Interweaving Chiral Spirals

Toru Kojo (Bielefeld U.)

with: **K. Fukushima, Y. Hidaka, L. McLerran, R.D. Pisarski**

References)

Quarkyonic Chiral Spirals (QCS)

Kojo-Hidaka-McLerran-Pisarski (NPA 843 (2010) 37)

Covering the Fermi surface with patches of QCSs

Kojo-Pisarski-Tsvelik (PRD 82 (2010) 074015)

A (1+1) dimensional example of Quarkyonic matter

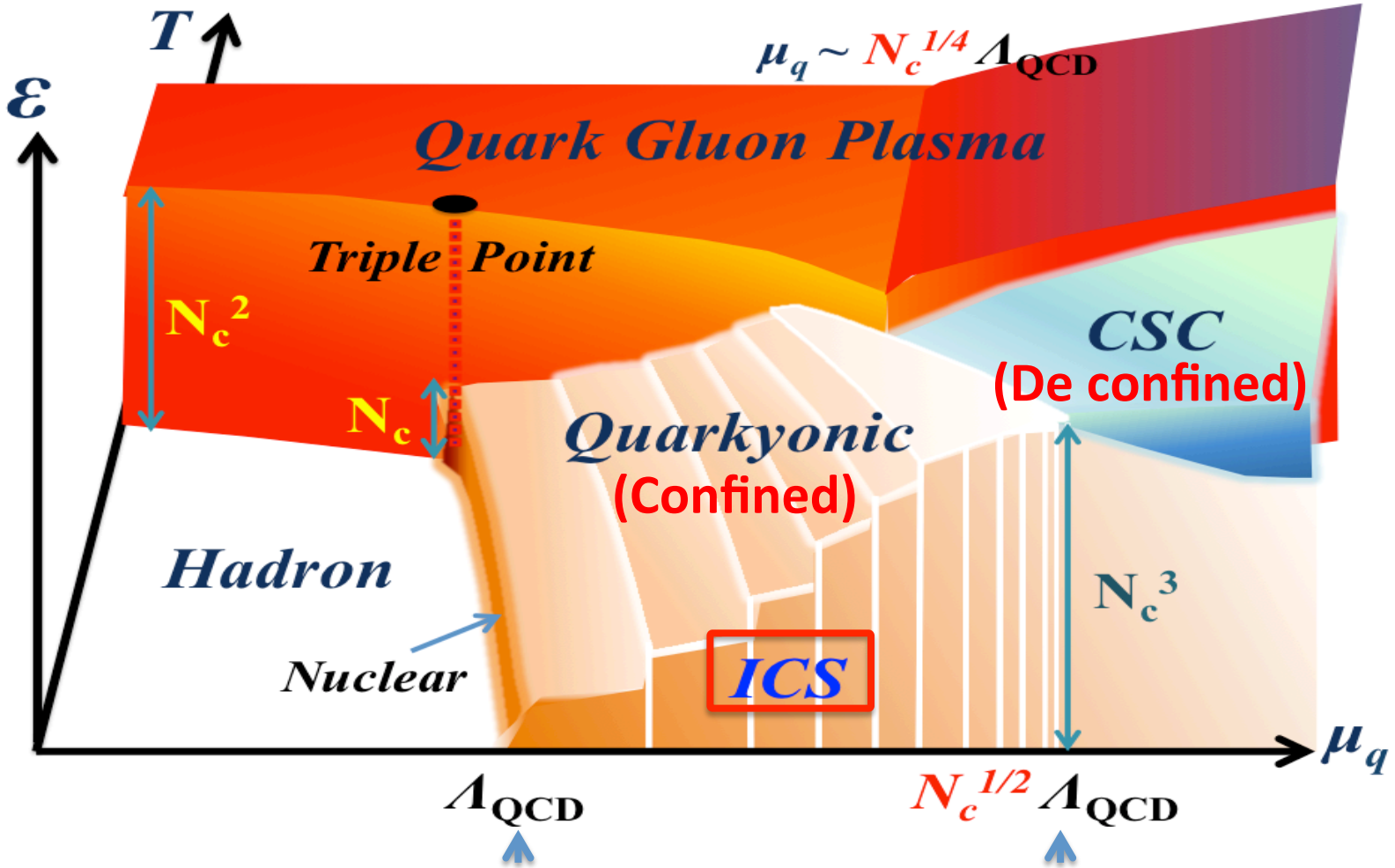
Kojo (NPA 877 (2012) 70)

Interweaving Chiral Spirals (ICS)

Kojo-Hidaka-Fukushima-McLerran-Pisarski (NPA 875 (2012) 94)

Recently conjectured Phase diagram

Kojo-Hidaka-Fukushima-McLerran-Pisarski (2011)

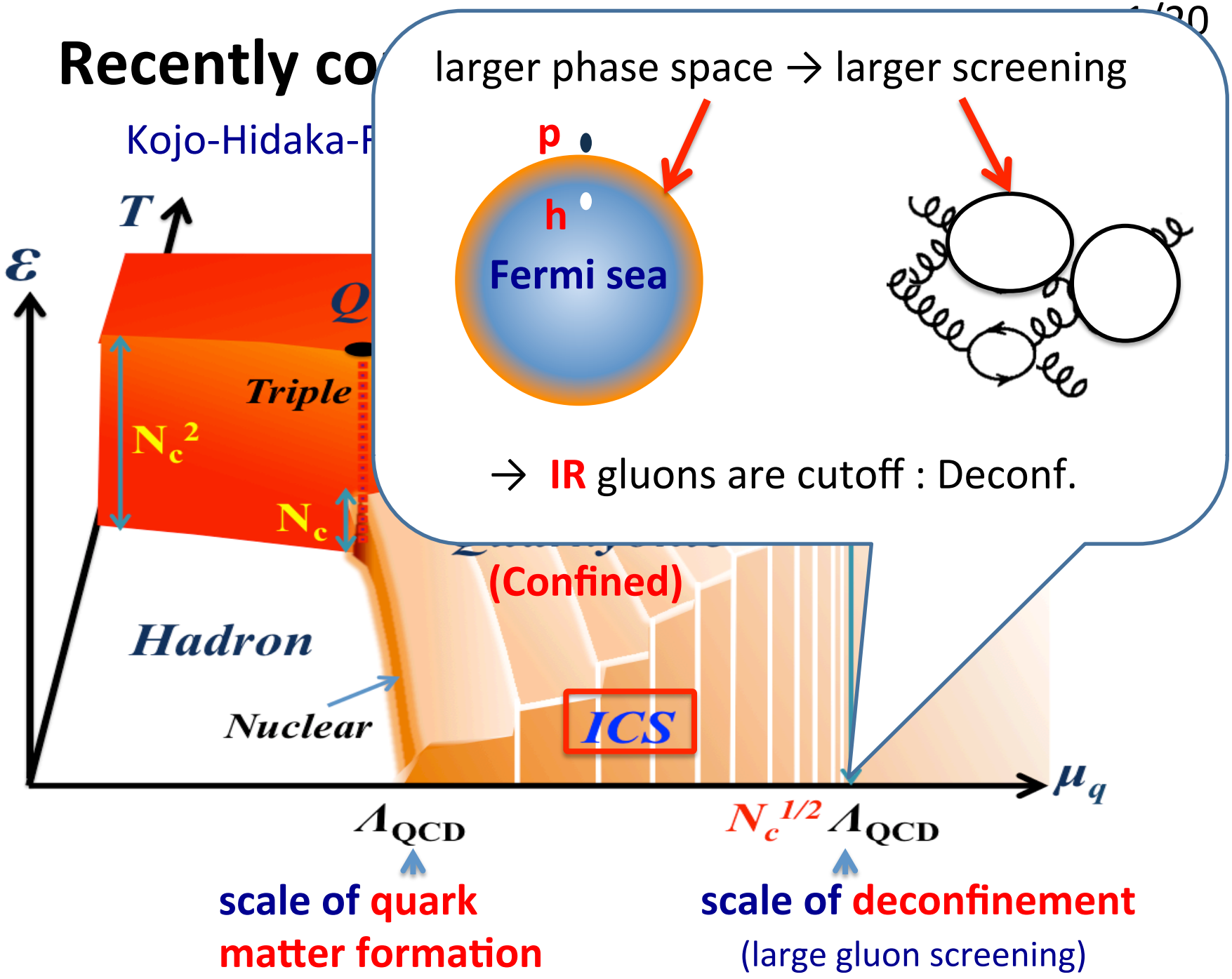


scale of quark matter formation

scale of deconfinement (large gluon screening)

Recently co

Kojo-Hidaka-F



Quarkyonic Matter: Basic picture

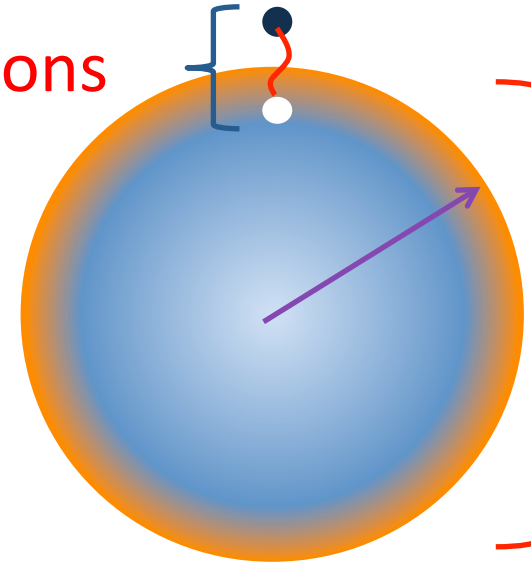
McLerran-Pisarski (2007)

hadronic excitations

Transport properties

Phase structures (condensation)

etc.



Bulk properties:

(EOS, Pressure, etc.)

Weakly int. quarks

Pauli-blocking.

forming color singlet B.G.

Quark Fermi sea + baryonic Fermi surface → Quark-yonic

(hadronic)

Domain of applicability :

$$M_N/N_c \;(\sim \Lambda_{QCD}) \;<\; \mu_q \;<\; \begin{cases} N_c^{1/2}\,\Lambda_{QCD} & (3+1)\ \mathrm{D} \\ \infty & (1+1)\ \mathrm{D} \end{cases}$$

(→ Ref: Kojo12)

This talk

I will just **assume** a strong non-perturbative force.

(*Not necessarily confining*)

large Nc & large μ → Fermi sea & non-pert. force.

→ semi-analytic understanding of CS.

