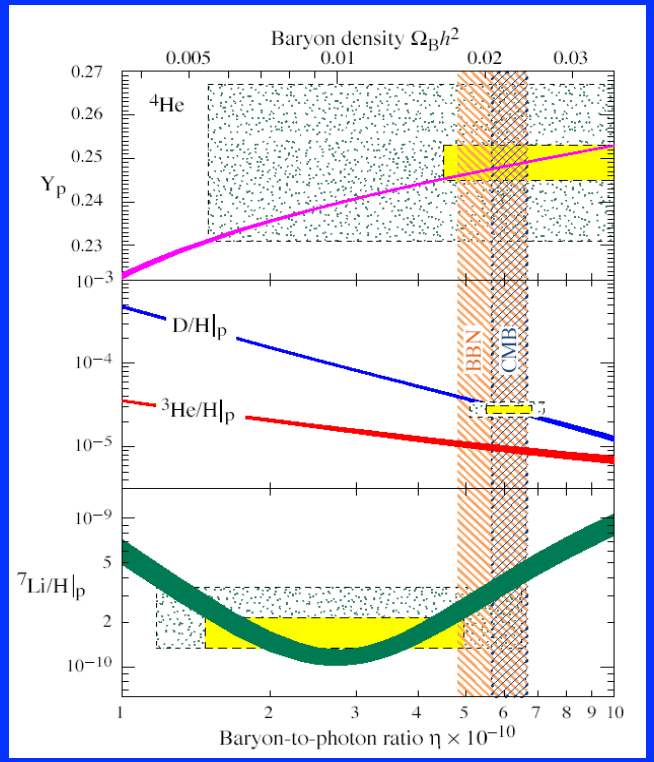
Problem: only 1 observable

→ Need to be convinced by a specific model:

Theory?, Experiment? (insight??) …

⇕

T < TeV scale? → EWBG



[Particle Data Group]

The basics

$$\eta_B = \frac{n_B}{s} = (8.7 \pm 0.3) \times 10^{-11}$$

Baryon number

C

CP

Equilibrium



Sakharov ‘67

SM



Sphalerons +

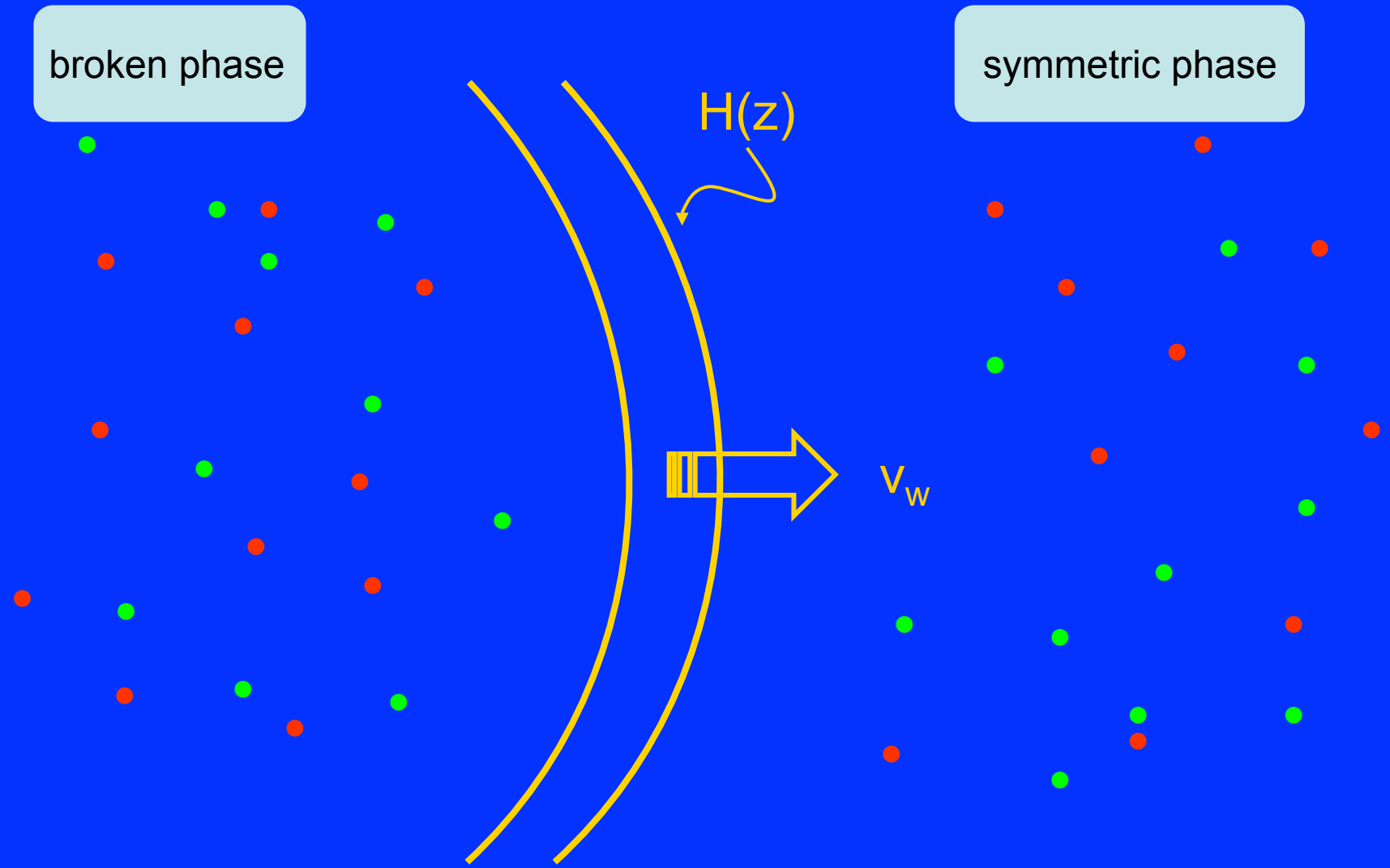
Gauge interactions +

Yukawa interactions ?

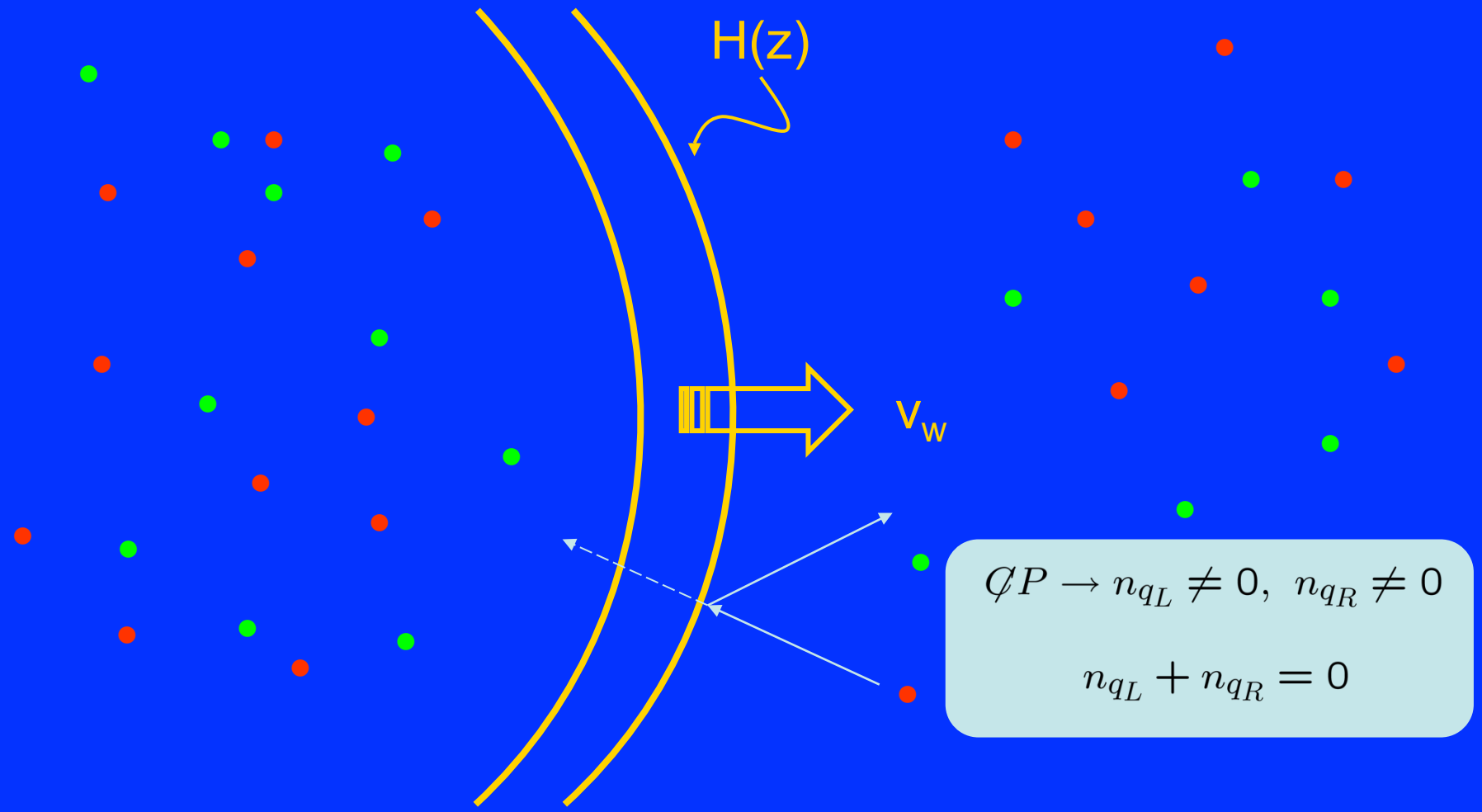
Electroweak phase transition ?

Kuzmin, Rubakov, Shaposhnikov ‘85

The mechanism



The mechanism



$\not{CP} \rightarrow n_{q_L} \neq 0, \; n_{q_R} \neq 0$

$n_{q_L} + n_{q_R} = 0$

The mechanism

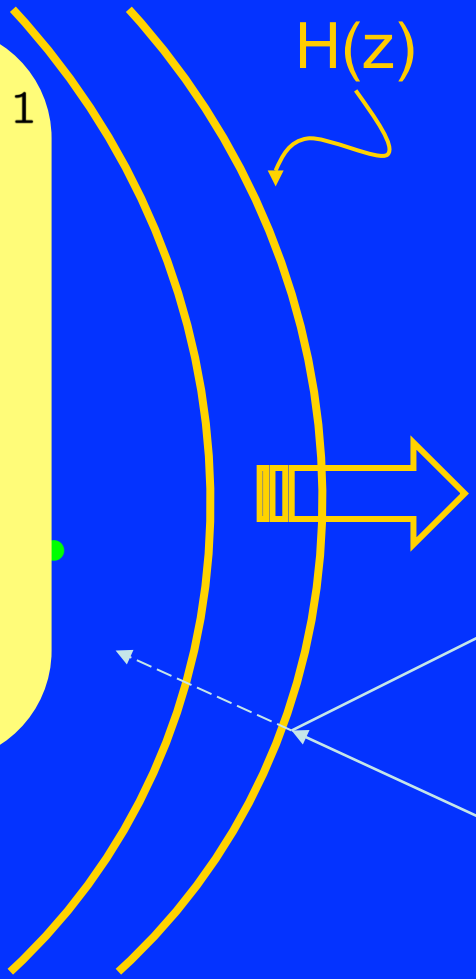
$\Gamma_{\rm sph} \sim T_c^4 \exp(-\kappa v_c/T_c) \ll 1$

$\Downarrow$

$\dfrac{v_c}{T_c} > 1.0$

$\Updownarrow$

"strong PT"



$\Gamma_{\rm sph} \sim T_c^4 \longrightarrow \not{B}$

diffusion

$\not{C}\not{P} \rightarrow n_{q_L} \neq 0, \; n_{q_R} \neq 0$

$n_{q_L} + n_{q_R} = 0$

Models

need to improve on the phase transition and CP violation

- 2HDM
- SM with higher-dimensional operators
- MSSM (+ singlets)

The 2HDM

$$V(H_1, H_2) = \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \mu_3^2 e^{i\phi} H_1^\dagger H_2 + \lambda_1|H_1|^4 + \ldots$$

→ 4 extra physical Higgs degrees of freedom: 2 neutral, 2 charged

→ CP violation, phase Φ (μ_3 breaks Z_2 symmetry softly)

→ there is a phase induced between the 2 Higgs vevs

$$v_1 = \langle H_1 \rangle, \qquad v_2 e^{i\theta} = \langle H_2 \rangle$$

early work:
Turok, Zadrozny ' 91
Davies, Froggatt, Jenkins, Moorhouse ' 94
Cline, Kainulainen, Vischer ' 95
Cline, Lemieux '96

simplified parameter choice:

1 light Higgs m_h → SM-like, so LEP bound of 114 GeV applies

3 degenerate heavy Higgses m_H → keeps EW corrections small

The phase transition

Evaluate 1-loop thermal potential:

loops of heavy Higgses generate a cubic term

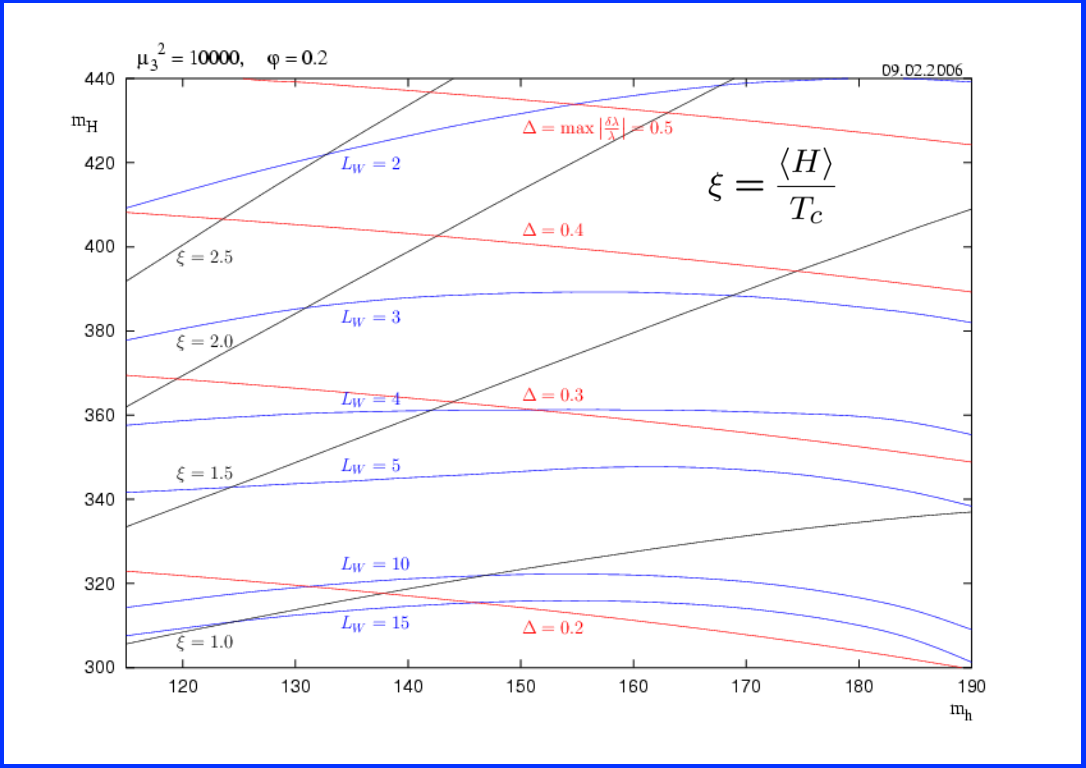
→ strong PT for

m_H>300 GeV

m_h up to 200 GeV

→ PT ~ independent of Φ

→ thin walls only for very strong PT (agrees with Cline, Lemieux ' 96)



[Fromme, S.H., Senuich ' 06]

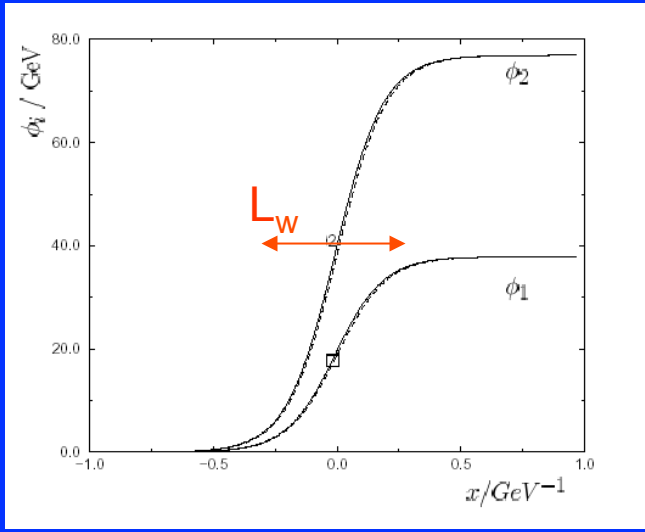
missing: 2-loop analysis of the thermal potential; lattice; wall velocity

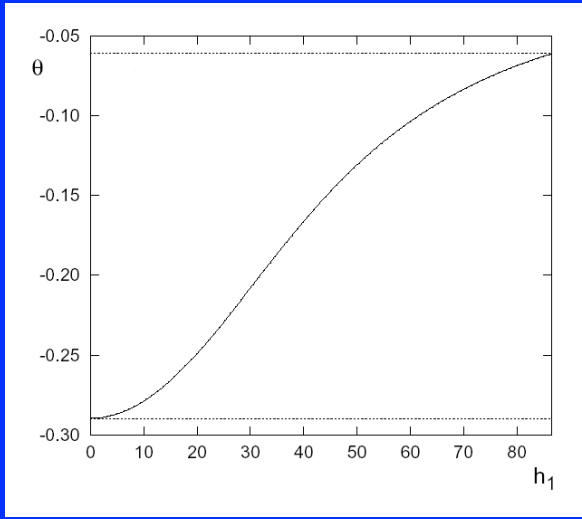
The bubble wall

Solve the field equations with the thermal potential → wall profile $\Phi_i(z)$

kink-shaped with wall thickness L_w

θ becomes dynamical





(numerical algorithm for multi-field profiles, T. Konstandin, S.H. ´06)

The baryon asymmetry

The relative phase between the Higgs vevs, θ, changes along the bubble wall

→ phase of the top mass varies

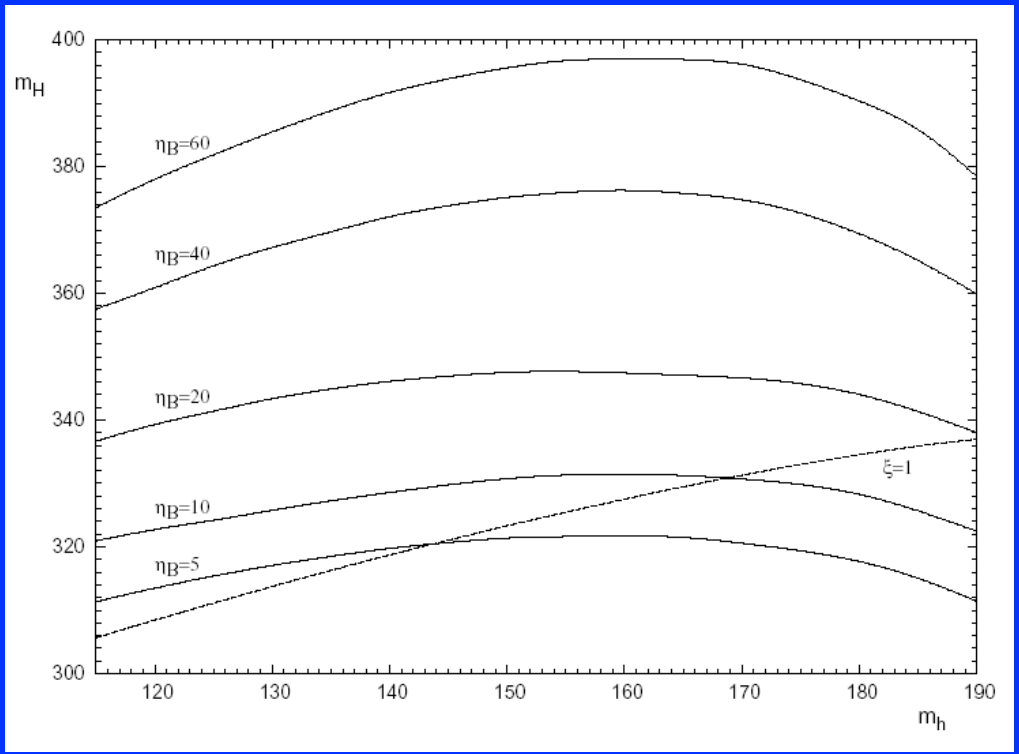
$\theta_t = \theta/(1+\tan^2\beta)$

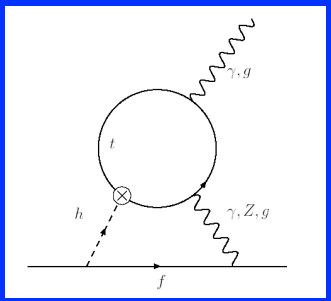
top transport generates a baryon asymmetry, but tanβ<10 (?)

→ only one phase, so EDMs can be predicted: here

$d_n = 0.1\ 10^{-26} - 7\ 10^{-26}$ e cm

exp. bound: $d_n < 3.0\ 10^{-26}$ e cm





η_B in units of 10^{-11}, φ=0.2

Moretti et al. '07: LHC could see a triple Higgs coupling Hhh

The baryon asymmetry

The relative phase between the Higgs vevs, θ, changes along the bubble wall

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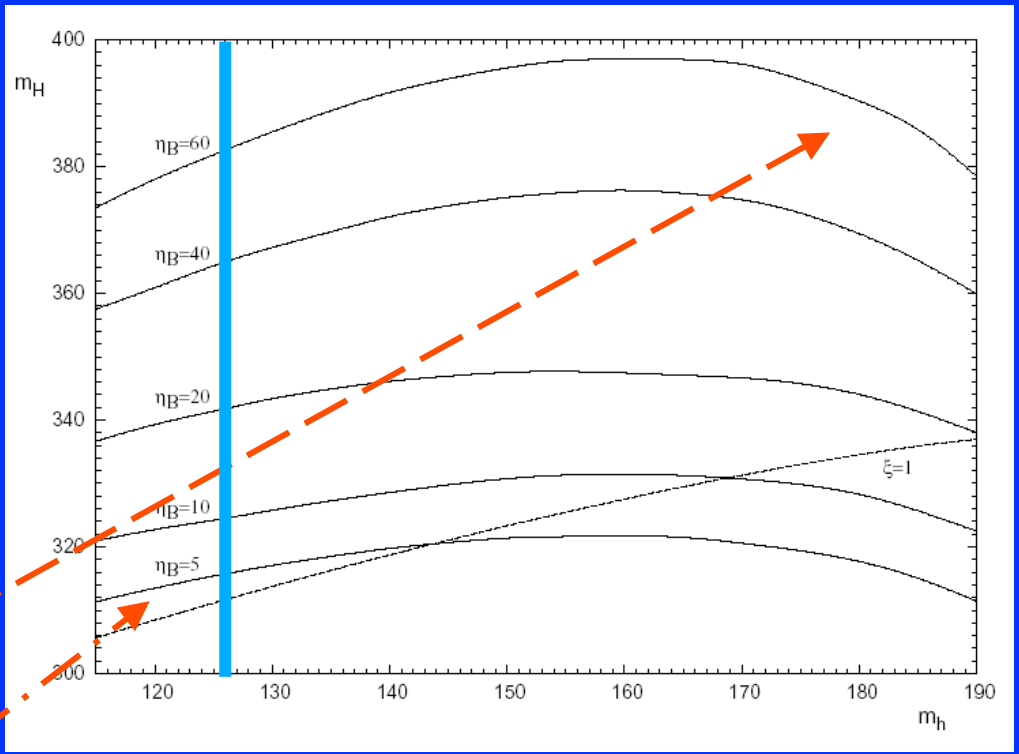
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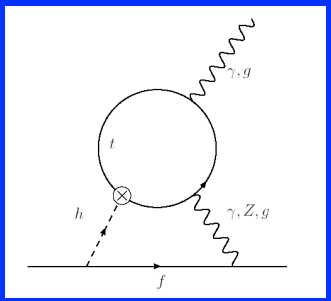
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η_B in units of 10^{-11}, φ=0.2

Moretti et al. '07: LHC could see a triple Higgs coupling Hhh

Inert 2HDM:

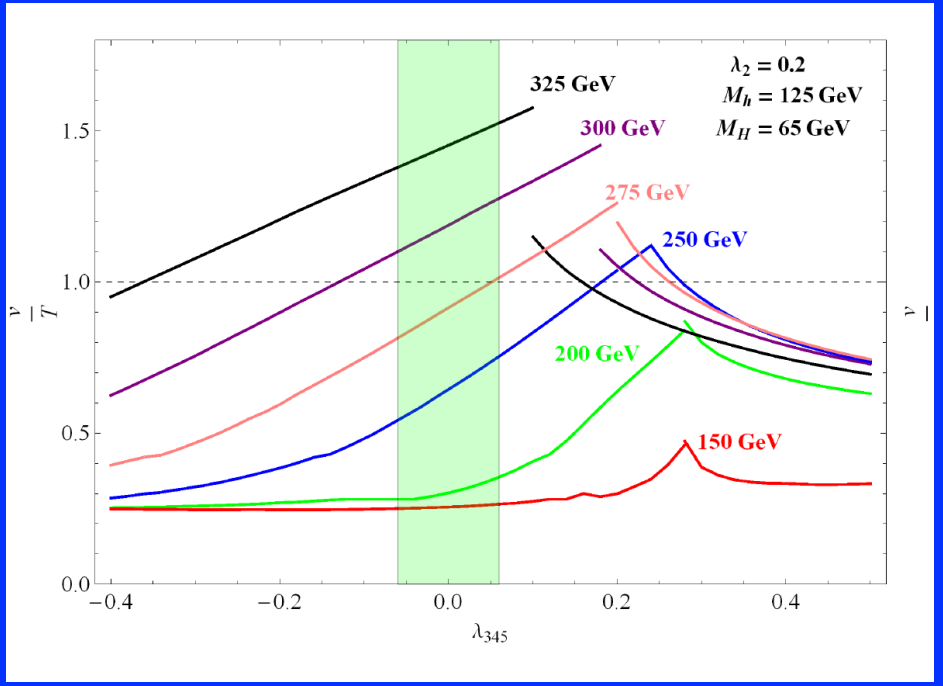
$$V(\Phi_1,\Phi_2) = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2$$
$$+\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\lambda_5}{2}\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\right]$$

doublet 2 does not get a vev

➔ Dark matter

CP violation from higher-dim. operators

$$\mathbb{Z}_2: \quad \Phi_1 \to \Phi_1\,, \qquad \Phi_2 \to -\Phi_2\,,$$
$$\mathbb{Z}_2': \quad \Phi_1 \to -\Phi_1\,, \qquad \Phi_2 \to \Phi_2\,, \qquad f_R \to -f_R\,,$$



[Gil, Chankowski, Krawczyk 2012]

SM + higher-dim. operators

$$V(H) = -\mu^2|H|^2 + \lambda|H|^4 + \frac{1}{M^2}|H|^6$$

Zhang '93

Grojean et al. '04

maybe related to strong dynamics at the TeV scale, such as technicolor or gravity?

(or simply comes from integrating out extra scalars)

two parameters, (λ, M) $\leftrightarrow$ (m_h, M)

λ can be negative $\rightarrow$ bump because of $|H|^4$ and $|H|^6$

$$\begin{aligned} V_{\rm eff}(\phi, T) = & \frac{1}{2}\left(-\mu^2 + \left(\frac{1}{2}\lambda + \frac{3}{16}g_2^2 + \frac{1}{16}{g_1}^2 + \frac{1}{4}y_t^2\right)T^2\right)\phi^2 \\ & - \frac{g_2^3}{16\pi}T\phi^3 + \frac{\lambda}{4}\phi^4 + \frac{3}{64\pi^2}y_t^4\phi^4\ln\left(\frac{Q^2}{c_F T^2}\right) \\ & + \frac{1}{8M^2}(\phi^6 + 2\phi^4T^2 + \phi^2T^4). \end{aligned}$$

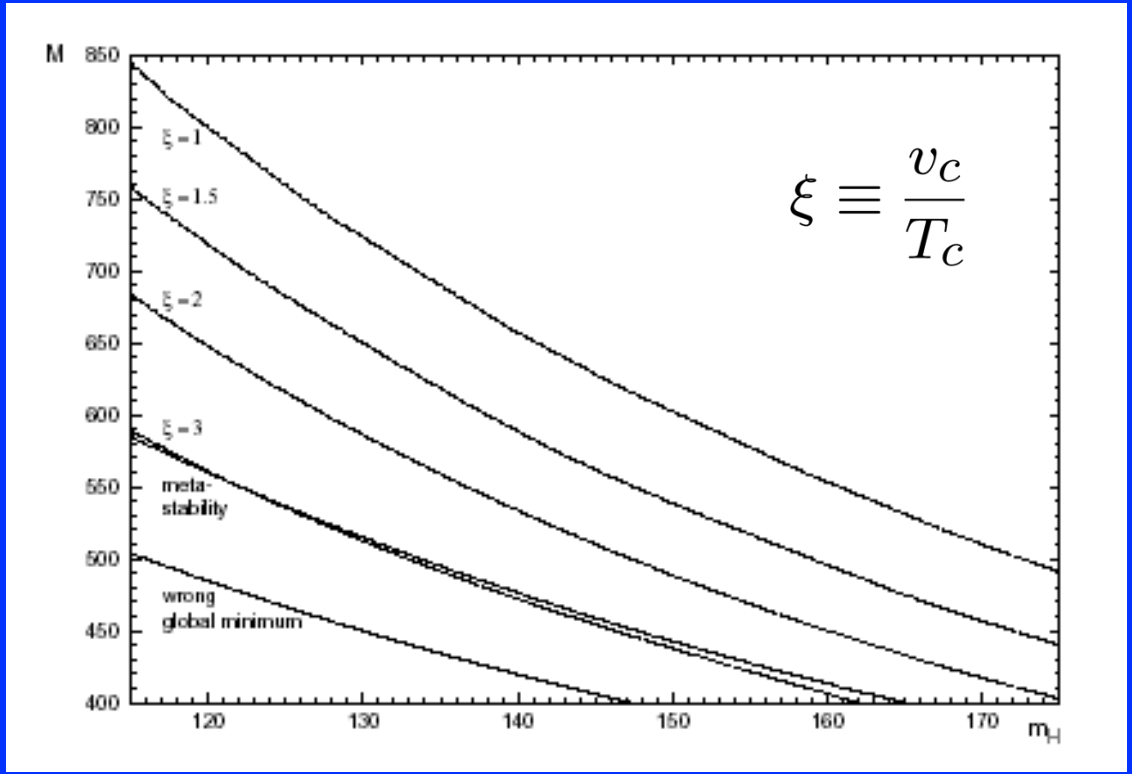
Results for the PT

Evaluating the 1-loop thermal potential:

strong phase transition for M<850 GeV up to m_h ~170 GeV

(LEP bound applies, m_h >114 GeV)

wall thickness $2 < L_w T_c < 16$



Bödeker, Fromme, S.H., Seniuch '04

Similar results, including Higgs cubic terms

Delaunay, Grojean, Wells '07

SM + higher-dim. operators

$$V(H) = -\mu^2|H|^2 + \lambda|H|^4 + \frac{1}{M^2}|H|^6$$

Zhang '93

maybe related to strong dynamics at the TeV scale, such as technicolor or gravity?