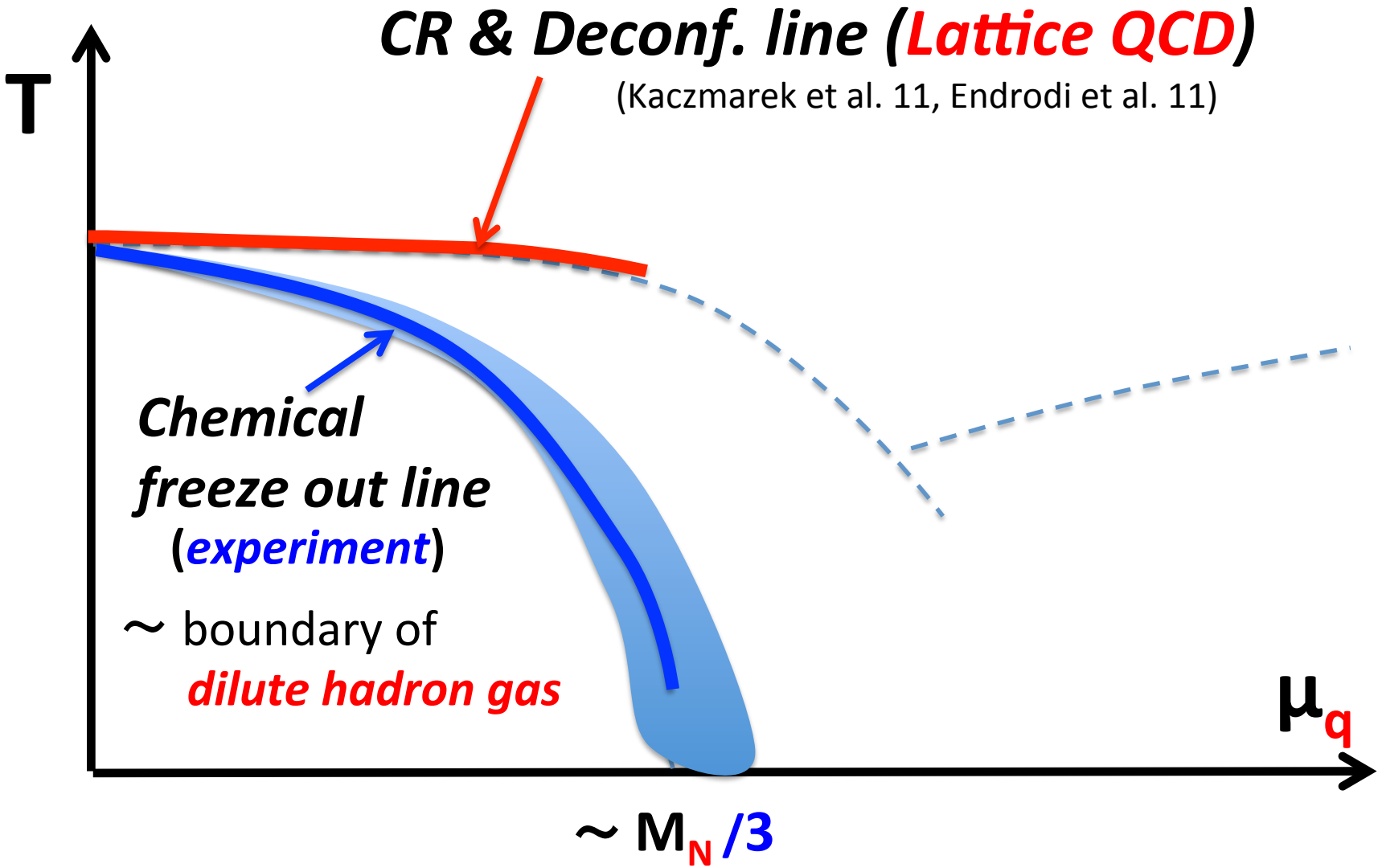
Contents

1, Possible Phenomenological impact of CS

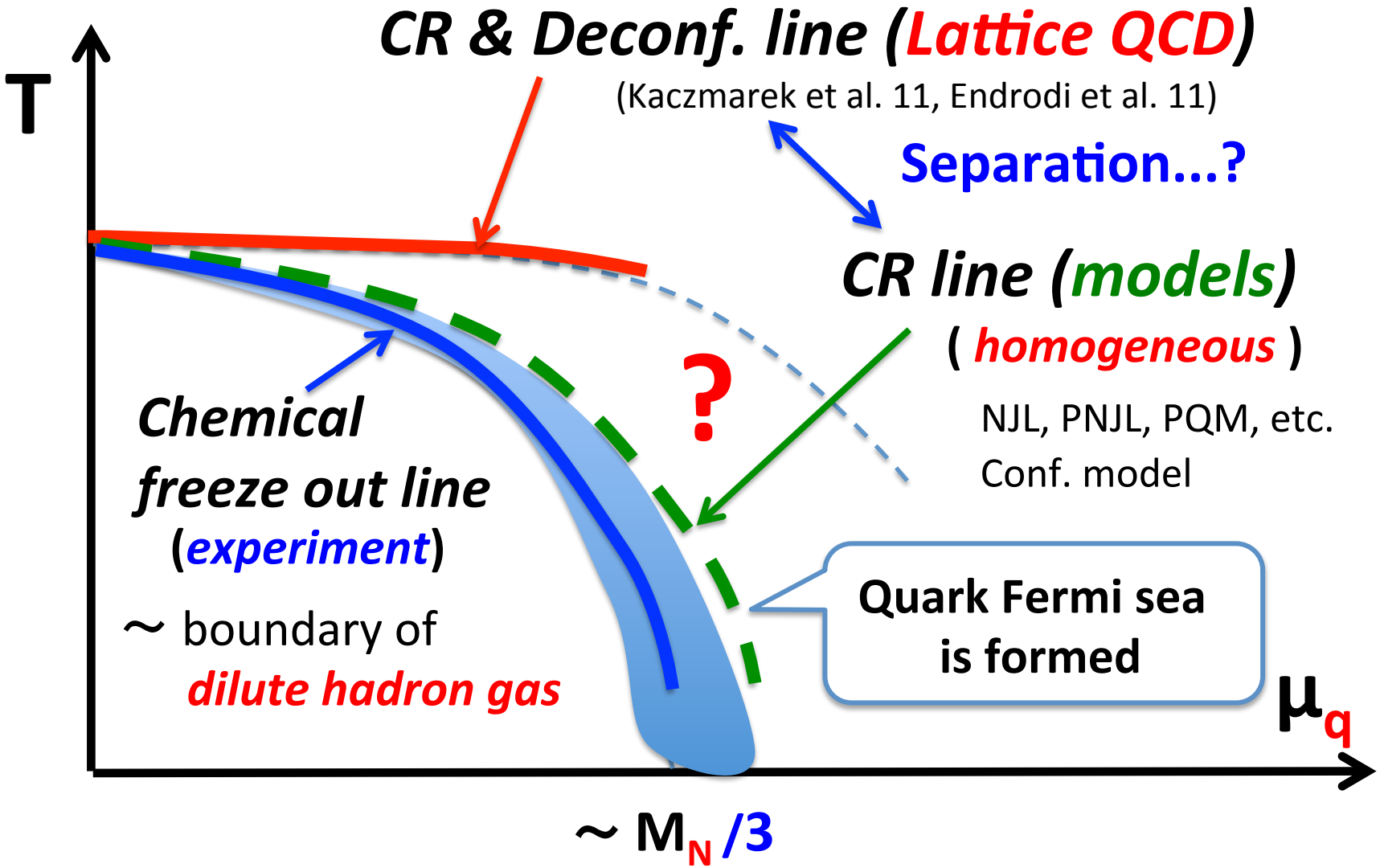
2, Quick review of Single Chiral Spiral (CS)

3, How to interweave many CSs

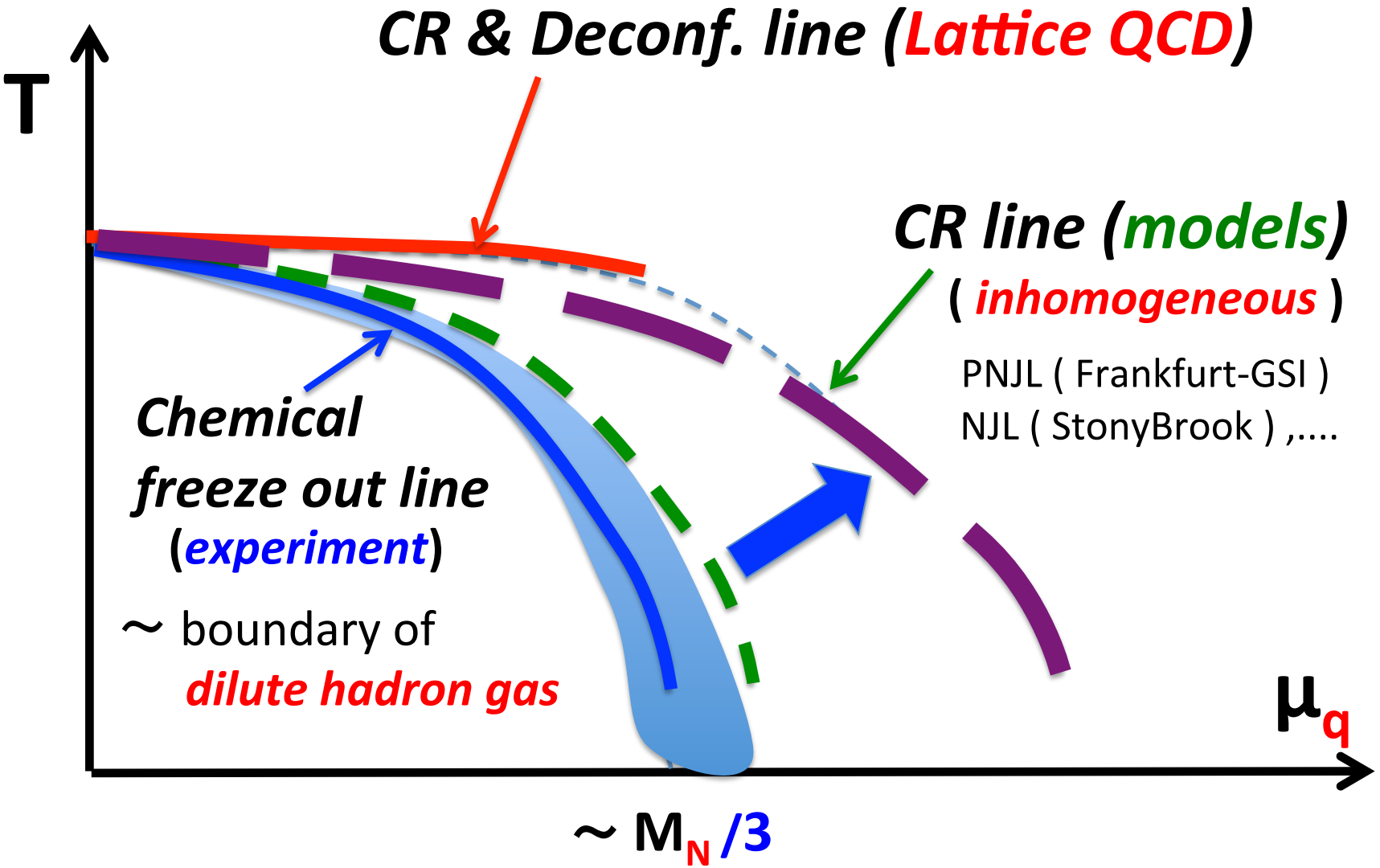
Chiral Restoration (CR) line



Chiral Restoration (CR) line



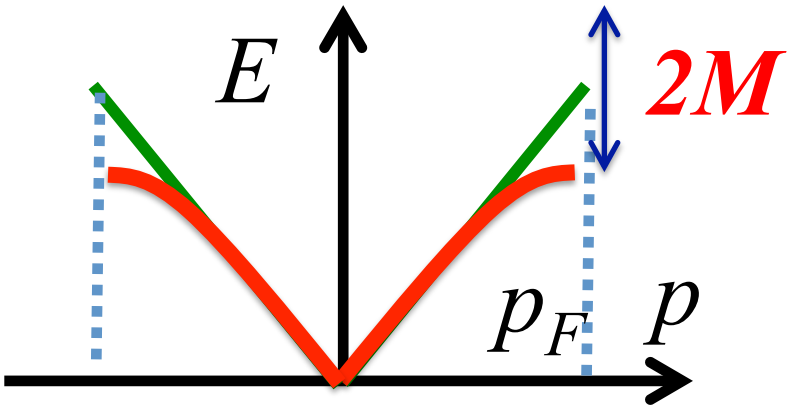
With inhomogeneous chiral condensate...