two parameters, (λ, M) ↔ (m_h, M)

λ can be negative → bump because of $|H|^4$ and $|H|^6$: M < ~800 GeV

CP violation:

$$\frac{x}{M^2}(H^\dagger H)Ht^c q$$

Zhang, Lee, Whisnant, Young '94

contributes to the top mass: $m_t = yH + \frac{x}{M^2}(H^\dagger H)H$

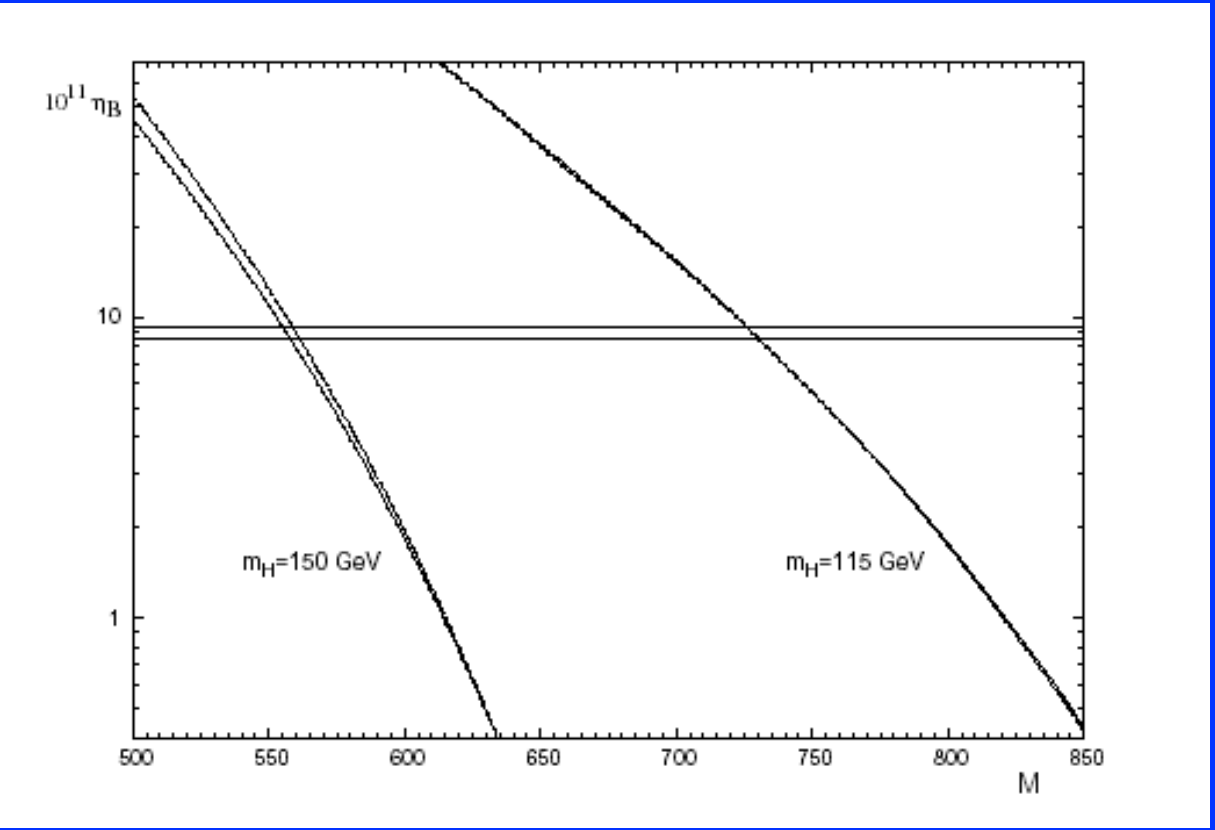
induces a varying phase in m_t if xy* is complex, with $\theta \sim \frac{|H|^2}{M^2}\frac{\mathrm{Im}(x)}{y}$

The baryon asymmetry

for Im(x)=1 and

v_w=0.01, 0.3

η_B inreases rapidly with smaller M because of the stronger PT



prediction for EDMs

with M. Pospelov, A. Ritz

→ testable with next generation experiments!

Fromme, S.H. '06

Phenomenology

1) operators which contribute to EW observables

must be suppressed by Λ>>M~TeV, e.g.

$$\frac{1}{\Lambda^2}(H^\dagger D_\mu H)^2$$

with Λ> 10 TeV → 1% tuning required?

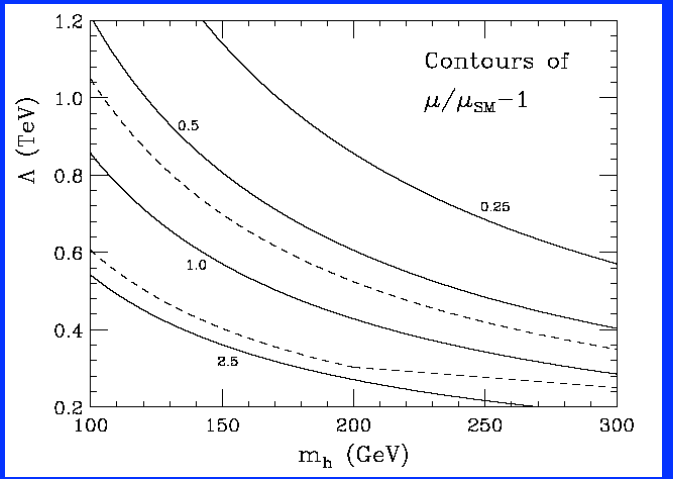
Grojean, Servant, Wells '04

2) deviations from the SM cubic Higgs

self coupling μH³

LHC: order unity test

ILC: 20%



Classic: The MSSM

strong PT from stop loops

→ right-handed stop mass ~100 GeV

left-handed stop mass ~1000 TeV

[poster by G. Nardini]

CP violation from varying chargino mixing

$$\theta(z) = \arg(M_2 \mu^*) f(H_1(z), H_2(z))$$

resonant enhancement of η for $M_2 \sim \mu$

chargino mass < ~300 GeV

large phases > 0.2 required

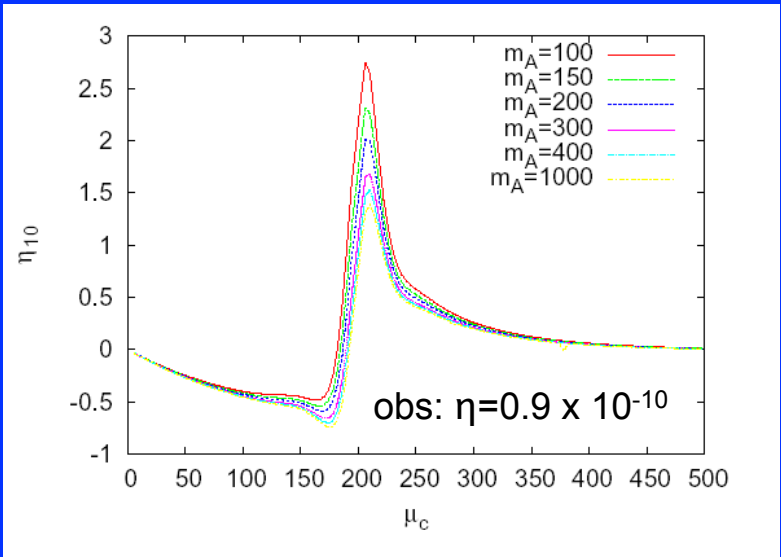
→ 1st and 2nd generation squarks

heavy to keep 1-loop EDMs small

→ "Split SUSY + light stop"

Konstandin, Prokopec, Schmidt, Seco '05

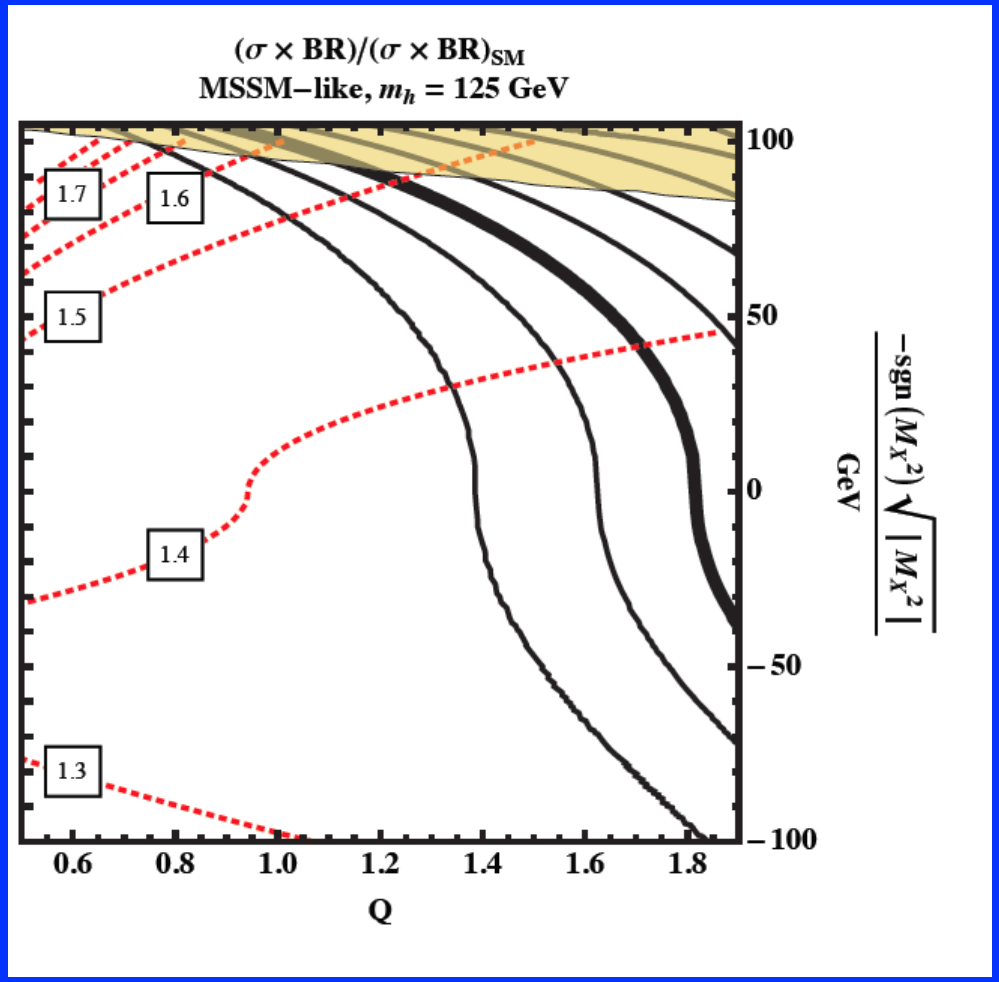
v_w=0.05, M_2=200 GeV, maximal phase



similar but somewhat more optimistic results in Carena, Quiros, Seco, Wagner '02 Cirigliano, Profumo, Ramsey-Musolf '06

→ scenario is tightly constrained!

Possible test: modified Higgs branching ratios, e.g. into two photons:



[Cohen, Morrissey, Pierce 2012]

[see also e.g. Curtin, Jaiswal, Meade 2012]

MSSM + "singlets"

singlets models contain cubic (SHH) terms

at tree-level → stronger PT

also new sources of CP violation

model building problems: domain walls vs.

destabilization of the weak scale

which model to take?

Z_3 symmetry (NMSSM)

$Z_{5,7}$ R-symmetries (nMSSM)

extra U(1)'s (ESSM, …)

fat Higgs…

Pietroni '92

Davies, Froggatt, Moorhouse '96

S.H., Schmidt '98

Bastero-Gil, Hugonie, King, Roy, Vespati '00

Kang, Langacker, Li, Liu '04

Menon, Morrissey, Wagner '04

S.H., Konstandin, Prokopec, Schmidt '06

Balazs, Carena, Freitas, Wagner '07

(Profumo, Ramsey-Musolf, Shaughnessy '07)

Carena, Shaw, Wagner '11

computation of bubble profiles?

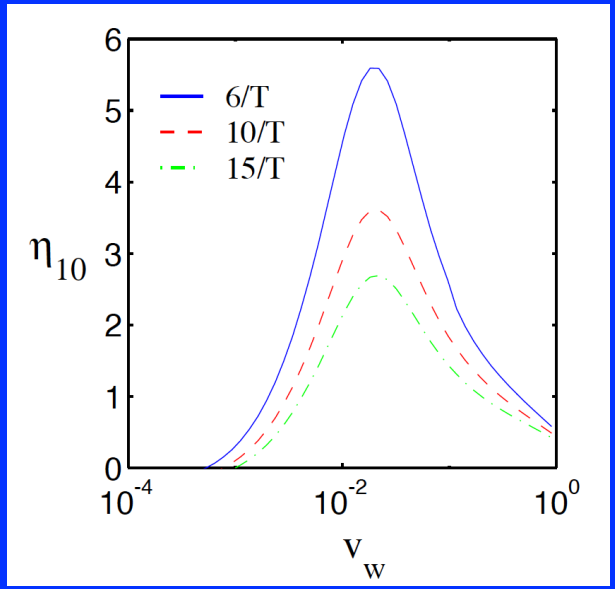
Konstandin, S.H. '06

problem with 1-loop EDM's remains!

Bubble velocity

Knowing the wall velocity is important as:

- Standard baryogenesis needs $v_w < v_s = 0.58$ (diffusion!)
- Strong gravitational wave signal only for $v_w > v_s$
- Sometimes the baryon asymmetry depends crucially on v_w



[Cline, Joyce, Kainulainen '01]

The wall velocity:

Friction with the plasma balances the pressure

Distinguish: supersonic vs. subsonic (v_s^2=1/3)

Standard model: v_w~ 0.35 - 0.45 for low Higgs masses [Moore, Prokopec '95]

MSSM: v_w~0.05 [John, Schmidt '00]

All other models: no detailed computations

**Recently: walls can run away, i.e. approach v_w=1 [Bodeker, Moore '09]

How to compute the wall velocity?

Main ingredients: pressure difference vs. plasma friction

Also important: reheating due to release of latent heat

Microscopic description: Moore, Prokopec '95 $(\partial_t + \dot{z}\partial_z + \dot{p}_z\partial_{p_z})f = \mathcal{C}[f]$

$$\Box\phi + V_T'(\phi) + \sum \frac{dm^2}{d\phi}\int \frac{d^3p}{(2\pi)^3\,2E}\delta f(p,x) = 0$$

$$f = \frac{1}{1+\exp\frac{E-E\delta T/T - p_z v - \mu}{T}}$$

(fluid ansatz)

$$\begin{aligned} a v_w\frac{\mu'}{T} + v_w\frac{\delta T'}{T} + \frac{1}{3}v' + F_1 &= -\Gamma_{\mu 1}\frac{\mu}{T} - \Gamma_{T1}\frac{\delta T}{T} \\ b v_w\frac{\mu'}{T} + v_w\frac{\delta T'}{T} + \frac{1}{3}v' + F_2 &= -\Gamma_{\mu 2}\frac{\mu}{T} - \Gamma_{T2}\frac{\delta T}{T} \\ b\frac{\mu'}{T} + \frac{\delta T'}{T} + v_w v' + 0 &= -\Gamma_v v \end{aligned}$$

$$F_1 = -\frac{v_w \ln 2}{9\zeta_3}\frac{(m^2)'}{T^2}, \qquad F_2 = -\frac{v_w\zeta_2}{42\zeta_4}\frac{(m^2)'}{T^2}$$

(force terms)

$(F = \dot{p}_z = -\partial_z E(z, p_z))$

→ Complicated set of coupled field equations and Boltzmann equations

need many scattering rates

SM: v ~ 0.35 - 0.45

Simplified approach: (Ignatius, Kajantie, Kurki-Suonio, Laine ‘94)

1) describe friction by a friction coefficient η

2) model the fluid by a fluid velocity and temperature

$$\frac{d^2\phi(x)}{dx^2} = \frac{\partial V(\phi,T)}{\partial\phi} + \eta\frac{\phi^2}{T_{s1}}v\gamma\frac{d\phi(x)}{dx}$$

$$(4aT^4 - T\frac{\partial V(\phi,T)}{\partial T})\gamma^2 v = C_1$$

$$(4aT^4 - T\frac{\partial V(\phi,T)}{\partial T})\gamma^2 v^2 + P_r - V(\phi,T) + \frac{1}{2}(\frac{d\phi}{dx})^2 = C_2$$

← 1 + 1 dimensions

3) Determine η from fitting the to the full result by Moore and Prokopec

[with Miguel Sopena, see also Megevand, Sanchez ’09]

→ the formalism should describe situations with SM friction well

→ study models with SM friction, but different potential, e.g. phi^6 model

→ same for the MSSM

→ such a model can be used for numerical simulations of the phase transition

[work in progress with Hindmarsh, Rummukainen, Weir]

Friction in relaxation time approximation:

$$-\frac{(m^2)'}{2E}v\beta\gamma\frac{e^{\beta\gamma(E-vp_z)}}{(e^{\beta\gamma(E-vp_z)}\pm 1)^2}+\frac{p_z}{E}\delta f'=-\frac{\delta f}{\tau}.$$

$$\frac{d^2\phi}{dz^2}+\frac{\partial V(\phi,T)}{\partial\phi}+\sum\frac{dm^2}{d\phi}\int\frac{d^3p}{(2\pi)^3 2E}\tau\beta\gamma v\frac{(m^2)'}{2E}\frac{e^{\beta\gamma(E-vp_z)}}{(e^{\beta\gamma(E-vp_z)}\pm 1)^2}=0.$$

tau needs to be fixed from the microscopic calculations

momentum integral can done numerically

➔ In essence gives η(v/T) successfully capturing the Boltzmann factor

MSSM: S.H., M. Sopena 2011

Input:

$\tan\beta$	m_U^2	v_w (John and Schmidt)	Fitted η
2	-60^2	0.060	4.58
	0	0.090	3.36
	$+60^2$	0.160	1.92
4	-60^2	0.080	4.35
	0	0.115	3.16
	$+60^2$	0.140	2.72
6	-60^2	0.085	4.65
	0	0.120	3.06
	$+60^2$	0.155	2.55

1-loop potential

m_Q=2000 GeV

Output:

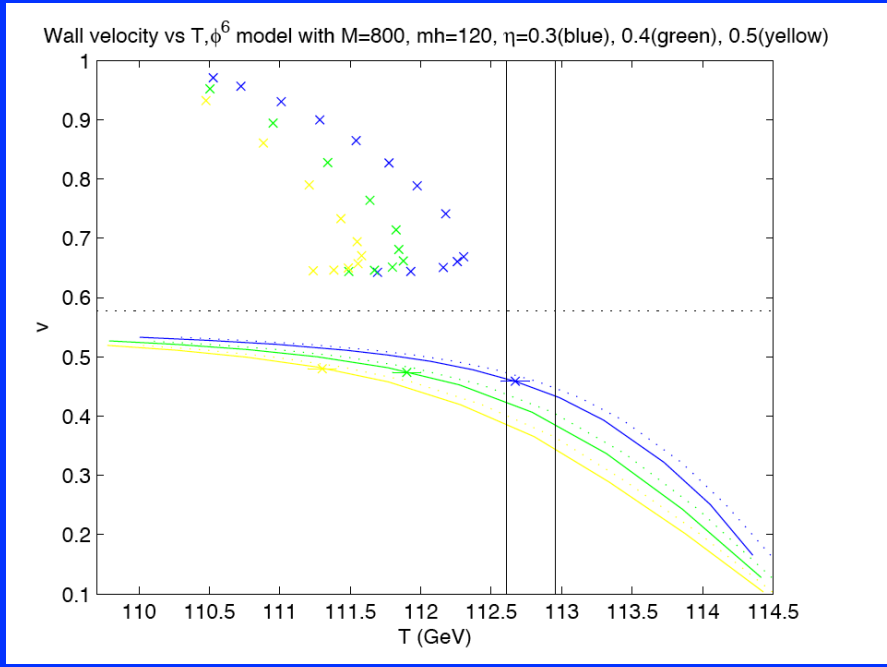
$\tan\beta$	T	$\xi = \frac{v}{T}$	Fitted η	v_w
4	T_n	0.92	6.10	0.044
	T_f	0.94		0.051
	T'_f	0.94		0.054
6	T_n	0.95	6.88	0.039
	T_f	0.96		0.045
	T'_f	0.96		0.047

2-loop potential

m_h=112 GeV

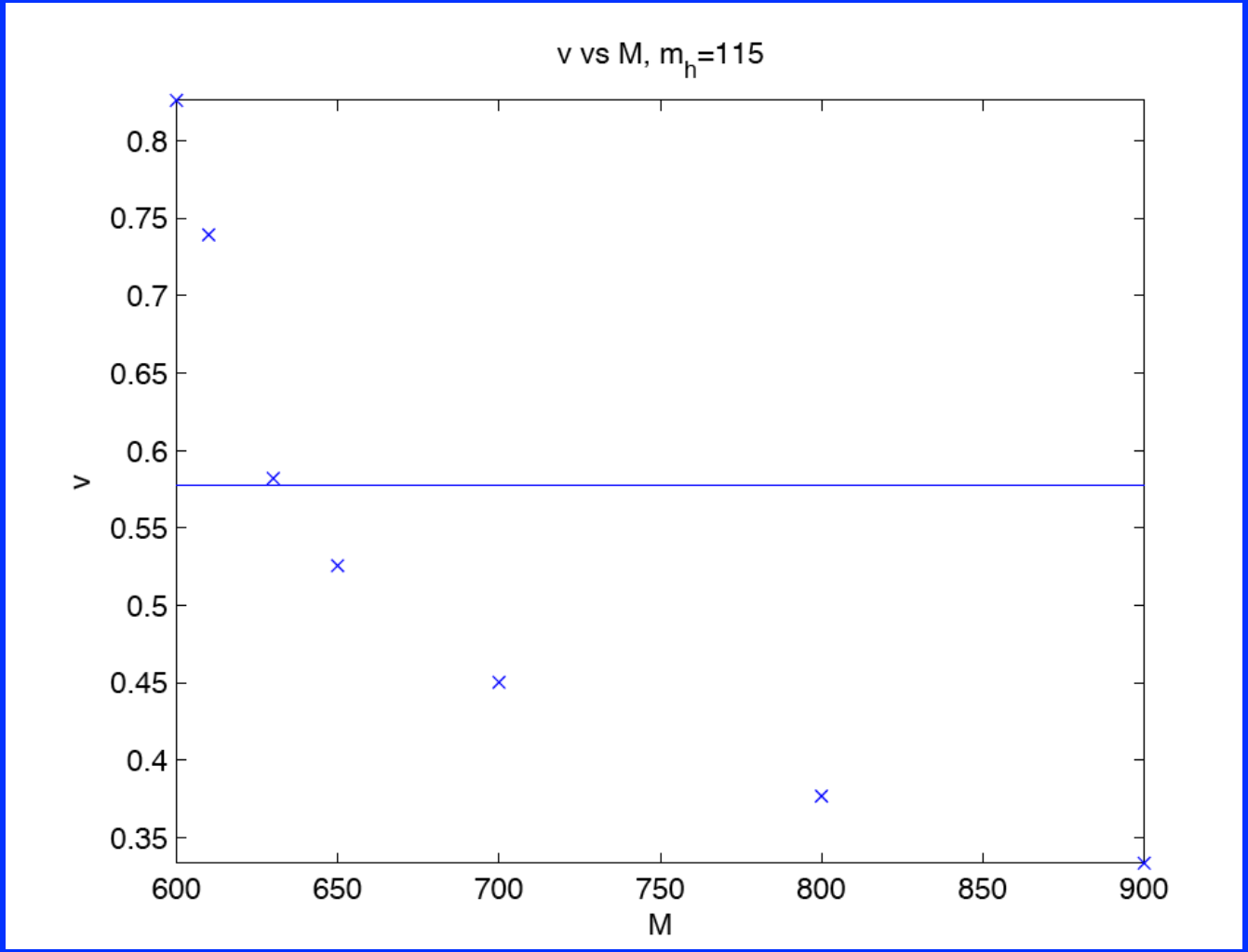
for m_h=125 we get v_w~0.03

Results for Φ^6 model



- for a moderately strong phase transition, the wall moves subsonic in the dim-6 model (good for baryon asymmetry, bad for GW's)
- relative mild dependence on the friction coefficient eta
- instable bubbles for some temperatures?

Wall velocity for m_h=115 GeV



SH, Sopena, to appear

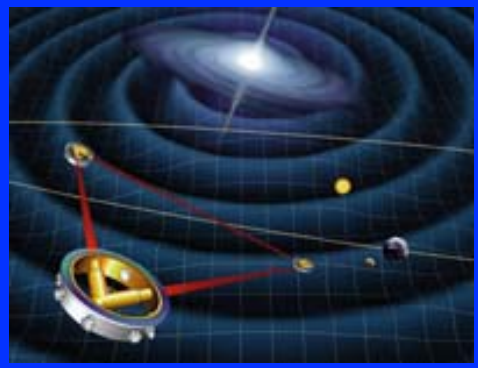
Gravitational waves

LISA: 2016?

sources of GW's: direct bubble collisions

turbulence

(magnetic fields)



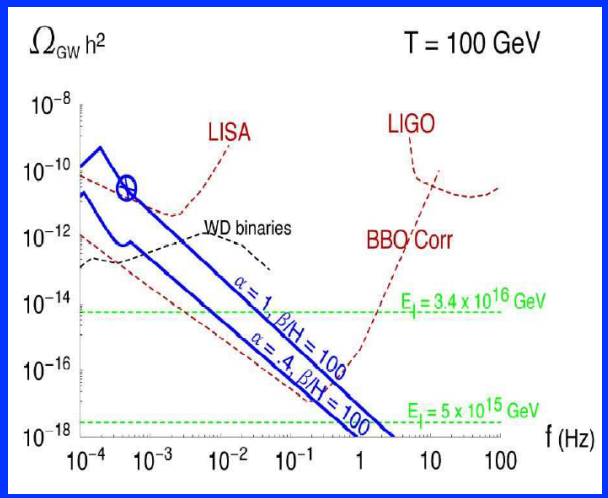
key parameters: available energy

$$\alpha = \frac{\text{latent heat}}{\text{radiation energy}}$$

typical bubble radius

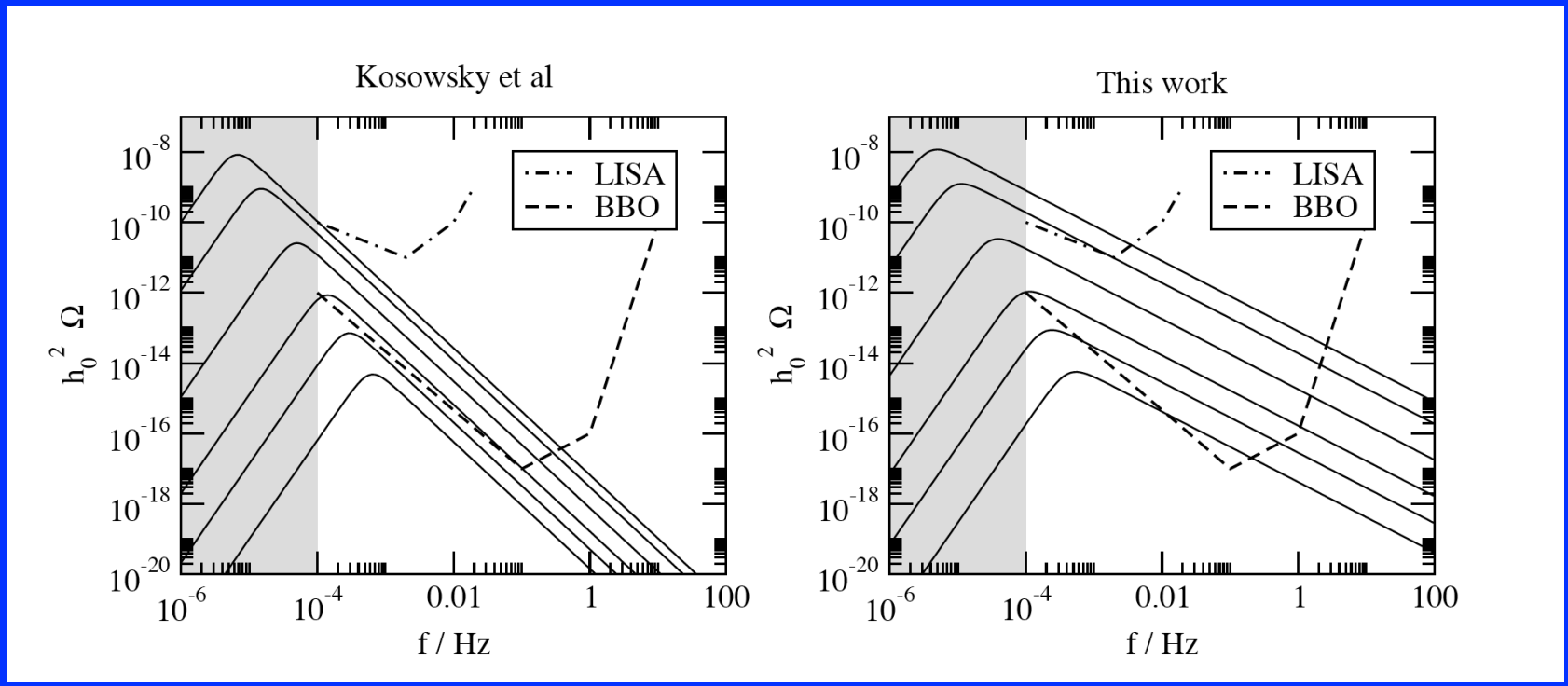
$$\langle R \rangle \propto v_b \, \tau \approx \frac{v_b}{\beta}.$$

v_b wall velocity



Grojean, Servant '06

Results in the Φ^6 model



GW ~ $f^{-1.8}$ $\Rightarrow$ GW ~ f^{-1}

(related to small bubbles??)