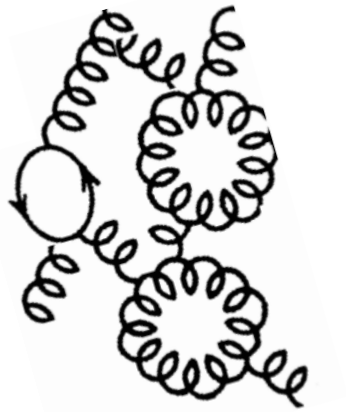
Impact on deconfinement line

***Mass gap* is opened *near the Fermi surface*:**

Quark dispersion with
Chiral Spiral background



suppression of
quark loops



→ ***Deconf. line should be shifted to higher μ.***

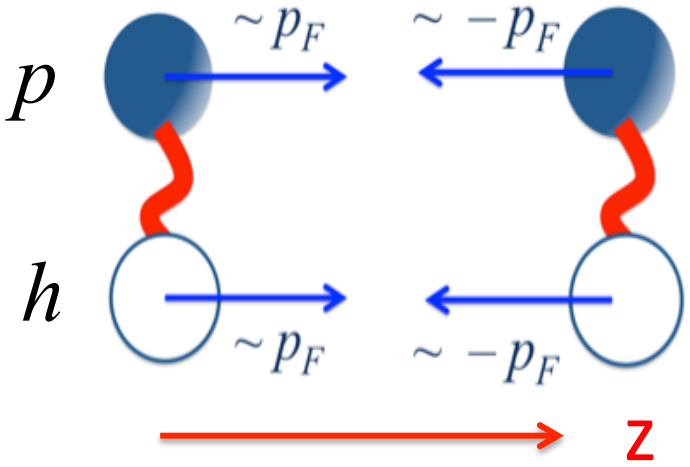
cf.) **Color singlet *diquark condensate* in *2-color QCD***

& Color singlet *pion condensate* in *isospin QCD*

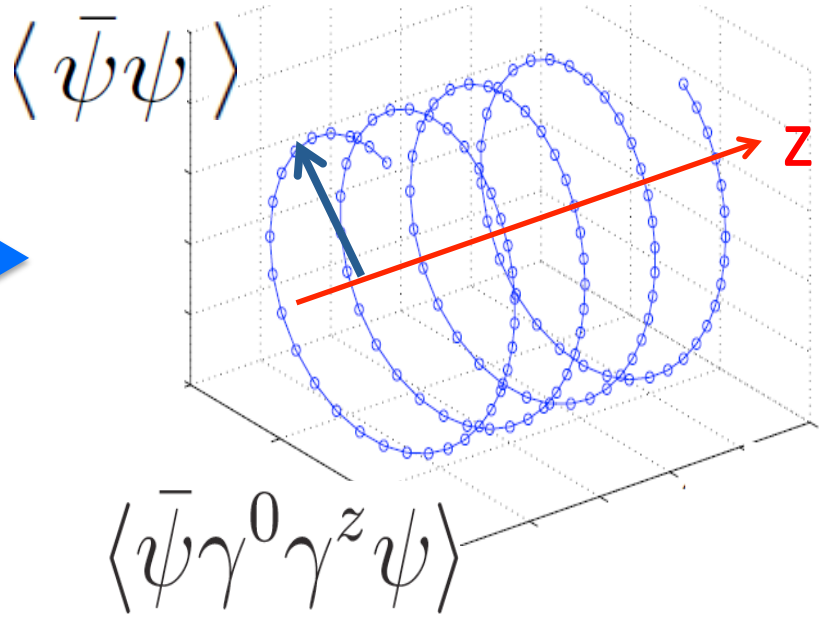
can do the similar job!

Single Chiral Spiral at large μ

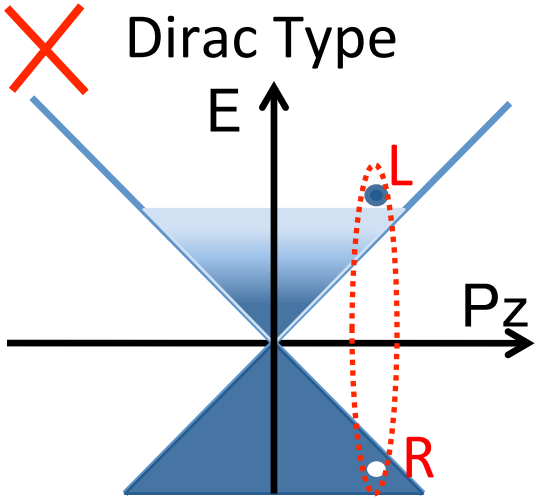
2 Chiral Density Waves



Single Chiral Spiral

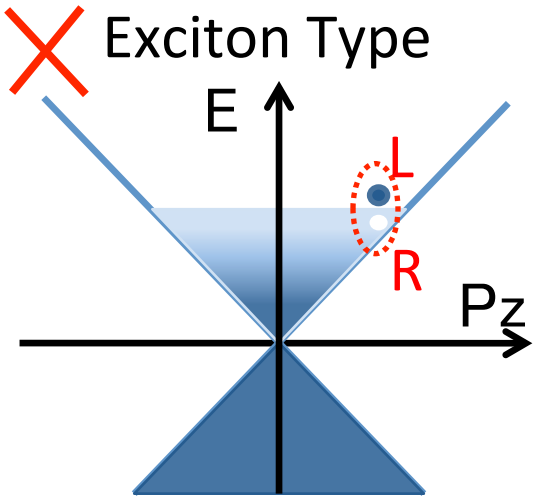


Candidates of Chiral Pairing (T=0)



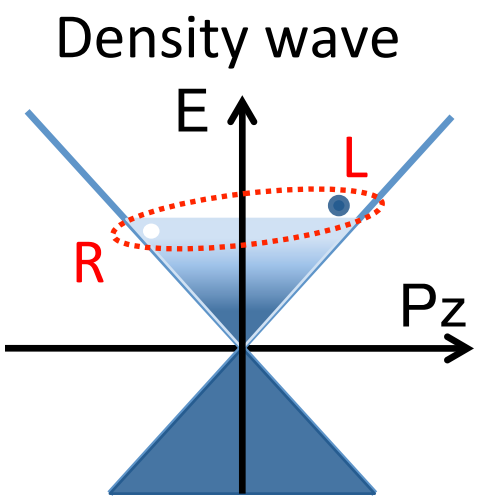
$P_{Tot}=0$ (uniform)

Kin. suppressed



$P_{Tot}=0$ (uniform)

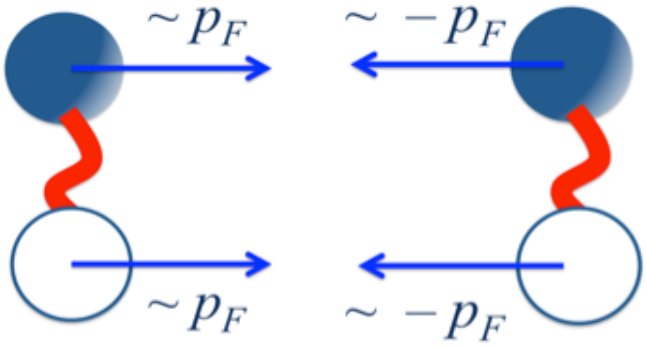
NOT favored by **int.**



$P_{Tot}=2\mu$ (non-uniform)

favored by **int.**

Co-moving pairs condense

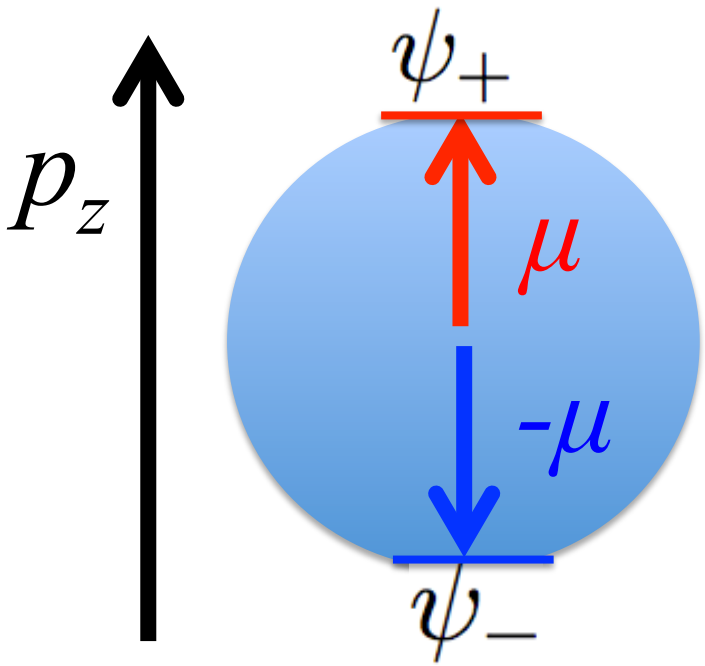


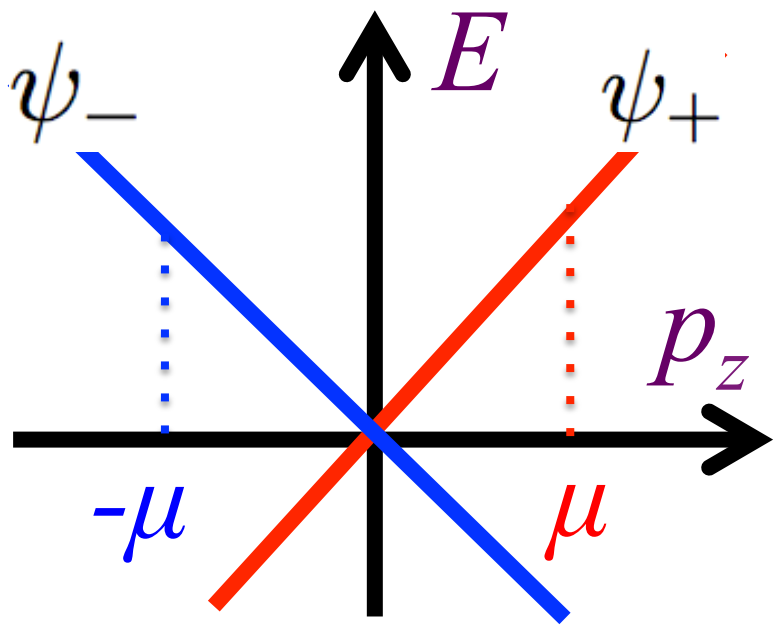
Single Chiral Spiral in z-direction

- ***Projection***: $\psi_\pm = \frac{1 \pm \gamma_0\gamma_z}{2}\psi$ ← like (1+1) D chirality

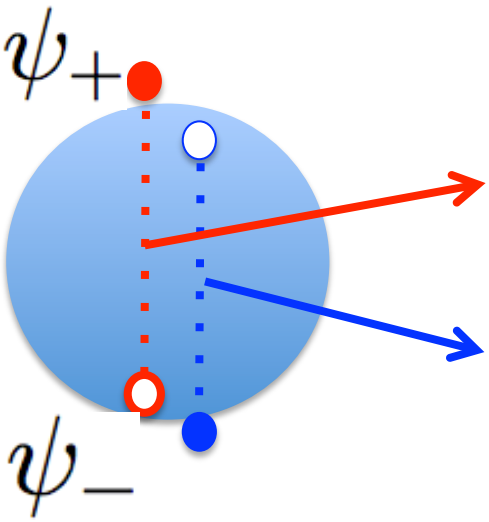
- ***Kin. terms***: $\mathcal{L}_{kin} \simeq \psi_\pm^\dagger \left(\mathrm{i}(\partial_0 \mp \partial_z) + \mu\right)\psi_\pm + \psi_\pm^\dagger \frac{\partial_\perp^2}{2\mu}\psi_\mp$

(near the Fermi surface) **longitudinal** **transverse**





Single Chiral Spiral in z-direction



$p \quad h$

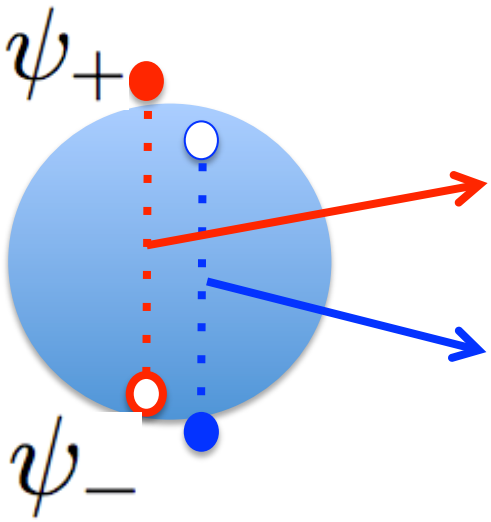
$$\langle \bar{\psi}_+ \psi_- \rangle = \Delta \, \mathrm{e}^{2ip_F z}$$

$$\langle \bar{\psi}_- \psi_+ \rangle = \Delta \, \mathrm{e}^{-2ip_F z}$$

Phase (due to finite mom.)

$(\Delta \sim \Lambda_{QCD}^{\,3})$

Single Chiral Spiral in z-direction



$p \quad h$

Phase (due to finite mom.)

$$\langle \bar{\psi}_+ \psi_- \rangle = \Delta \, \mathrm{e}^{2ip_F z}$$

$$\langle \bar{\psi}_- \psi_+ \rangle = \Delta \, \mathrm{e}^{-2ip_F z}$$

($\Delta \sim \Lambda_{QCD}^3$)

$$\psi_\pm = \frac{1 \pm \gamma_0\gamma_z}{2}\psi$$

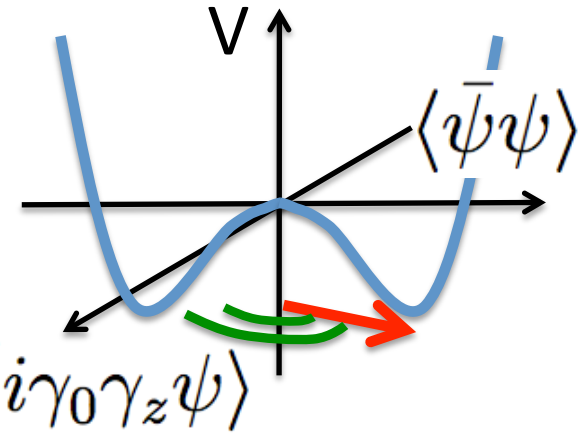
Linear combination :

space-dep.

Sum : $\langle \bar{\psi}\psi \rangle = \Delta \cos(2p_F z)$

P-odd

diff : $\langle \bar{\psi} i\gamma_0\gamma_z \psi \rangle = \Delta \sin(2p_F z)$



2 CDWs → Single Chiral Spiral

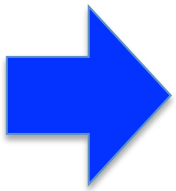
Other facts: Quick summary

1, Fermi surface effects enhance the size of mass gap compared to vac. one. (as far as we turn off the screening)

2, At leading μ: Quark number density is uniform.

(even though chiral density is spatially oscillating)

3, Effects of *perturbative* 1/μ corrections:

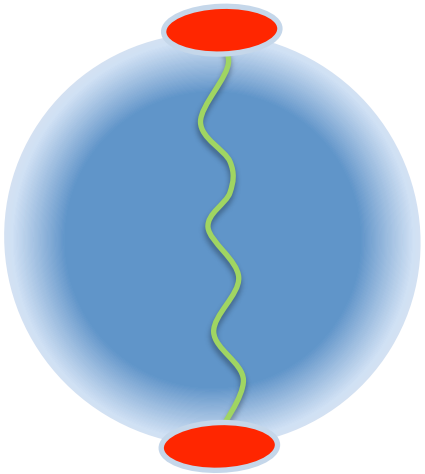


***Plane wave* spirals → *Solitonic* spirals.**

***Quark number density* starts to modulate.**

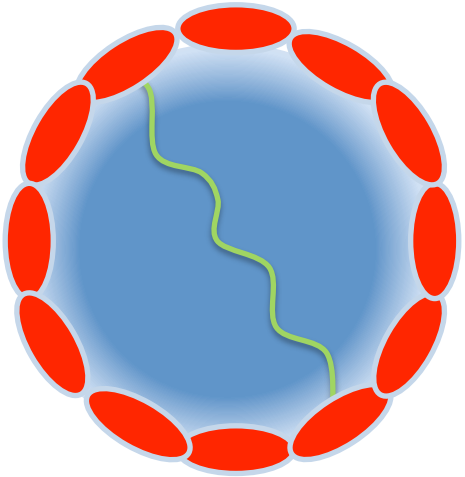
→ *Solitons* of *quark number density*.

Interweaving Chiral Spirals



theoretically possible ?





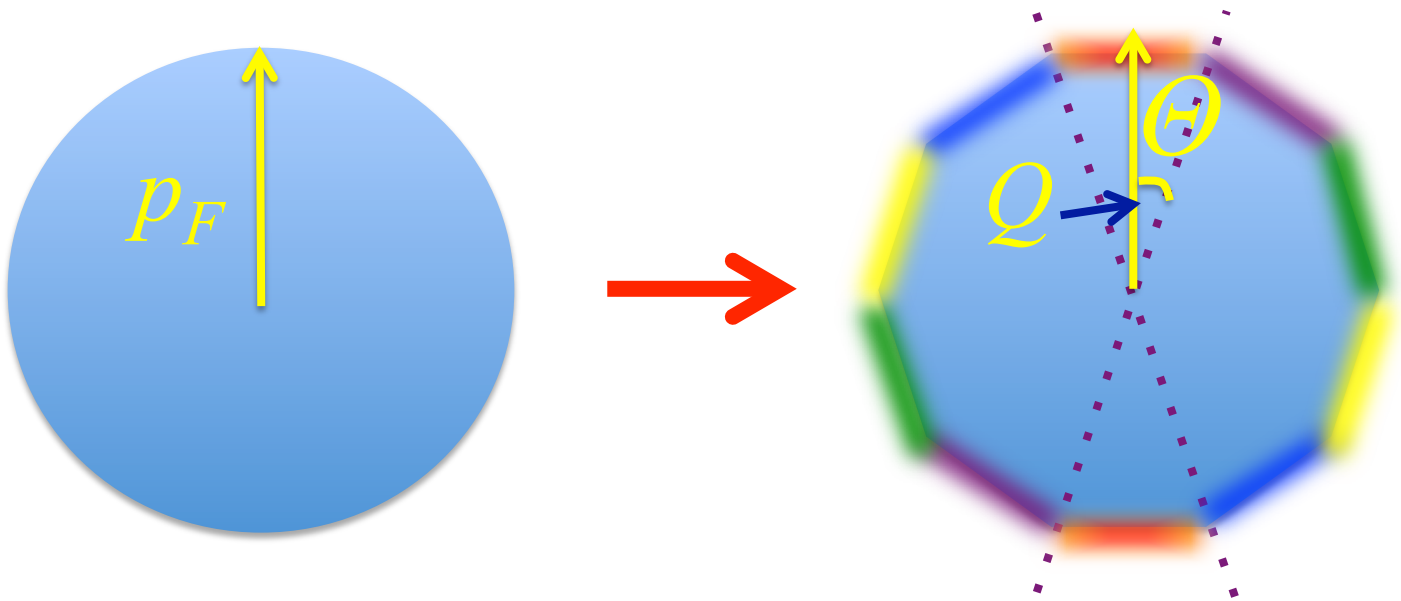
Single Chiral Spiral

Multiple Chiral Spirals

Our study: (2+1) D Example

1, Divide the Fermi surface into N_p patch domains

(N_p: variational parameter)



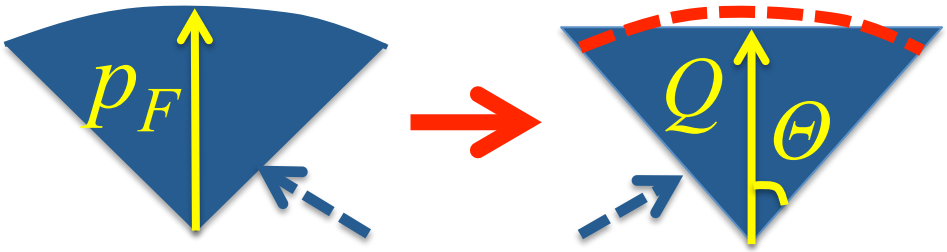
2, *Each* patch domain has one CS → Compute it.

3, Compute interactions b.t.w. CSs.

4, Optimize $N_p \sim 1/\Theta$ → Find the ground state

Energetic gain v.s. cost

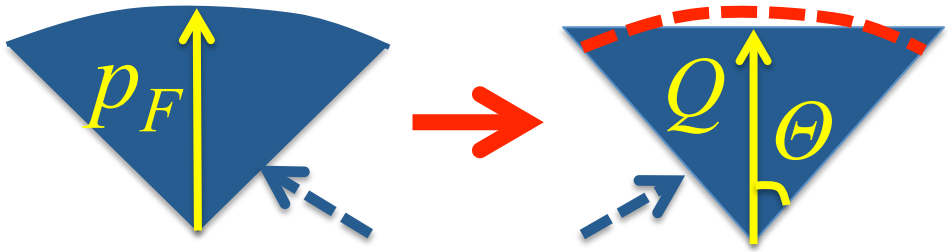
- **Cost : Deformation**
  (dominant for **large** Θ)



equal vol. (particle num.)

Energetic gain v.s. cost

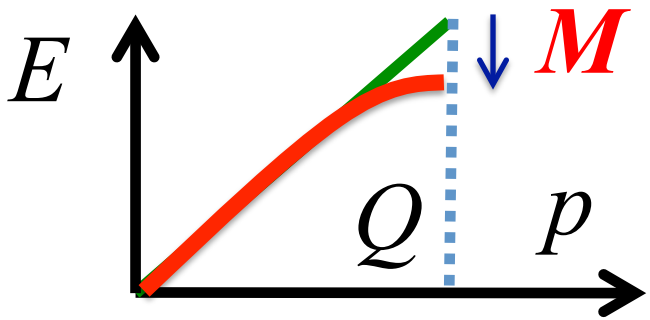
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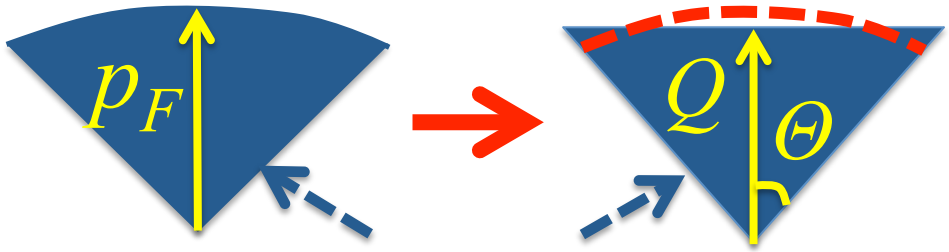
- **Gain : Mass gap origin**

Condensation effects



Energetic gain v.s. cost

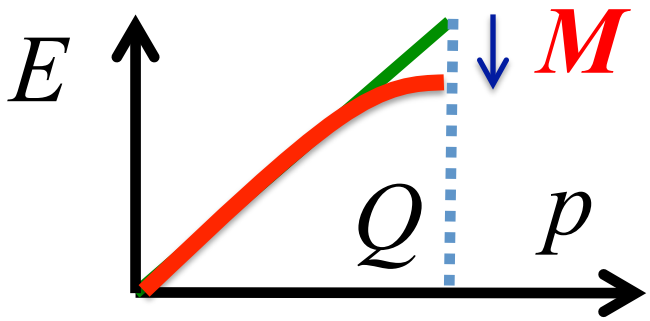
- **Cost : Deformation**
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- **Gain : Mass gap origin**

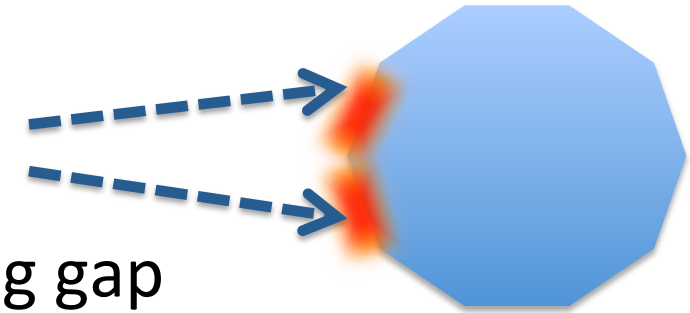
Condensation effects



- **Cost : Interactions among CSs**
  (dominant for **small** Θ)

(Model dep. !!)