T. Konstandin, S.H. '08

Summary

viable models for a strong first order phase transition:

- ►2HDM: m_H>~300 (transport by tops)
- ►SM with a dim-6 Higgs potential for M<800 GeV

  (EDMs similar to 2HDM)
- ►MSSM: light stop for the phase transition (very constrained now!)

  transport by the charginos (instead of tops)

  severe constraints from EDMs
- ►Singlet models (NMSSM): many possiblities

  cubic terms in the tree-level potential induce a strong phase transition

  EDM constraints somewhat relaxed (or totally absent for transitional ~~CP~~)
- ►Gravitational waves: considerable uncertainties (f^{-1} fall off!), probably difficult to detect

what else is the LHC going to find??

Outlook

- extended models have a large parameter space which is typically only partially explored
- take into account additional constraints from dark matter, electroweak bounds, EDMs, etc.…
- establish a closer link to collider physics
- computation of the wall velocities in extended models
- more fancy models, such as Wilson line Higgs,…

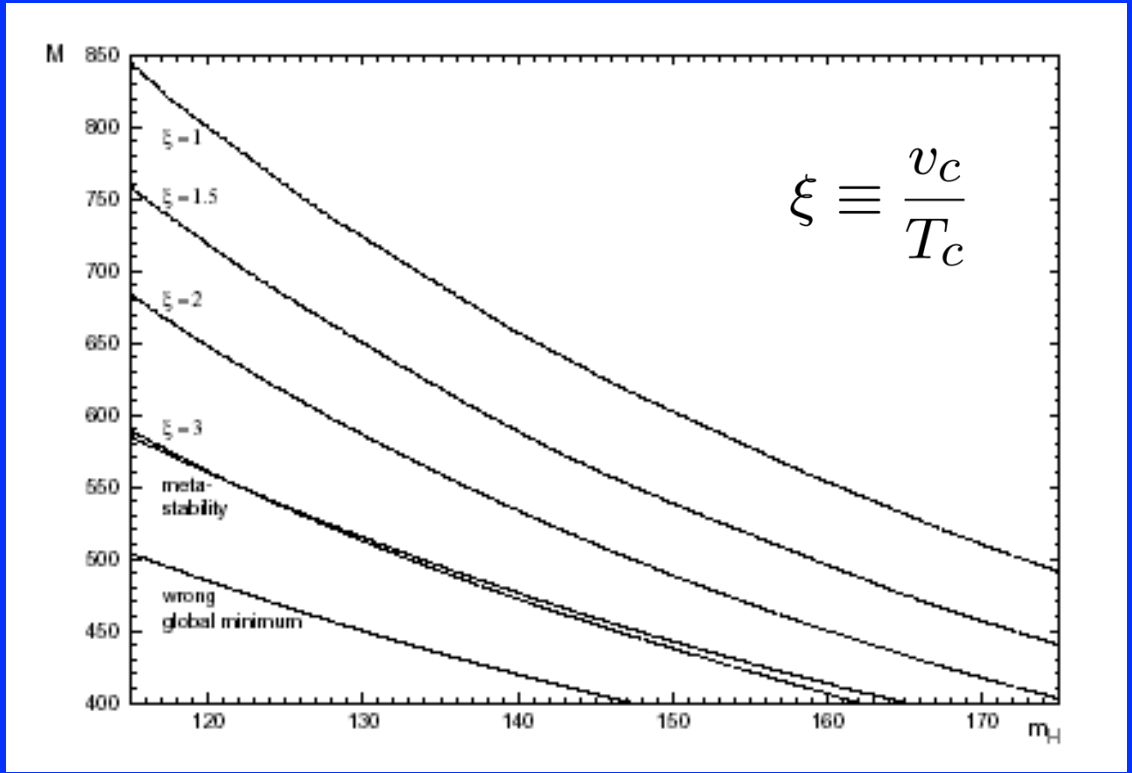
Results for the PT

Evaluating the 1-loop thermal potential:

strong phase transition for M<850 GeV up to m_h ~170 GeV

(LEP bound applies, m_h >114 GeV)

wall thickness $2 < L_w T_c < 16$



Bödeker, Fromme, S.H., Seniuch '04

The baryon asymmetry

$$\eta_B = \frac{n_B}{s} = (8.7 \pm 0.3) \times 10^{-11}$$

WMAP, SDSS '06

Reasonable agreement between CMBR+LSS and primordial nucleosynthesis

→ we understand the universe up to T~MeV

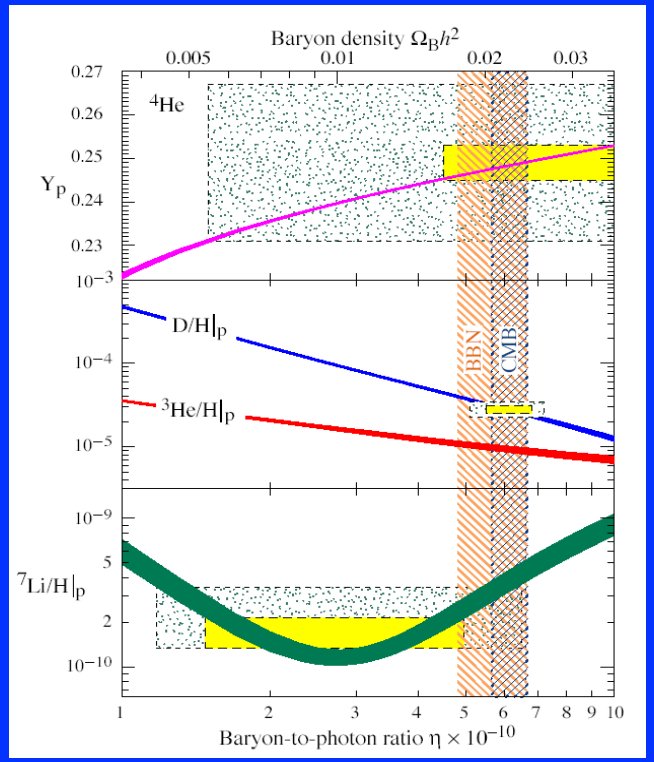
Can we repeat this success for the baryon asym.?

Problem: only 1 observable

→ Need to be convinced by a specific model:

Theory? Experiment? Belief? …

T < TeV scale? → EWBG



[Particle Data Group]

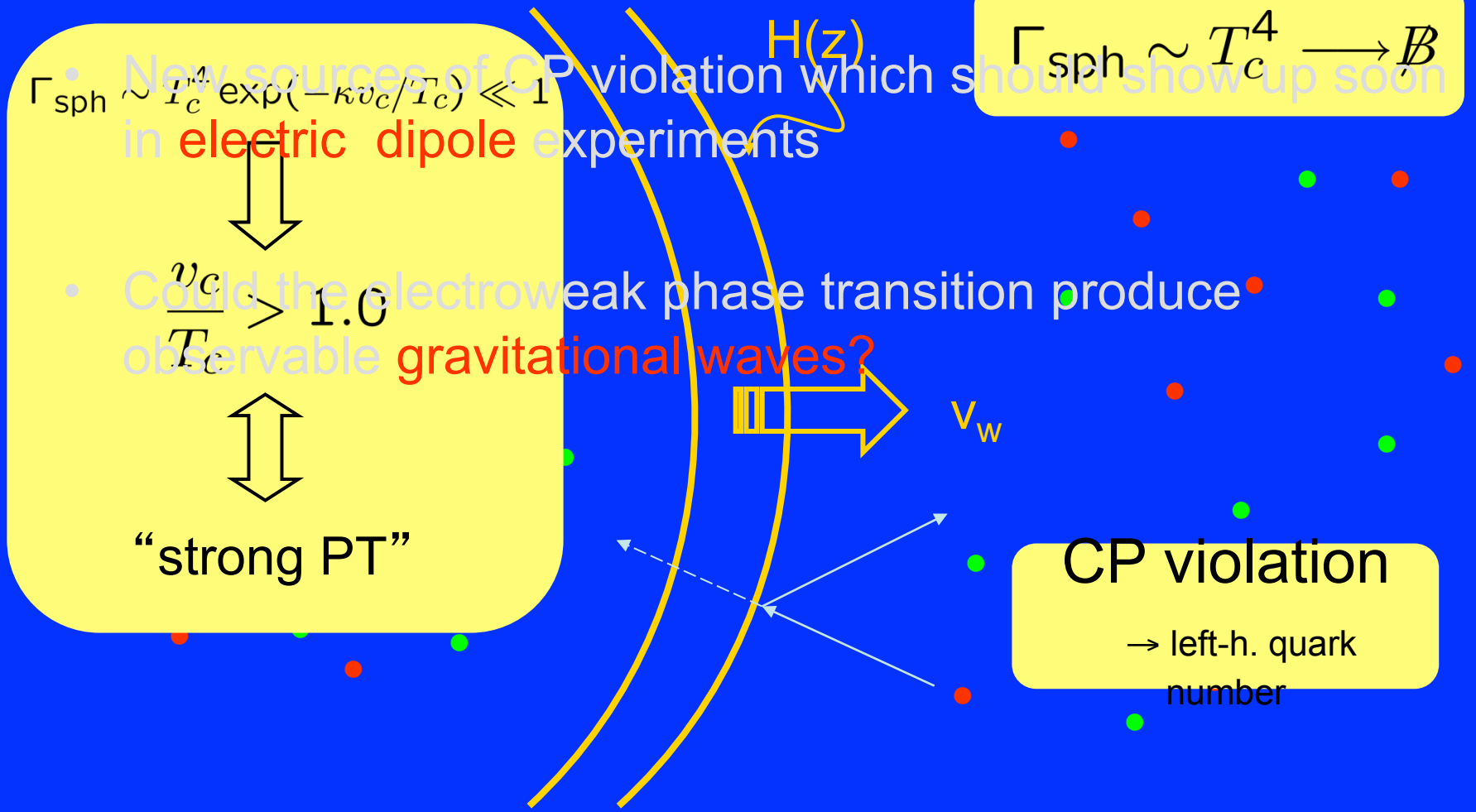
Why is it interesting?

There are testable consequences:

- New particles (scalars?!) at the LHC (Higgs sector is crucial!)

If confirmed, it would constrain the early universe up to T~100 GeV (nano sec.), like nucleosynthesis does for the MeV-scale (min.)

The mechanism



- New sources of CP violation which should show up soon in electric dipole experiments
- Could the electroweak phase transition produce observable gravitational waves?

Transport and CP violation

The interaction with the bubble wall induces a force on the particles, which is different for particles and antiparticles if CP is broken

$$(\partial_t + \dot{z}\partial_z + \dot{p}_z\partial_{p_z})f = \mathcal{C}[f]$$

collision terms, many?

Force: $\dot{p}_z = -\partial_z E(z, p_z)$

$$\begin{aligned} E_\pm &= E_0 \pm \Delta E_0 \\ &= \sqrt{p^2 + m^2} \pm \theta' \frac{m^2}{2(p^2 + m^2)} \end{aligned}$$

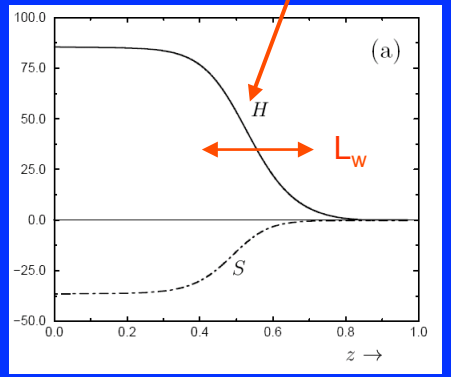
Joyce, Prokopec, Turok ' 95
Cline, Joyce, Kainulainen ' 00

„thick“

Phase in fermion mass can vary along the wall (wall width L_w)

e.g. because the phase between the Higgs vevs changes:

$$M(z) = m(z)e^{i\theta(z)}$$



The phase transition

Evaluate 1-loop thermal potential:

loops of heavy Higgses generate a cubic term

→ strong PT for

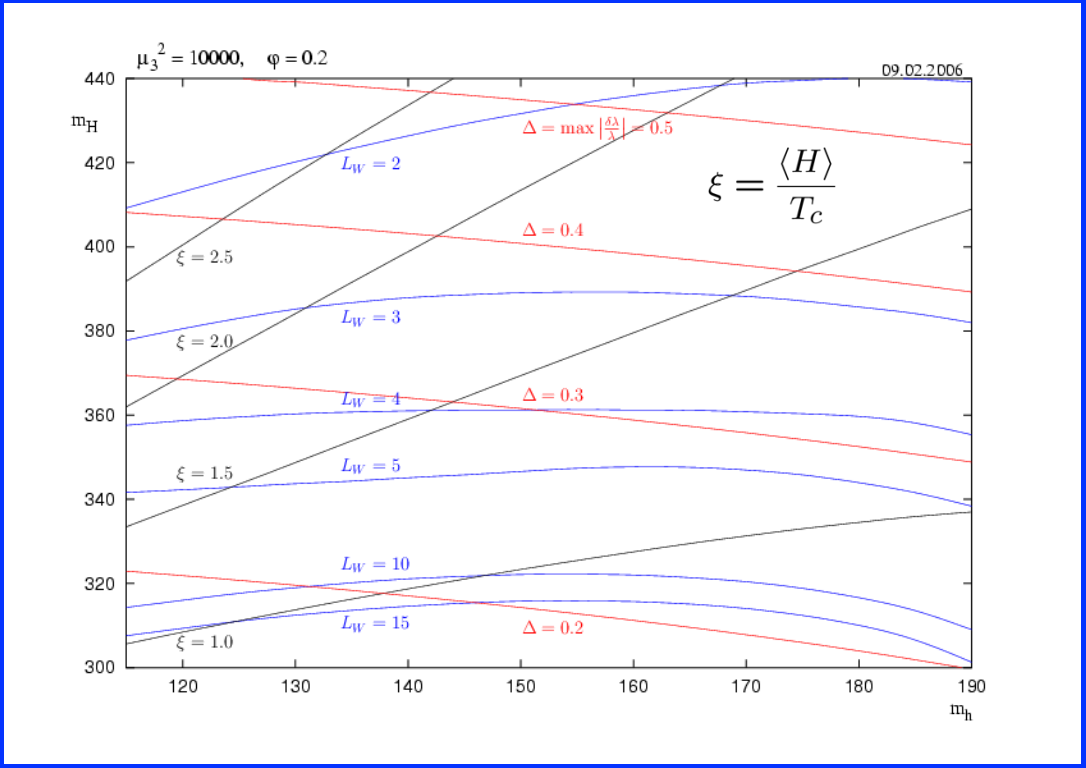
m_H>300 GeV

m_h up to 200 GeV

→ PT ~ independent of Φ

→ thin walls only for very strong PT

(agrees with Cline, Lemieux ’96)



Fromme, S.H., Senuich ‘06

The baryon asymmetry

The relative phase between the Higgs vevs, θ, changes along the bubble wall

→ phase of the top mass varies

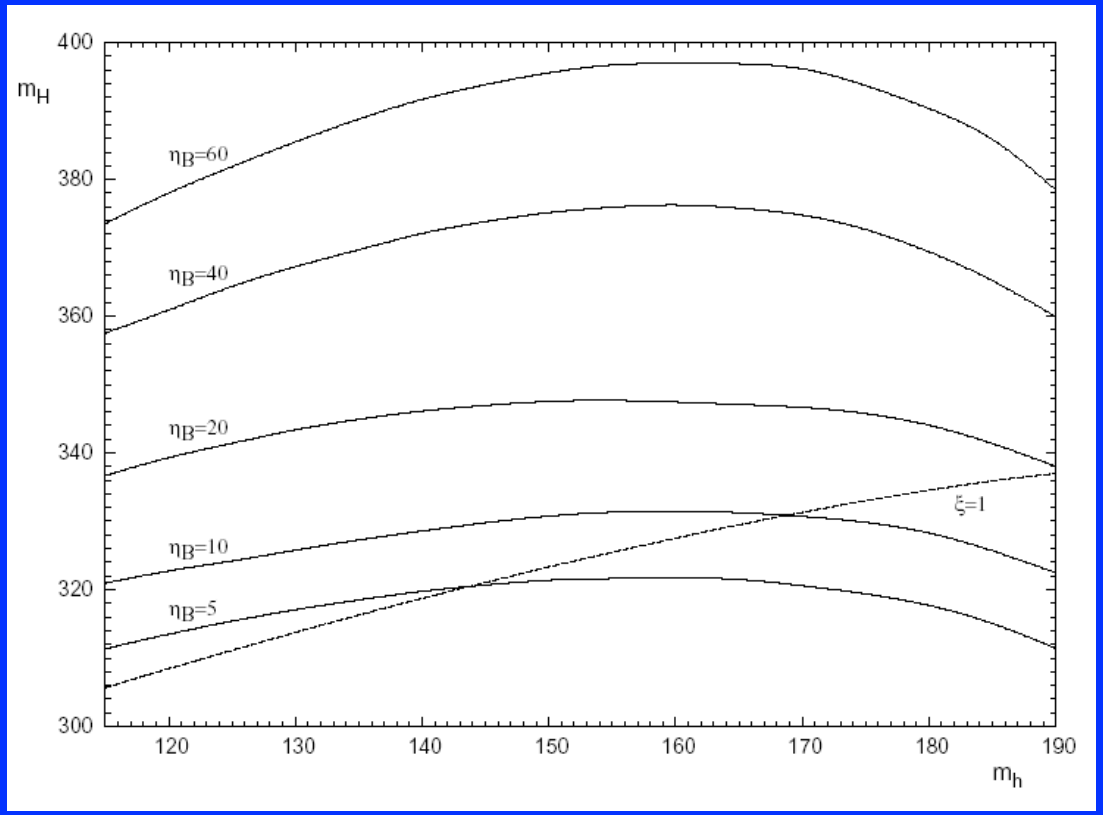
$\theta_t = \theta/(1+\tan^2\beta)$

top transport generates a baryon asymmetry, but tanβ<10 (?)