Condensate – Condensate int.



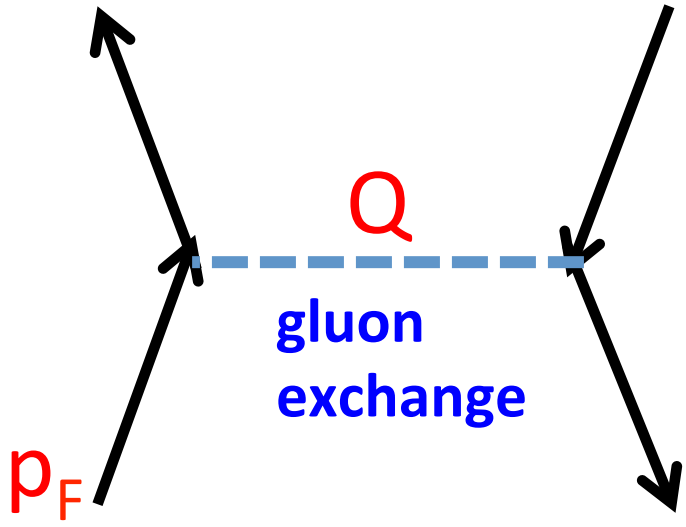
→ destroy one another, reducing gap

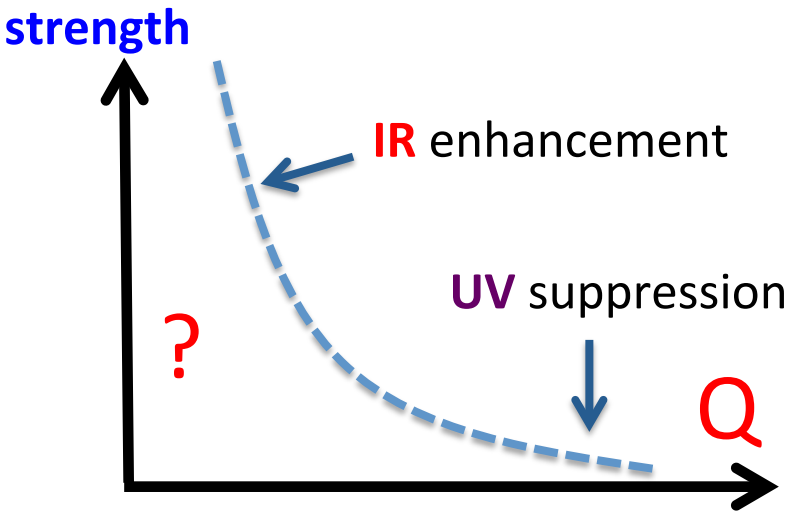
A schematic non-confining model

Strength of interactions is determined by

Momentum transfer, **NOT** by **quark momenta.**

→ **Even at high density, int. is strong** for **some processes**.



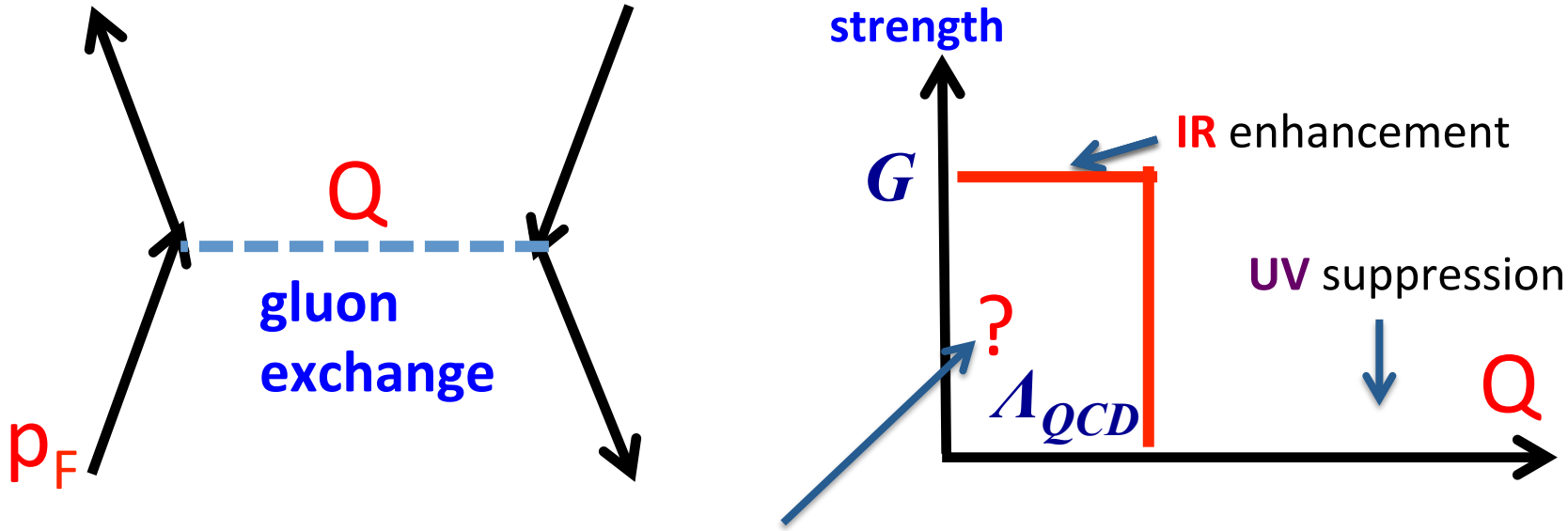


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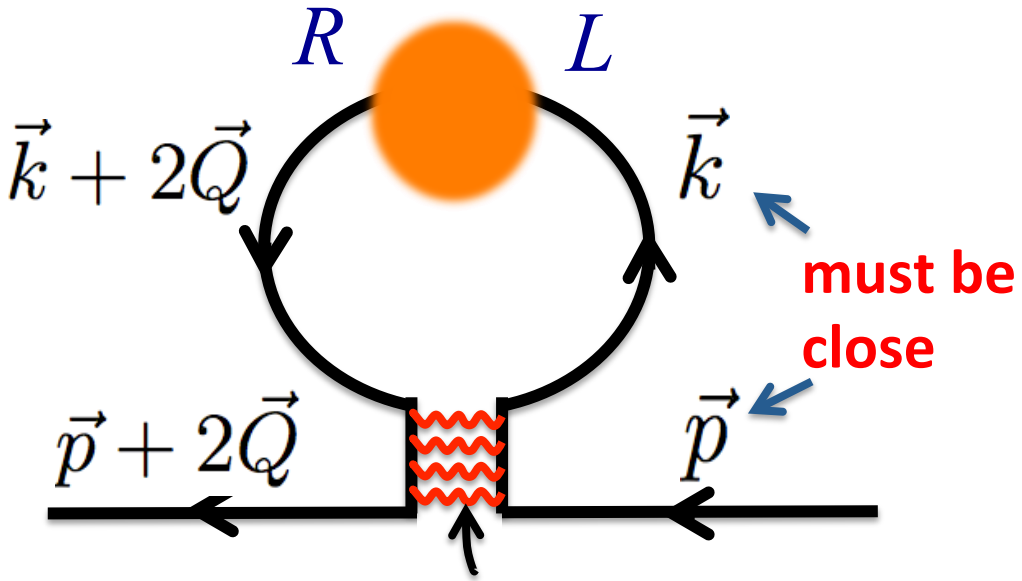
- Detailed form in the **deep IR region: We don't care**

Consequences of the model

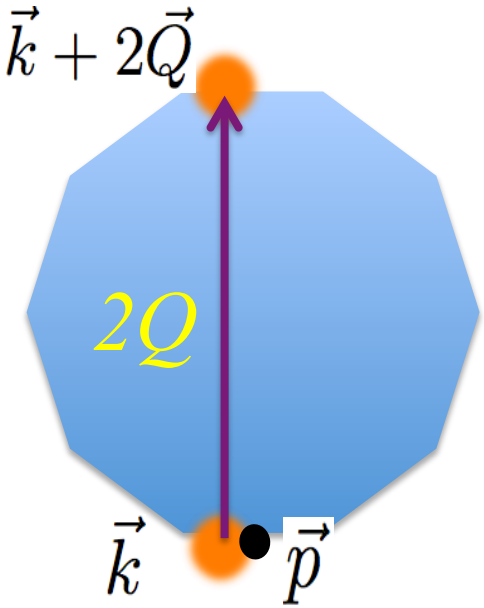
Contributions to the mass gap at leading Nc:

Inhomogeneous condensate

$$\langle \bar{\psi}_R(\vec{k}+2\vec{Q})\psi_L(\vec{k})\rangle$$



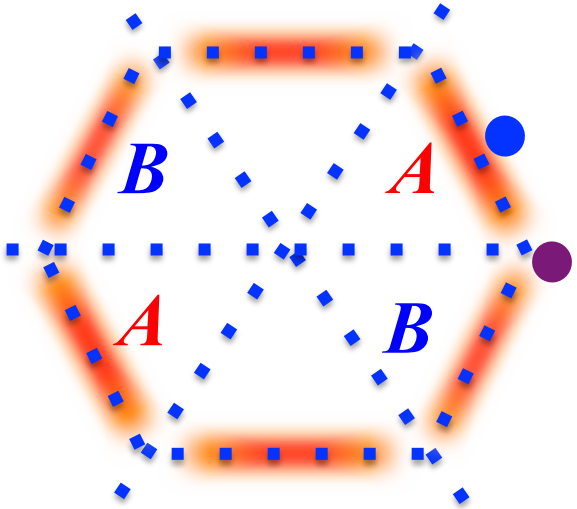
Strong for small mom. transfer



Quark-condensate int. is *Local* in *momentum space* !

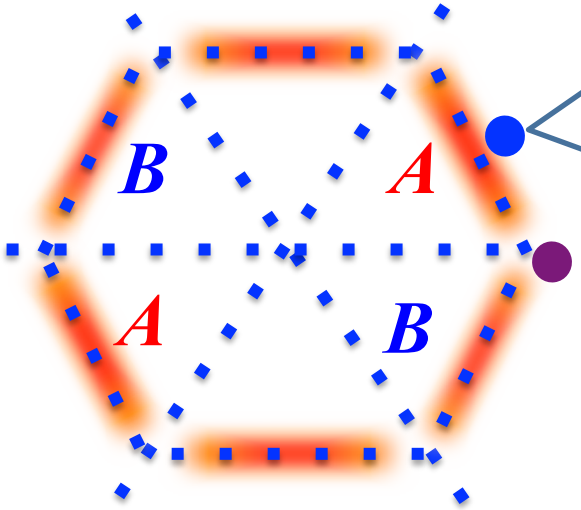
Condensate & gap distributions

Condensate contribute to the quark mass gap
only if their momentum domains are close one another.

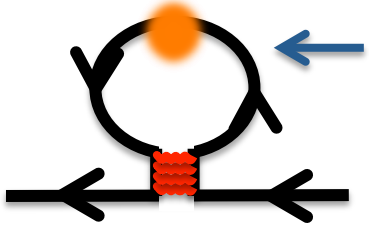


Condensate & gap distributions

Condensate contribute to the quark mass gap
only if their momentum domains are close one another.



Quarks Away from the patch boundary:

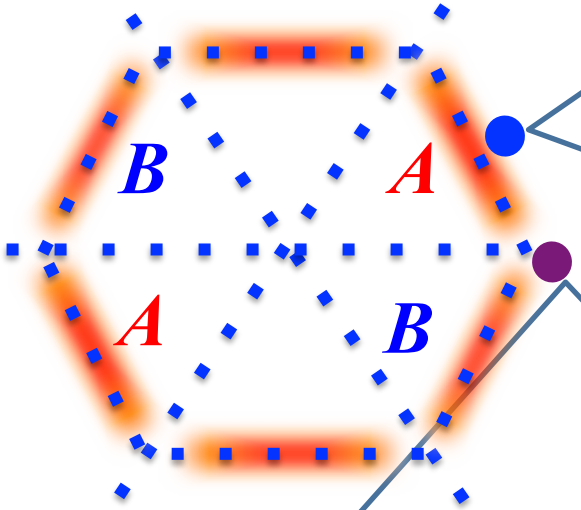


feel only **one** CS (A)
condensate in the gap eq.

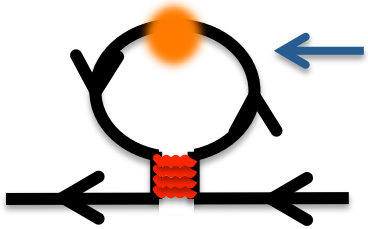
The gap eq. can be solved within one patch treatment.

Condensate & gap distributions

Condensate contribute to the quark mass gap
only if their momentum domains are close one another.



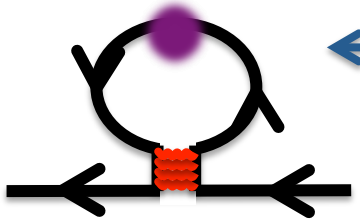
Quarks Away from the patch boundary:



feel only **one** CS (A)
condensate in the gap eq.

The gap eq. can be solved within one patch treatment.

Quarks Near the patch boundaries:

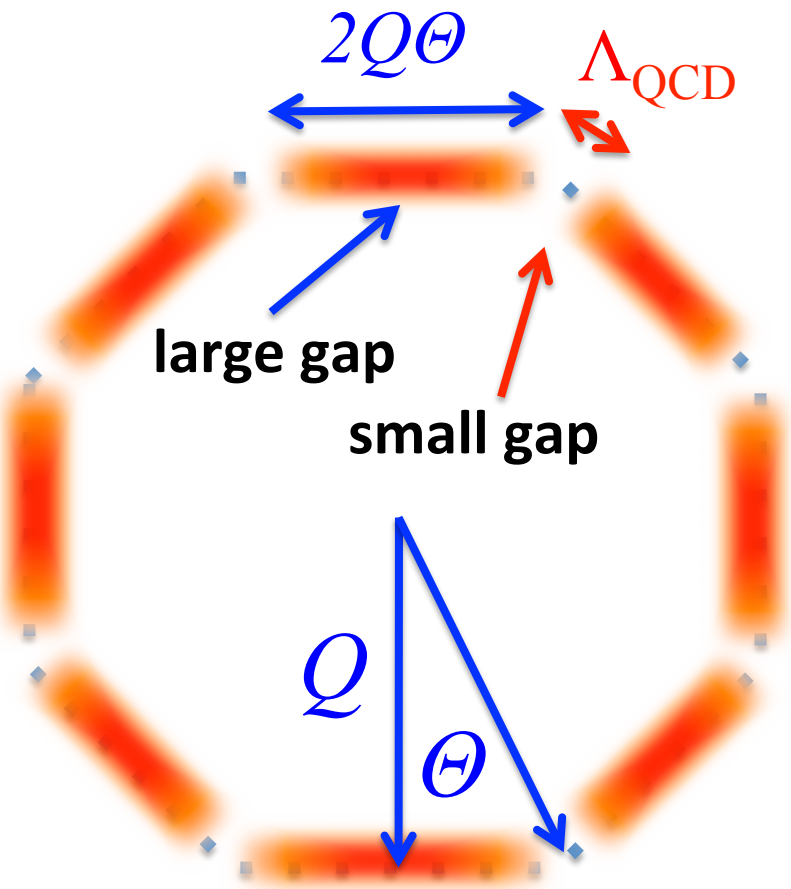


feel **Two** CSs (A & B)
The gap eq. involve **Two** CSs background.

Results: reduction of the gap & condensate

Too many CSs is not favored

Gap or Condensate distribution



If Θ is **too small**

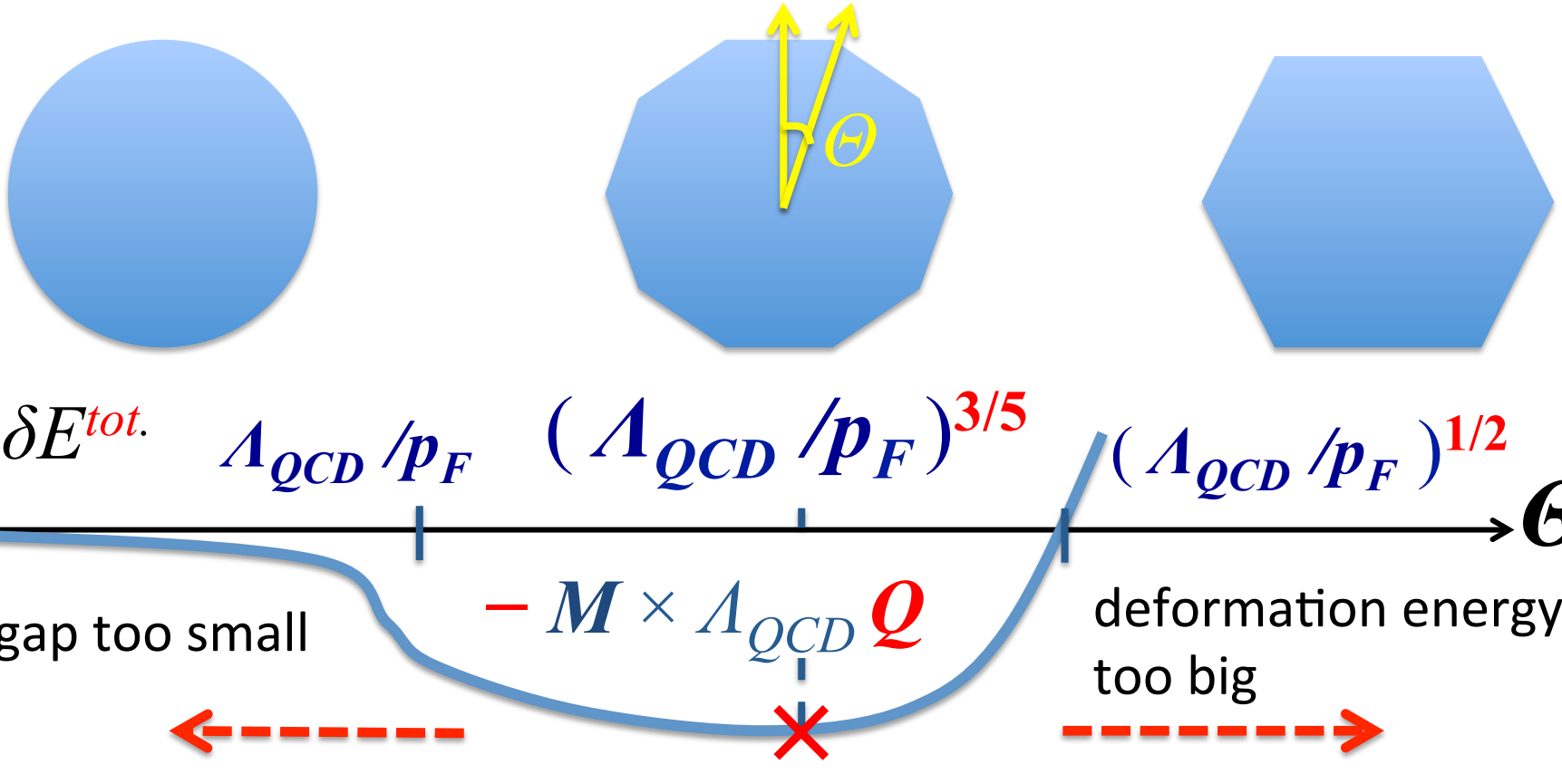
↓

The domain of **large** condensate or gap decreases.

↓

We cannot reduce the energy of the system so much.

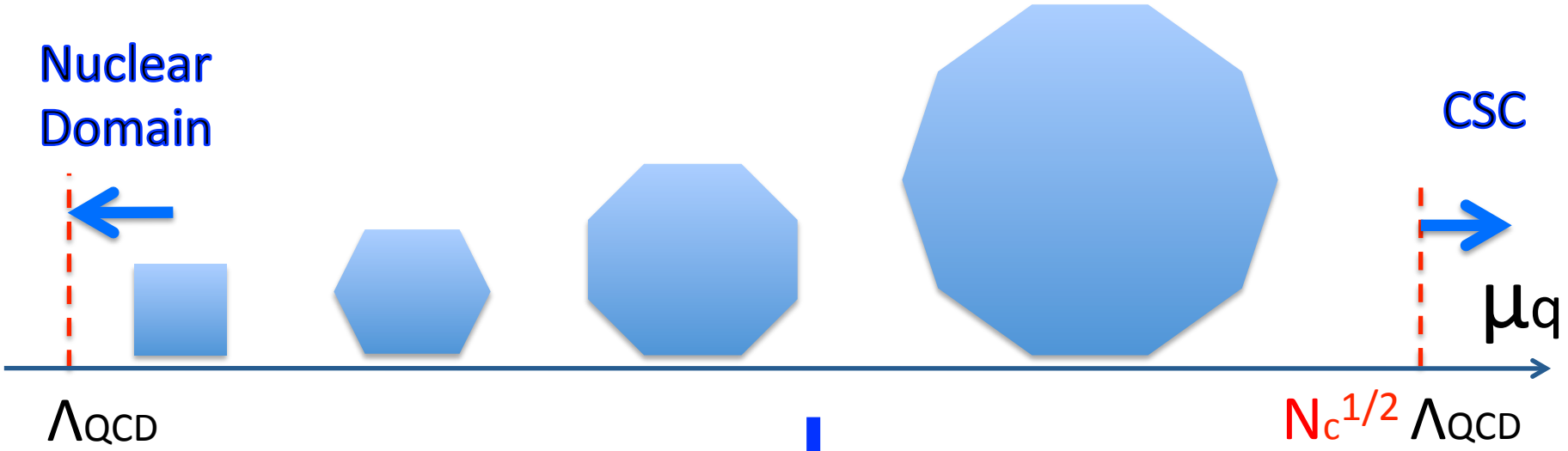
Energy Landscape (for fixed p_F)