→ only one phase, so EDMs can be predicted: here

$d_e = 4\ 10^{-30} - 0.3\ 10^{-27}$ e cm

$d_n = 0.1\ 10^{-26} - 7\ 10^{-26}$ e cm



η_B in units of 10^{-11}, φ=0.2

Moretti et al. '07: LHC could see a triple Higgs coupling Hhh

$$f_{\rm turb} = 3.4\times10^{-3}{\rm mHz}\, u_s \left[\frac{1}{RH_*}\right]\left[\frac{T_*}{100{\rm GeV}}\right]\left[\frac{g_*}{100}\right]^{1/6}$$

$$h_0^2\Omega_{\rm turb} = 1.4\times10^{-4} u_s^5 (RH_*)^2 \left[\frac{g_*}{100}\right]^{-1/3}$$

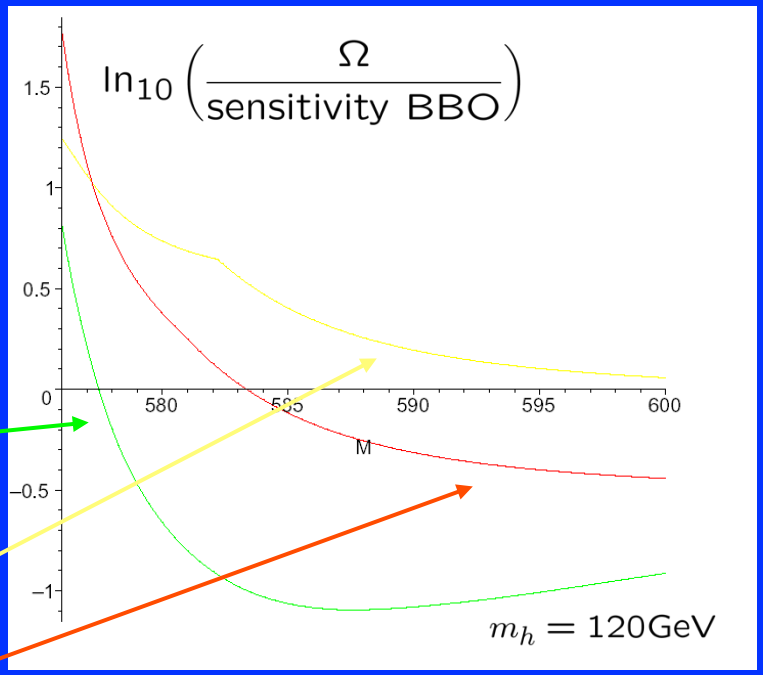
Turbulence: Kosowsky, Mack, Kahniashvili '01

Dolgov, Grasso, Nicolis '02

Turbulence: Caprini, Dürrer '06

Collisions: Kamionkovski, Kosowsky, Turner '93

Φ^6 model:



[in progess with T. Konstandin]

*: quantities at the PT temperature

u_s: eddy velocity (dep. on α)

strong PT: α~1, but RH_* gets larger (fewer bubbles, growing larger)

→ stronger signal, but at smaller f !!

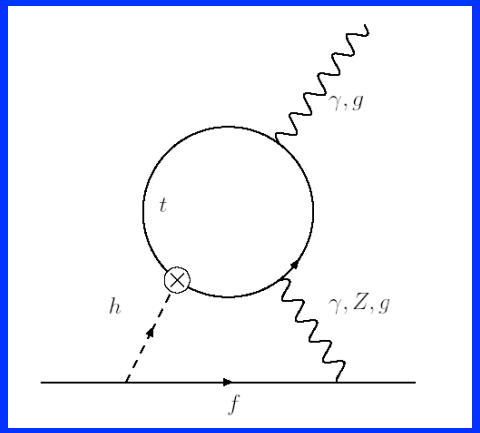
factor 100 between KMK and CD!

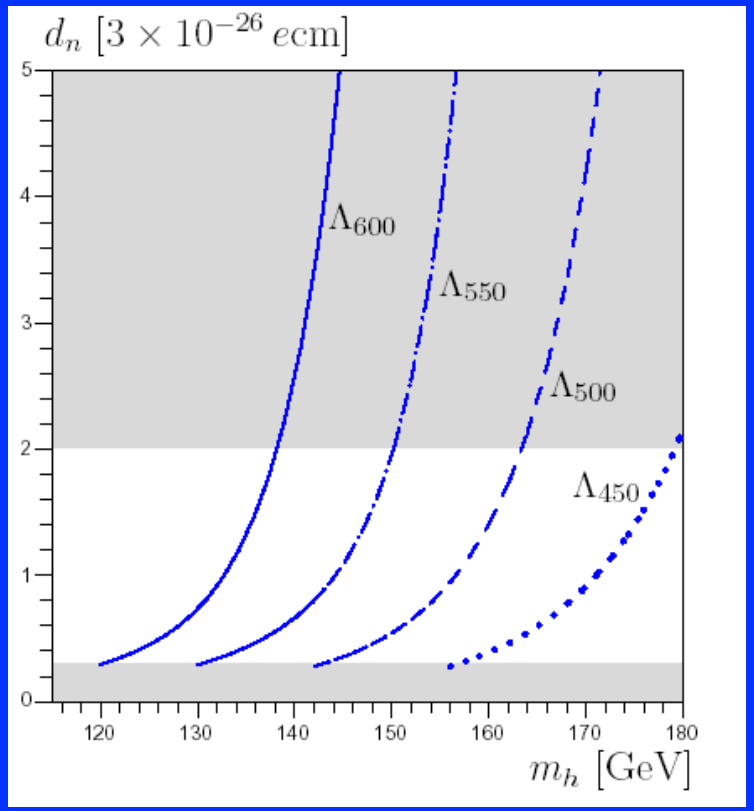
if CD correct, chance for the BBO

LISA can detect GW's only at points extremely close to metastability

EDM from dim-6

x adjusted to get $\eta=\eta_{obs}$:





Barr, Zee '90

Experimental bounds:

$d_e<1.6\ 10^{-27}$ e cm

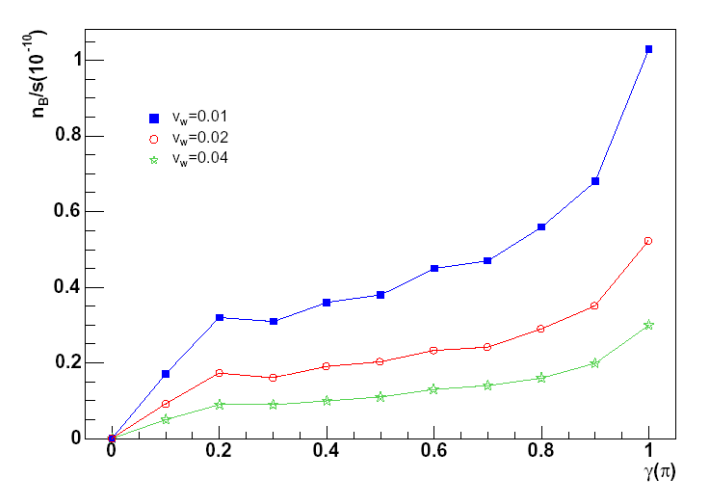
$d_n<3.0\ 10^{-26}$ e cm

S.H., Pospelov, Ritz '06

Extra U(1)´s

Kang, Langacker, Li, Liu ’04

$$W_H = hSH_dH_u + \lambda S_1 S_2 S_3$$



$$m^2_{S_1 S_2} \equiv |m^2_{S_1 S_2}| e^{i\gamma}$$

thin wall approximation used,
tau lepton contribution only

Ham, Oh ’07

$$W \approx h_t Q H_2 t^c_R + \lambda N H_1^T \epsilon H_2$$

Strong phase transition possible

No computation of the BAU

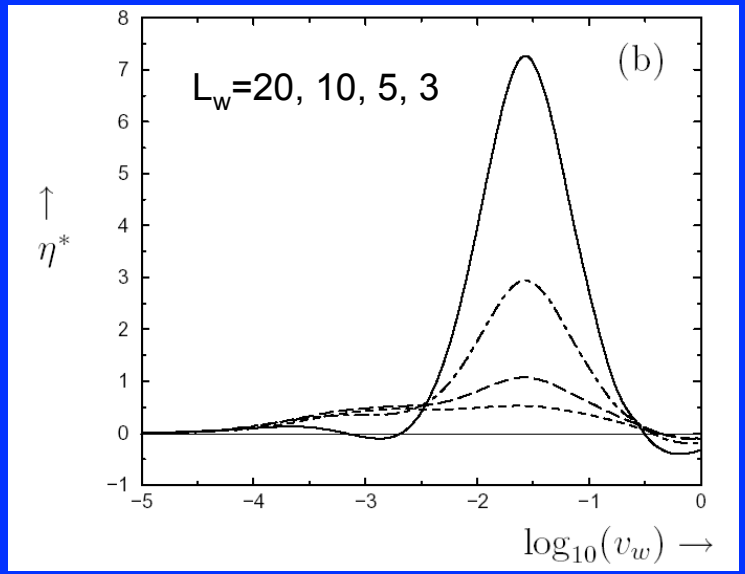
Examples have large λ=0.7, 0.8

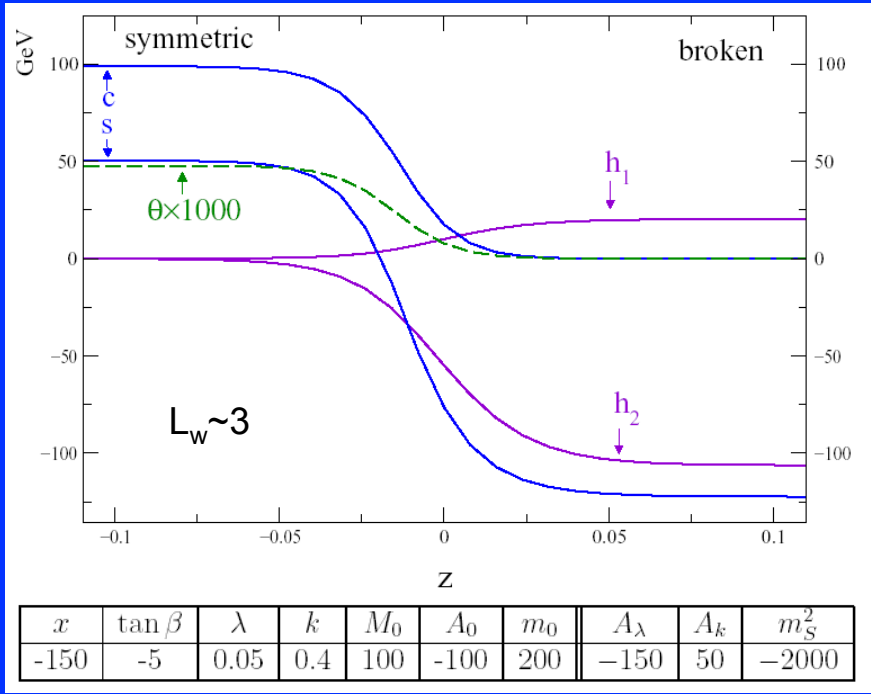
Transitional CP violation

in the general singlet model the broken minimum can be CP conserving, but the symmetric minimum violates CP

→ CP violating wall profile

CP conservation at T=0





x	$\tan\beta$	λ	k	M_0	A_0	m_0	A_λ	A_k	m_S^2
-150	-5	0.05	0.4	100	-100	200	−150	50	−2000

S.H., John, Laine, Schmidt '99

S.H., Schmidt '00

Dynamics of the transition

At the critical temperature T_c the two minima are degenerate

Bubble nucleation starts at $T< T_c$ with a rate

$$\Gamma = A\,T^4\,e^{-S_3/T},$$

Where the bubble energy is

$$S_3 = 4\pi \int dr\, r^2 \left[\frac{1}{2}\left(\frac{d\varphi}{dr}\right) + V(\varphi, T)\right]$$

The bubble configuration follows from

(with γ=2 and appropriate BC's)

$$\frac{\partial^2\varphi}{\partial\varrho^2} + \frac{\gamma}{\varrho}\frac{\partial\varphi}{\partial\varrho} = V'(\varphi)$$

This bounce solution is a saddle point, not a minimum
→ difficult to compute for multi field models (one field: shooting)

For a algorithm see Konstandin, S.H. '06: take γ=0 (energy conserved), modify the potential to obtain a minimization problem, continue to γ=2

Compute the bubble configuration as function of T

$\rightarrow S_3(T)$

Define nucleation temperature T_n when the probability for nucleating one bubble per horizon volume becomes 1

This happens for $S_3(T)/T \sim 130\text{-}150$ (close to T_c the bubbles would have to be very large with huge energy)

The bubbles expand and fill space to a fraction 1-exp(-f):

$$f(T_x) \simeq \frac{4\pi H^3}{3} \int_{T_x}^{\infty} R^3(T_x, T) dP.$$

$$dP = A \frac{T^4}{H^4} e^{-S_3/T} \frac{dT}{T}.$$

$$R(T_x, T) = v_b \frac{T_x}{H(T_x)} \left(\frac{1}{T_x} - \frac{1}{T} \right)$$

Define the end of the phase transition T_f (i.e. when the bubbles bubbles collide) to occur when f=1

Bubble radius:

Define $\langle R \rangle$ from the bubble volume distribution at T_f (Dolgov et al. '02)

alternative:

$$\frac{\beta}{H_*} = -T_* \frac{d}{dT}\left(\frac{S_3}{T}\right)\Big|_{T_*} \qquad \langle R \rangle \approx 3\frac{v_b}{\beta}.$$

note: for large supercooling (strong phase transition) β becomes T-dependent (can become even negative)

Latent heat:

$$\epsilon_* = -\Delta V + T_* \left.\frac{\partial V}{\partial T}\right|_{T_*} \qquad \alpha = \frac{30\epsilon_*}{\pi^2 g_* T_*^4}.$$

The wall velocity:

Friction with the plasma balances the pressure

Distinguish: supersonic vs. subsonic (v_s^2=1/3)

Standard model: v_b~-0.3-0.5 for low Higgs masses [Moore, Prokopec '95]

MSSM: v_b~0.05 [John, Schmidt '00]

All other models: no detailed computations

For very strong phase transitions: bubbles become supersonic, velocity dominated by hydrodynamics (neglect friction) [Steinhardt '82]

$$v_b(\alpha) = \frac{1/\sqrt{3} + \sqrt{\alpha^2 + 2\alpha/3}}{1+\alpha}.$$

(for sufficiently large α > few %)

Gravitational waves:

Bubble collisions ("detonations"): [Kamionkowski, Kosowsky, Turner '93]

(see also Caprini, Dürrer, Servant '07)

$$h_0^2\Omega_{\rm det} \simeq 1.2\times 10^{-7}\kappa^2 \langle R\rangle^2 H_*^2 \left[\frac{\alpha}{\alpha+1}\right]^2 \left[\frac{v_b^2}{0.24+v_b^3}\right]^2 \left[\frac{100}{g_*}\right]^{1/3}.$$

$$\kappa(\alpha) \simeq \frac{1}{1+0.715\alpha}\left[0.715\alpha + \frac{4}{27}\sqrt{\frac{3\alpha}{2}}\right]$$

"efficiency factor", assumes supersonic bubbles!