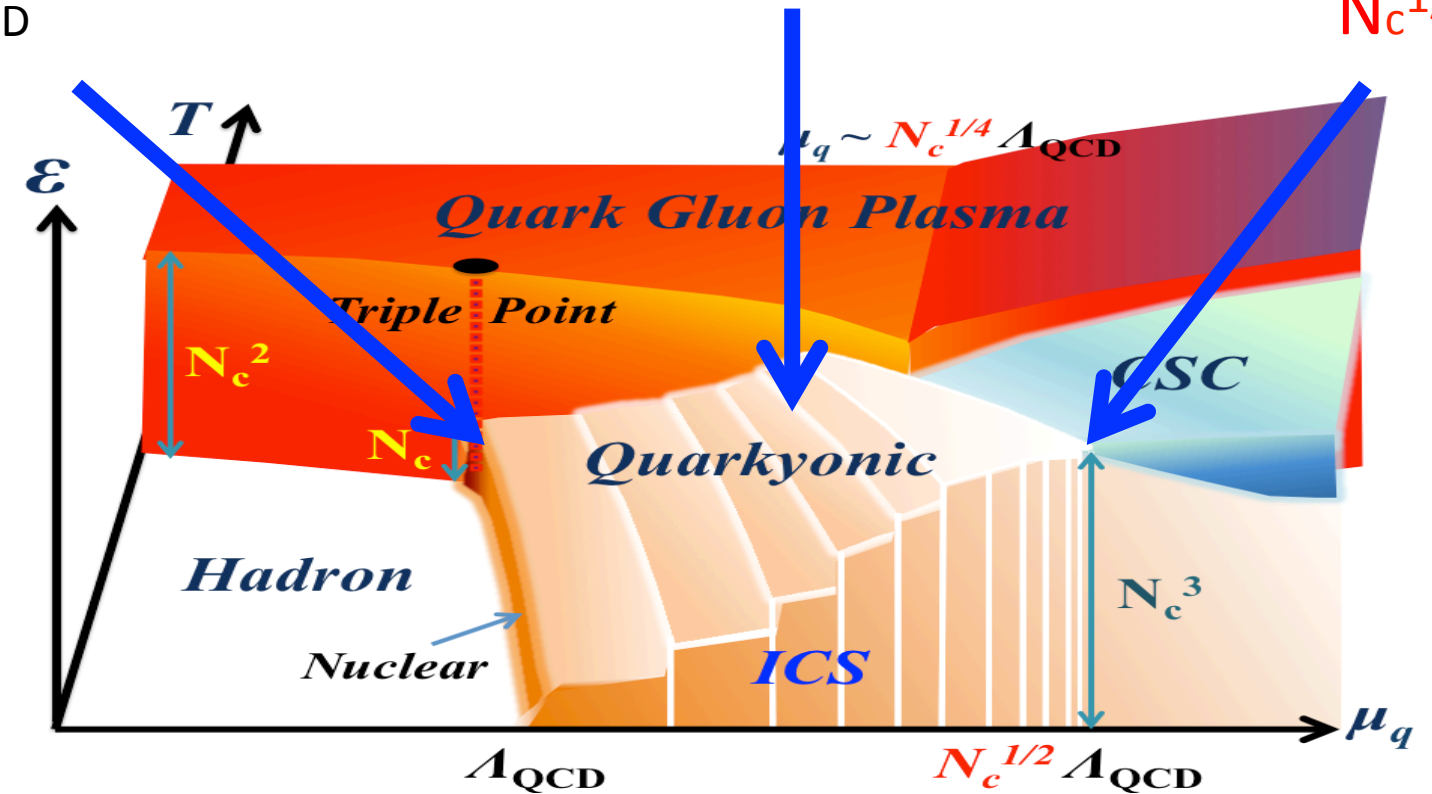


$$N_p \sim 1/\Theta \sim (p_F/\Lambda_{QCD})^{3/5}$$

- **Patch num. depends upon density.**

Sequential Phase transition





Summary

- 1, ***Quarkyonic*** Matter

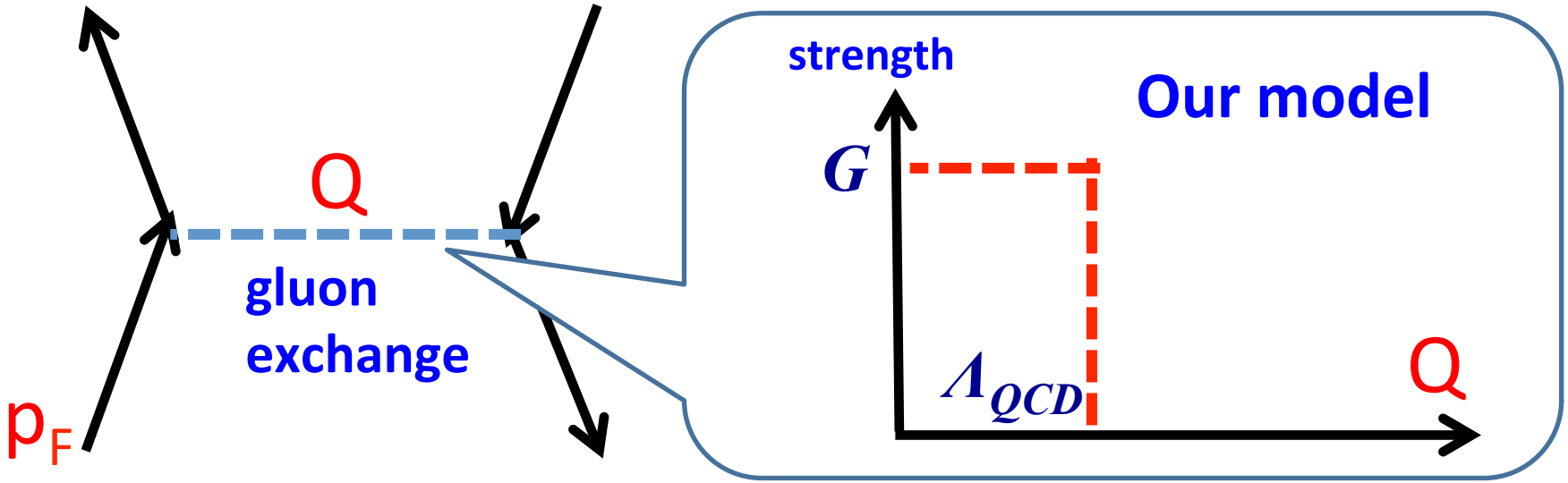
  → Quark matter with **confined** condensates & excitations.

- 2, The CS creates **Quark Mass gap near the Fermi surface**.

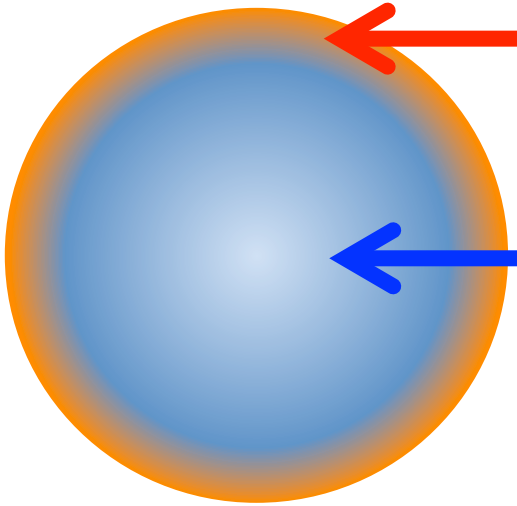
  (→ **shifts** Chiral & Deconf. transition lines to higher density)

- 3, The ICS is likely to be possible only for the theory with **IR enhancement** & **UV suppression** of the int.

Other facts : Quick summary



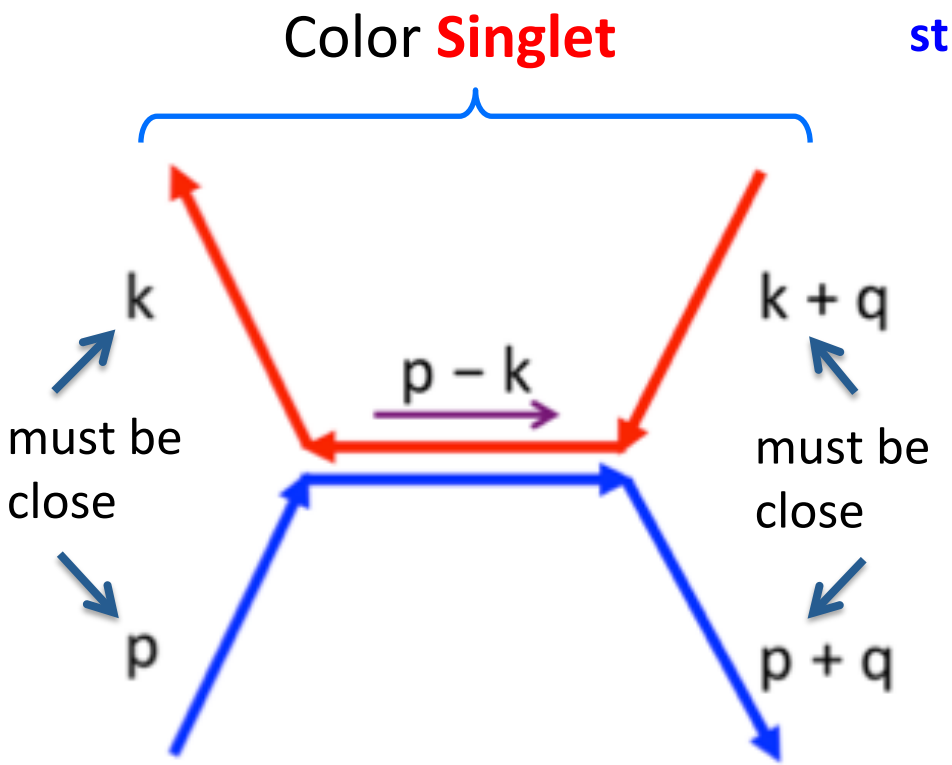
At MF
(large Nc)

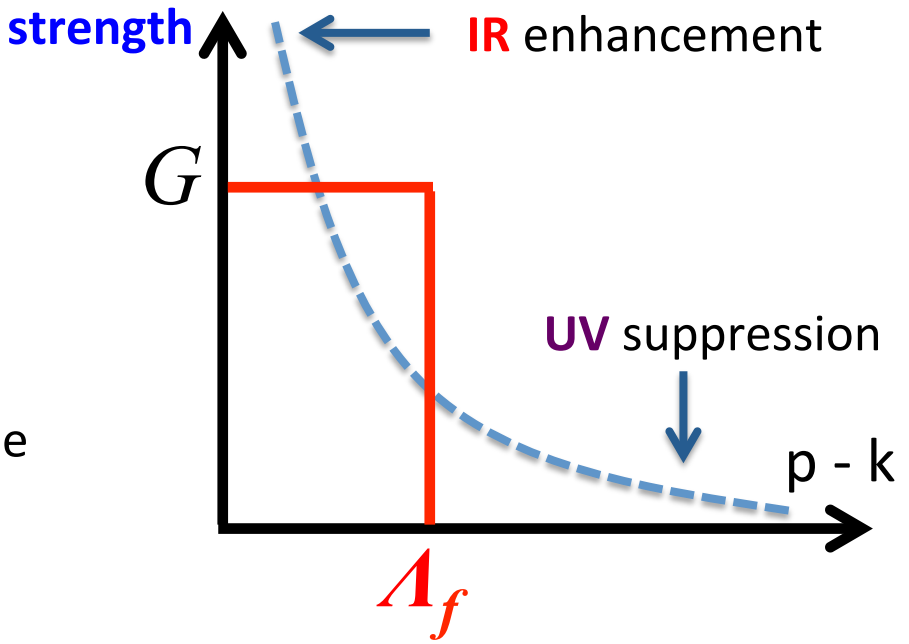


Non-pert. quarks (gapped)

Free quarks
(Chiral symmetric)

A crude model with asymptotic freedom





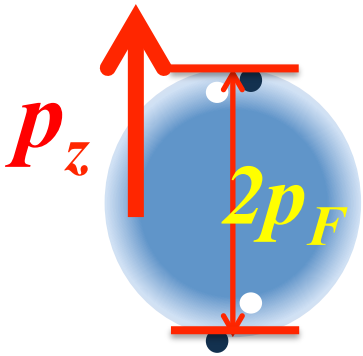
$$\theta_{p,k} \equiv \theta\left(\Lambda_f^2 - (\vec{p} - \vec{k})^2\right)$$

- ex) **Scalar** - **Scalar** channel

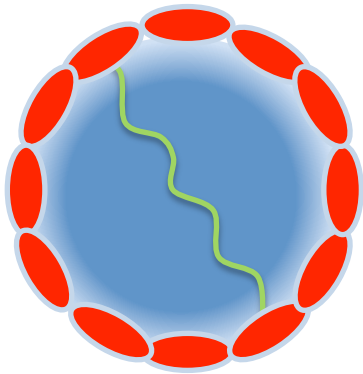
$$\frac{G}{N_c}\int dx^0 \int_{q,p,k} \left(\bar{\psi}(\vec{p}+\vec{q})\psi(\vec{p})\right)\left(\bar{\psi}(\vec{k})\psi(\vec{k}+\vec{q})\right)\theta_{p,k}$$

Interweaving Chiral Spirals

So far we have considered only
the Chiral Spiral in **one direction**.