Turbulence: [Kosowsky, Mack, Kahniashvili '01, Dolgov et al. '02]

$$h_0^2\Omega_{\rm turb} \simeq 5.6\times 10^{-6} u_S^5 \langle R\rangle^2 H_*^2 \left[\frac{100}{g_*}\right]^{1/3}.$$

$$u_S \simeq \sqrt{\frac{\kappa\alpha}{4/3+\kappa\alpha}}.$$

(eddy velocity)

[Caprini, Dürrer '02]

$$h_0^2\Omega_{\rm Richardson} \simeq 6.7\times 10^{-6} u_S^4 \langle R\rangle^2 H_*^2 \left[\frac{100}{g_*}\right]^{1/3}.$$

(Different modeling of time correlation in turbulence)

Frequencies:

$$f_{\rm det} \simeq 1.6 \times 10^{-2}{\rm mHz}\, v_b \frac{1}{\langle R\rangle\, H_*} \frac{T_*}{100\,{\rm GeV}} \left[\frac{g_*}{100}\right]^{1/6},$$

$$f_{\rm turb} \simeq 1.7 \times 10^{-2}{\rm mHz}\, u_S \frac{1}{\langle R\rangle\, H_*} \frac{T_*}{100\,{\rm GeV}} \left[\frac{g_*}{100}\right]^{1/6},$$

$$f_{\rm Richardson} \simeq 1.7 \times 10^{-2}{\rm mHz} \frac{1}{\langle R\rangle\, H_*} \frac{T_*}{100\,{\rm GeV}} \left[\frac{g_*}{100}\right]^{1/6}.$$

Transport equations

We want to write down a set of Boltzmann equations

The interaction with the bubble wall induces a force on the particles, which is different for particles and antiparticles if CP is broken

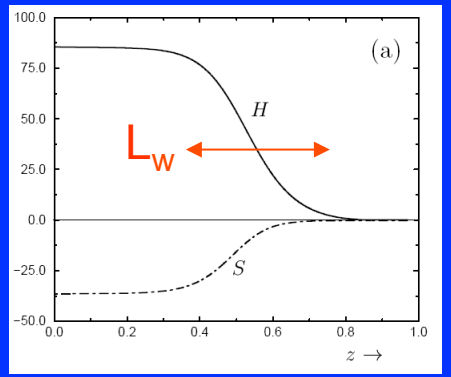
$$(\partial_t + \dot{z}\partial_z + \dot{p}_z\partial_{p_z})f = \mathcal{C}[f]$$

collision terms

Compute the force from dispersion relations

$$\dot{p}_z = -\partial_z E(z, p_z)$$

z is the coordinate along the wall profile
H(z)~tanh(z/L_w) with wall width L_w



WKB approximation

Elektroweak bubbles have typically thick walls, i.e. $L_w >> (T_c)^{-1}$
$(L_w)^{-1} << p$ for a typical particle in the plasma

Compute the dispersion relation via an expansion in $1/(L_w T_c)$

Consider a free fermion with a complex mass

$$M(z) = m(z) e^{i\theta(z)}$$

$$(i\partial - P_L M(z) - P_R M^*(z))\psi = 0$$

$$\psi \sim \exp(-iEt - i\int^z p_z(z')dz')$$

$$\begin{aligned} E_\pm &= E_0 \pm \Delta E_0 \\ &= \sqrt{p^2+m^2} \pm \theta' \frac{m^2}{2(p^2+m^2)} \end{aligned}$$

Joyce, Prokopec, Turok ' 95
Cline, Joyce, Kainulainen ' 00

more rigorous, using the Schwinger-Keldysh formalism:
Kainulainen, Prokopec, Schmidt, Weinstock ' 01-' 04
Konstandin, Prokopec, Schmidt, Seco '05

alternative:
Carena, Moreno, Quiros, Seco, Wagner '00

only a varying θ contributes!

no effect for scalars in LO!

Diffusion equations

Fluid ansatz for the phase space densities: $f_i = \frac{1}{e^{(E_i - v_i p_z - \mu_i)/T} \pm 1}$

to arrive at diffusion equations for the μ's

$$-(D_i \mu_i'' + v_w \mu_i') + \Gamma_{ij} \mu_j = S_i$$

diffusion constant — wall velocity — interaction rates — CP violating source terms

relevant particles: top, Higgs, super partners,…

interactions: top Yukawa interaction
strong sphalerons
top helicity flips (broken phase)
super gauge interactions (equ.)

Step 1: compute n_{B_L} $(= -n_{B_R})$

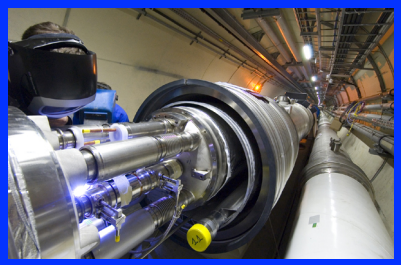
Step 2: switch on the weak sphalerons

$$\eta_B \sim \Gamma_{\rm ws} \int^{\infty} dz \; n_{B_L}(z)$$

Why is it interesting?

There are testable consequences:

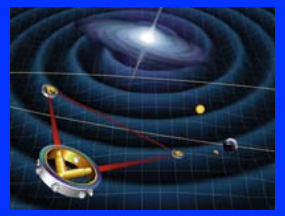
- New particles (scalars?!) at the LHC
  (Higgs sector is crucial!)



- New sources of CP violation which should show up soon in electric dipole experiments
  new bound for electron edm $<1.0\times10^{-27}$ ecm (IC group)



- Could the electroweak phase transition produce observable gravitational waves?



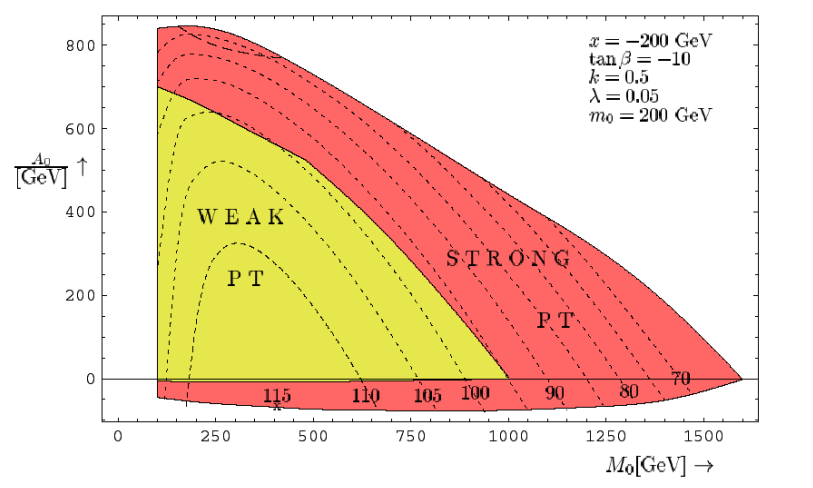
If confirmed, it would constrain the early universe up to T~100 GeV

(nano sec.), like nucleosynthesis does for the MeV-scale (min.)

Strong phase transition

singlet model without discrete symmetries

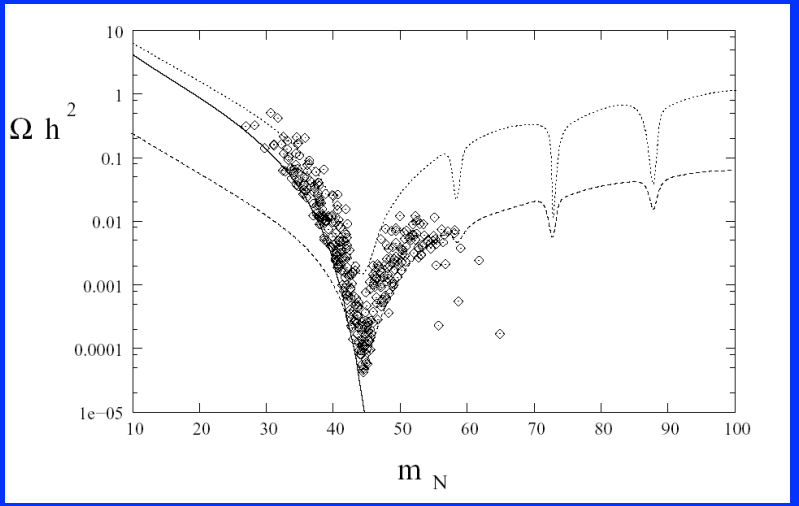
$$W = \lambda S H_1 H_2 + \frac{k}{3} S^3 + \mu H_1 H_2 + rS$$



S.H.,Schmidt '00

nMSSM

$$W_{nMSSM} = \lambda \hat{S} \hat{H}_1 \cdot \hat{H}_2 + \frac{m_{12}^2}{\lambda} \hat{S}$$



Menon, Morrissey, Wagner ' 04

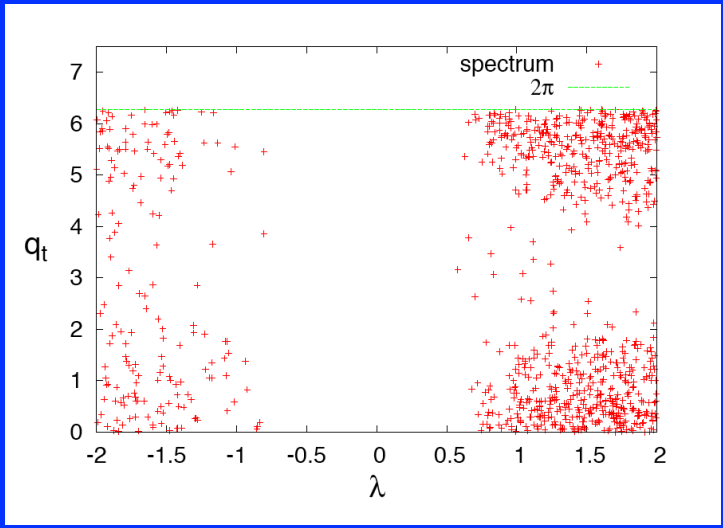
(S.H., Konstandin, Prokopec, Schmidt '06)

Baryogenesis in the nMSSM

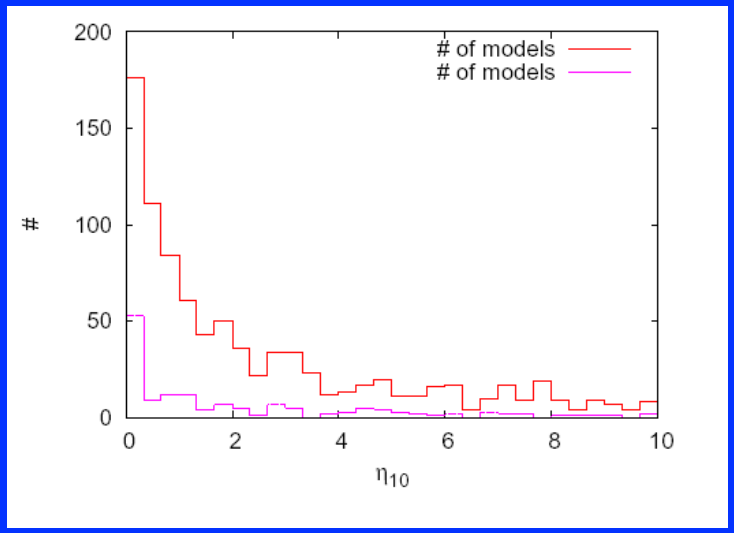
λ above Landau pole prefered:

(and tan β ~1)

CP violation in $t_S e^{iq} S$ (phase in μ parameter induced, not constant along the bubble wall)



EDM constraints with 1TeV sfermions (1. & 2. generation):

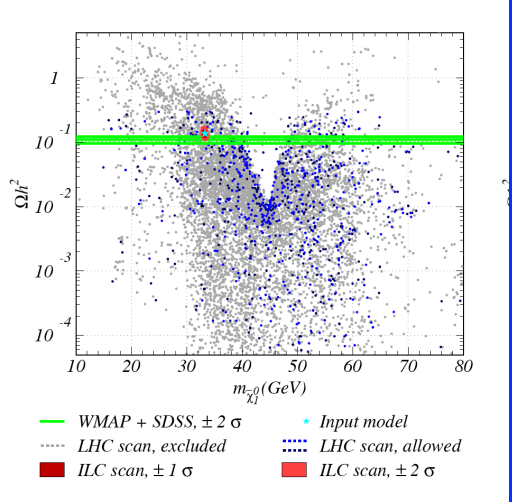


S.H., Konstandin, Prokopec, Schmidt '06

Colliders vs. cosmology: nMSSM

[Balazs, Carena, Freitas, Wagner '07]

Dark matter:



(problem: large error on neutralino mass at LHC)

Baryogenesis:

Presence of light charginos could be shown, especially at ILC

LHC could see a Higgs signal, but difficult to separate the different states (ILC!)

ILC could determine crucial parameters for the phase transition A_λ, t_s, m_s at 10-20% (still not sufficient to establish a strong PT)

EDMs should (probably) be seen by next generation experiments

→ predicts new physics at LHC

Keep in mind: model dependence!! (only an example case)

(Also the non-SUSY singlet models have been studied recently, e.g. Profumo et al. '07)

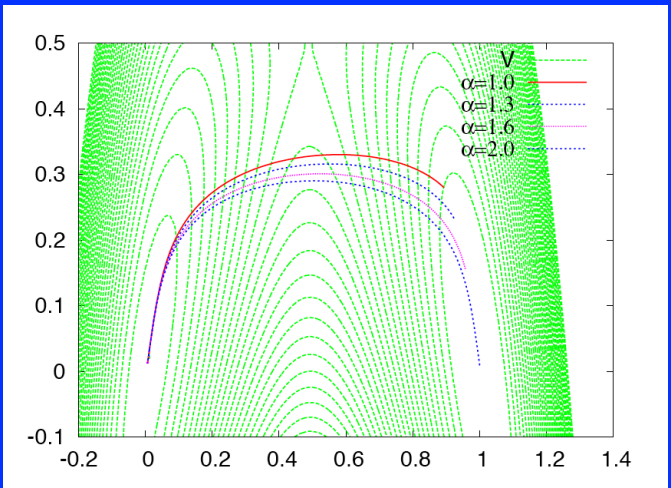
LISA has its maximal sensitivity at frequencies related to the electroweak phase transition:

Observable signal? (direct signal of the phase transition!)

Are the parameters α and <R> correlated in specific models?

What does this mean for the GW signal?

How important is the precise knowledge of the bubble configurations?



Konstandin, S.H '06

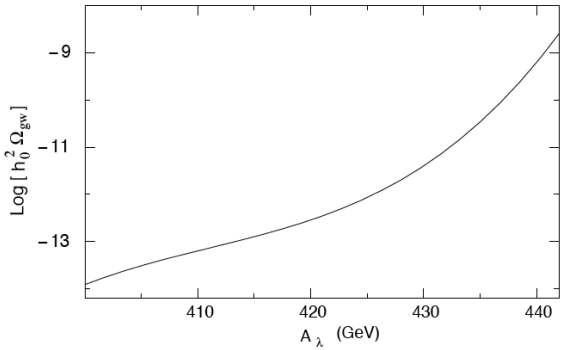
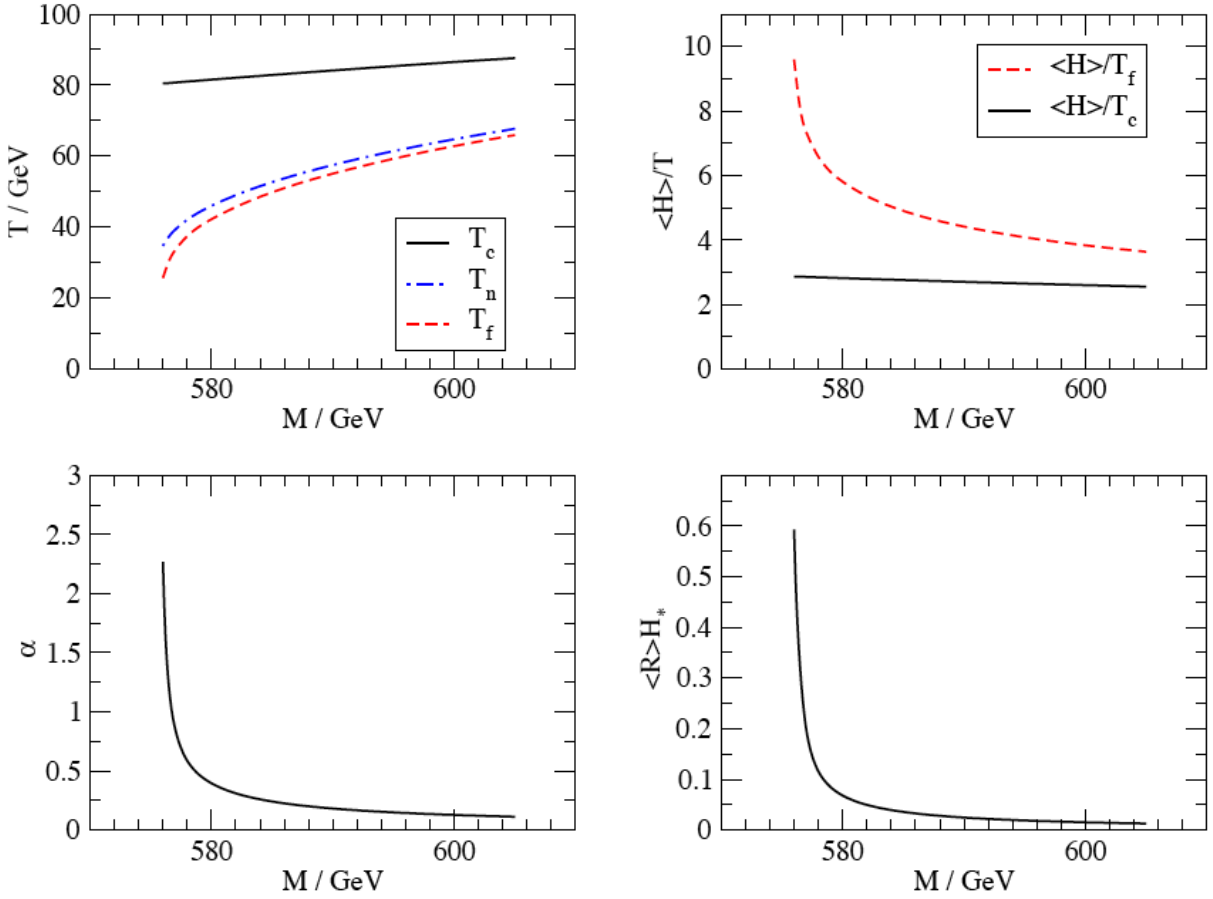


Figure 13: $h_0^2\Omega_{\rm gw}$ from turbolence as a function of A_λ for $A_k = 480$ GeV, $x = 350$ GeV, $\lambda = 0.83$, $k = 0.67$ and $\tan\beta = 2$.

Apreda, Maggiore, Nicolis, Riotto '01

Key parameteres of the phase transition: Φ^6 model, m_h=120 GeV



S. H. & Konstandin '07

Compute as function of temperature: bubble configurations → E

nucleation rate Γ~exp(-E)

bubble distribution → R

$$f_{\rm turb} = 3.4\times10^{-3}{\rm mHz}\, u_s \left[\frac{1}{RH_*}\right]\left[\frac{T_*}{100{\rm GeV}}\right]\left[\frac{g_*}{100}\right]^{1/6}$$

$$h_0^2\Omega_{\rm turb} = 1.4\times10^{-4} u_s^5 (RH_*)^2 \left[\frac{g_*}{100}\right]^{-1/3}$$

Turbulence: Kosowsky, Mack, Kahniashvili '01

Dolgov, Grasso, Nicolis '02

Turbulence: Caprini, Dürrer '06

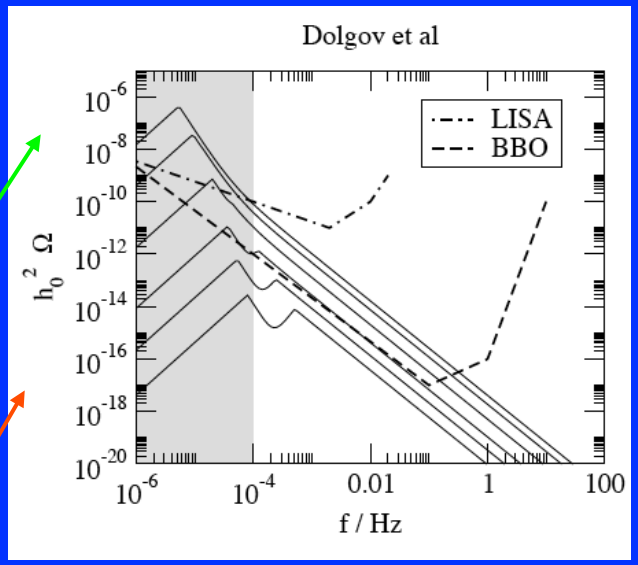
Collisions: Kamionkovski, Kosowsky, Turner '93

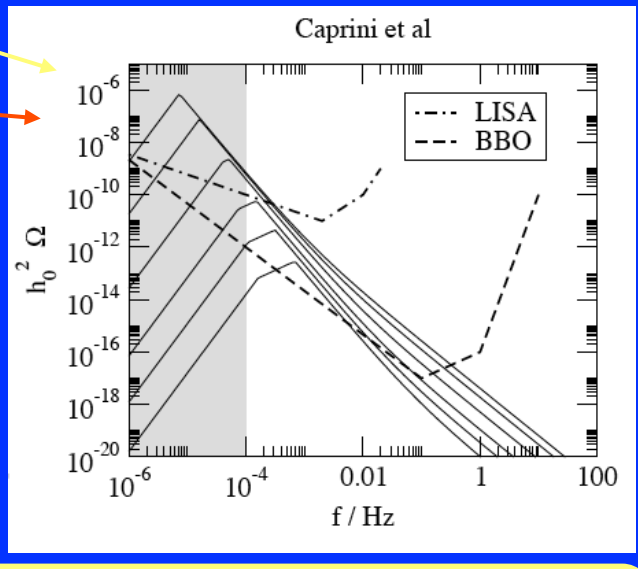
*: quantities at the PT temperature

u_s: eddy velocity (dep. on α)

strong PT: α~1, but RH_* gets larger
(fewer bubbles, growing larger)

→ stronger signal, but at smaller f !!

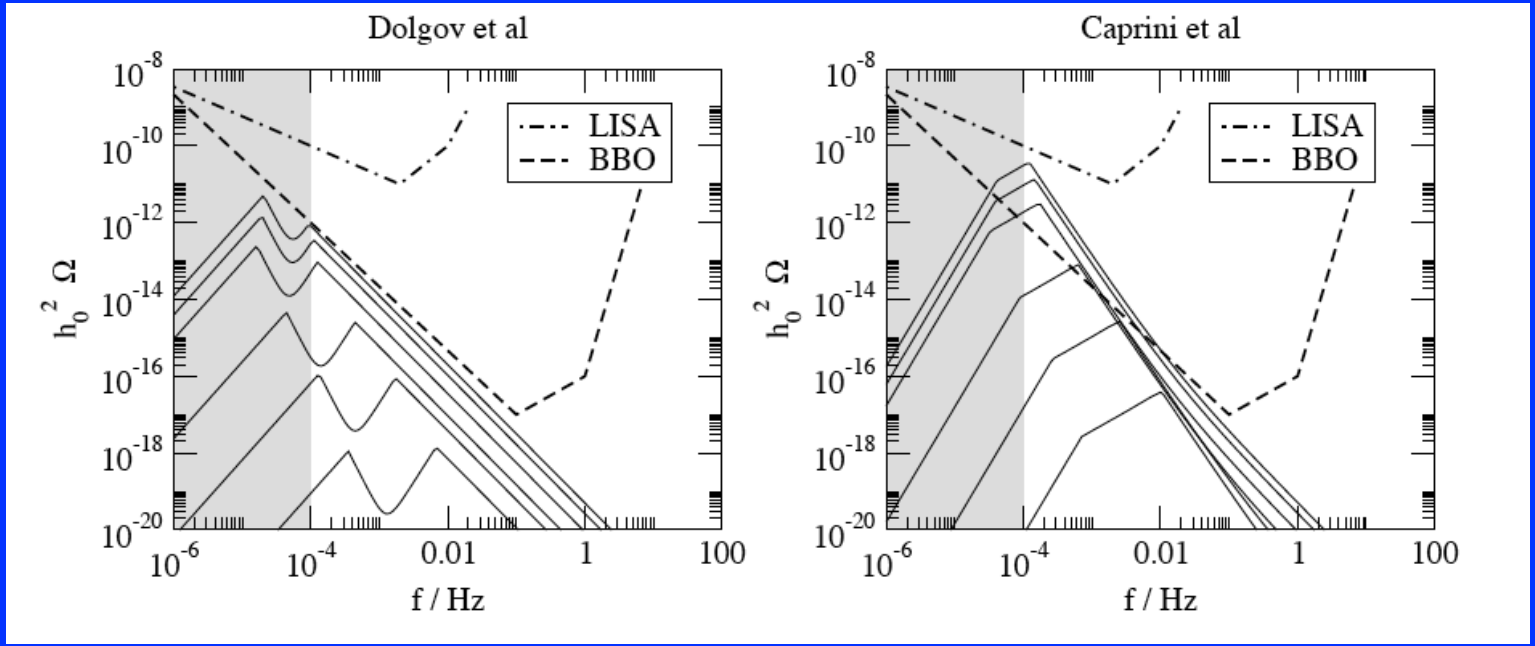




discrepancy between Dol. and CD!

double peak structure?

Gravitational waves in the nMSSM



S. H., Konstandin '07

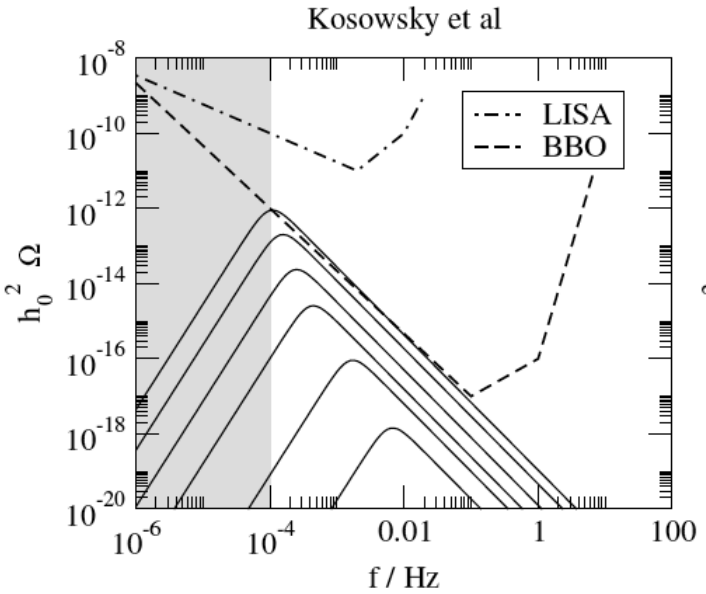
Very similar to Φ^6 case, but slightly weaker signal (tuning?)

Cannot confirm the much larger results of Apreda, Maggiore, Nicolis, Riotto '01 (important to compute the bubbles shapes correctly!)

Baryogenesis: strong GW signal only for $v_b > v_s$ → no diffusion. Baryogenesis would at least look different

Revisiting bubble collisions

Use “envelope approximation”



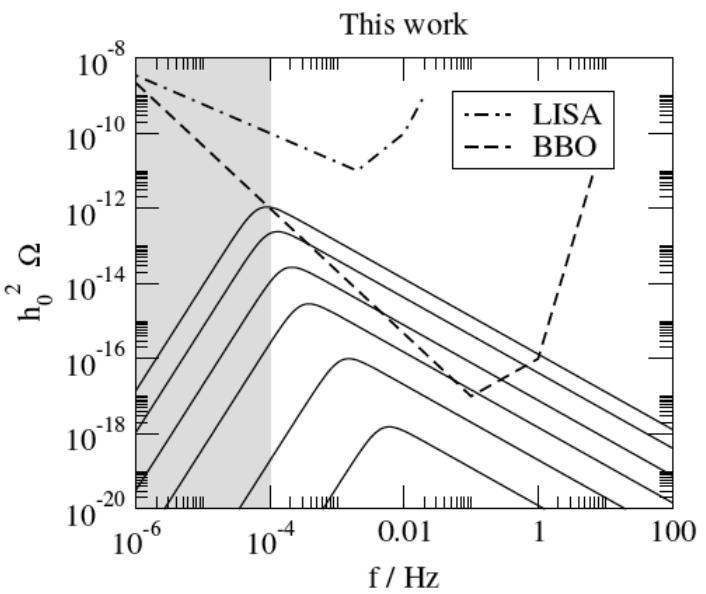


FIG. 3: Several spectra of gravitational radiation according to the old and new formulas. The

$GW \sim f^{-1.8} \Rightarrow GW \sim f^{-1}$

(related to small bubbles??)

T. Konstandin, S.H ‘08