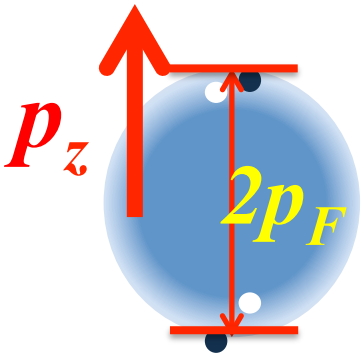


Is it possible to have CSs in multiple directions?



Interweaving Chiral Spirals

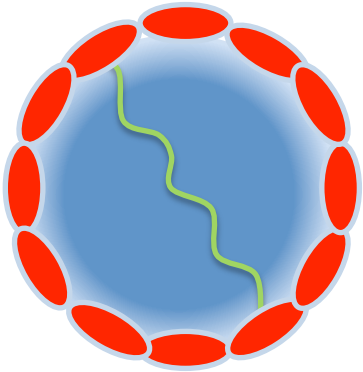
So far we have considered only
the Chiral Spiral in **one direction**.



Is it possible to have CSs in multiple directions?

YES!

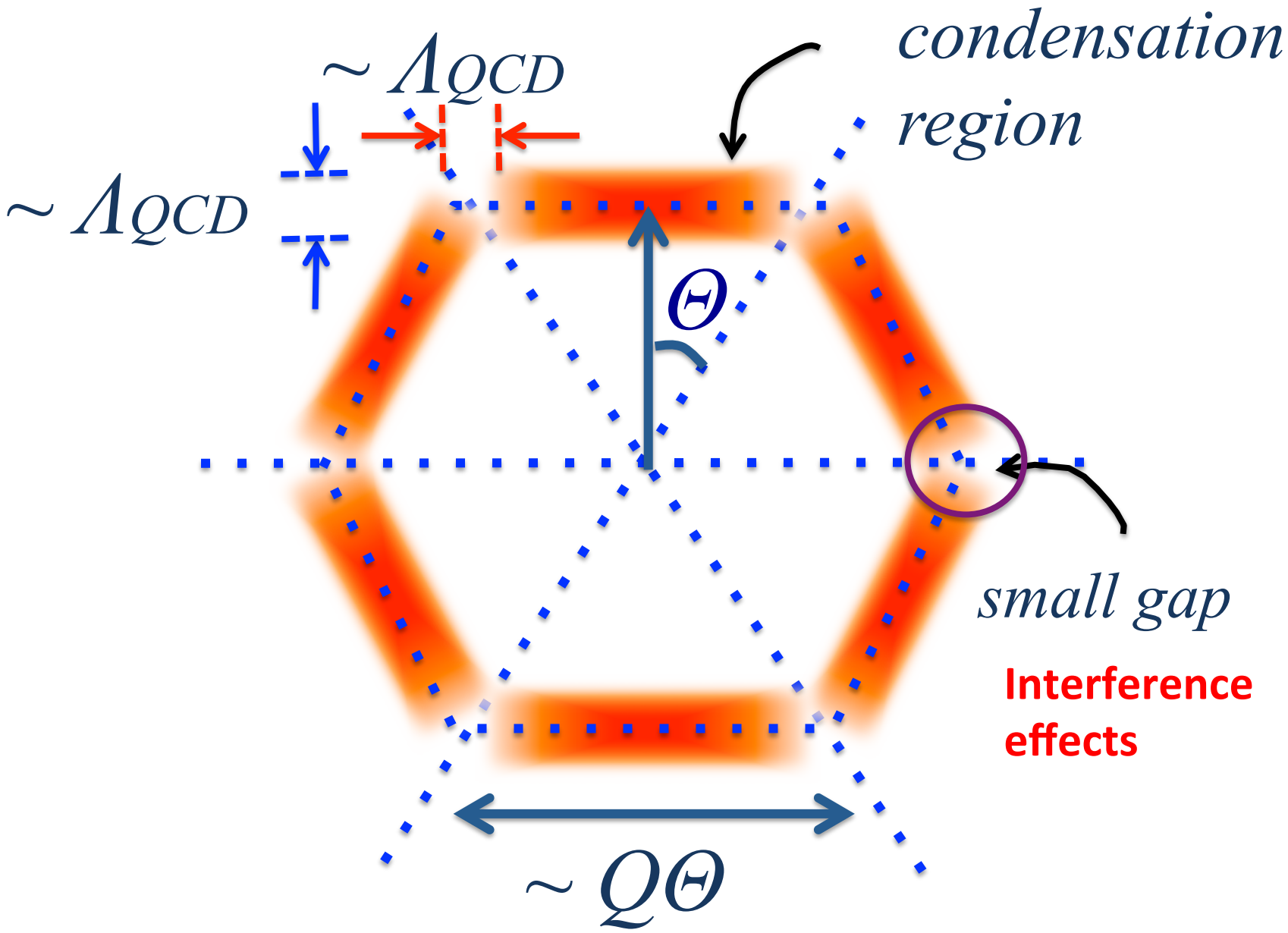
(**if** theory is **asymptotic free**)



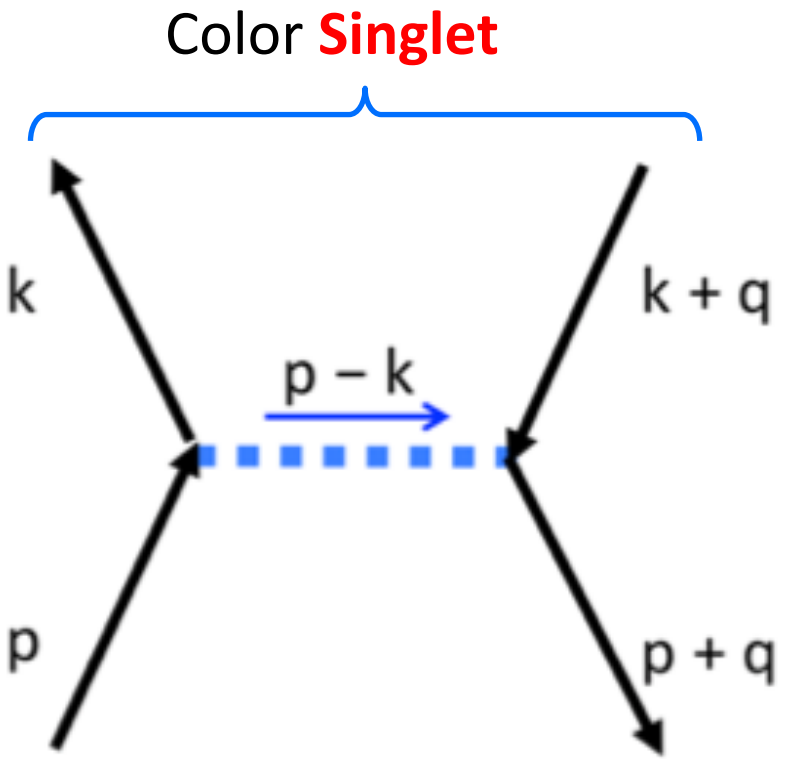
Pairs around the **entire Fermi surface** can condense.

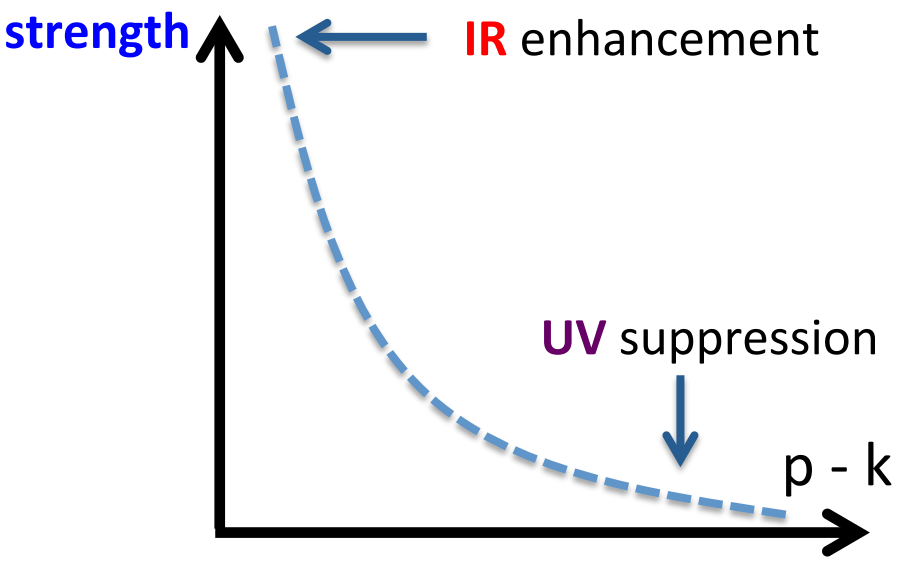
The system acquires very large energetic gain.

Gap distribution will be



A crude model with asymptotic freedom

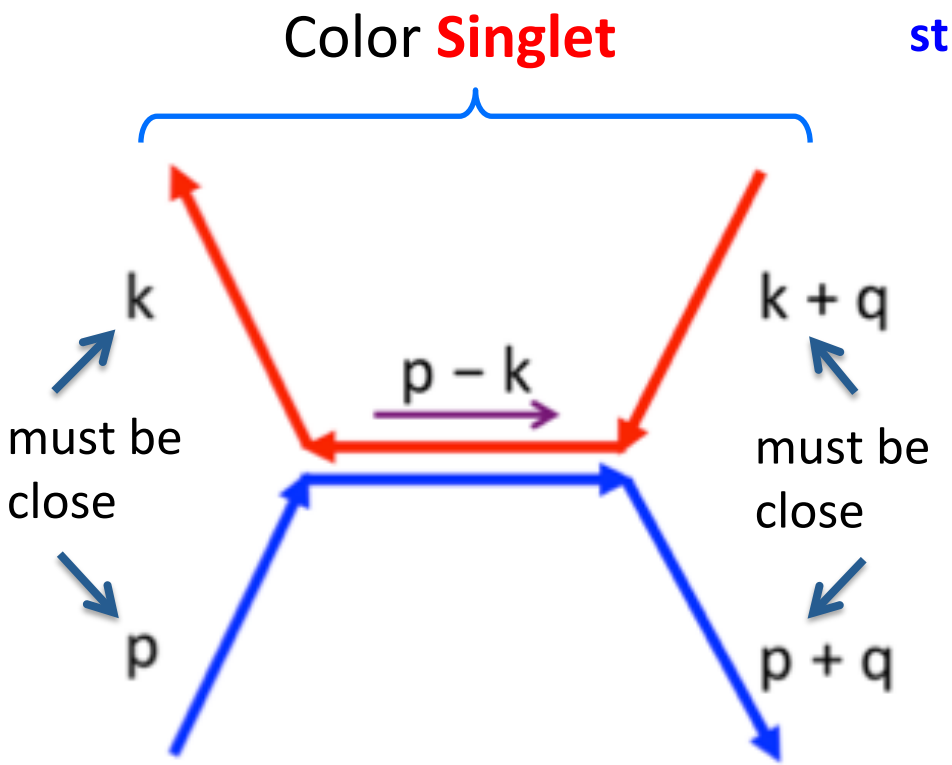


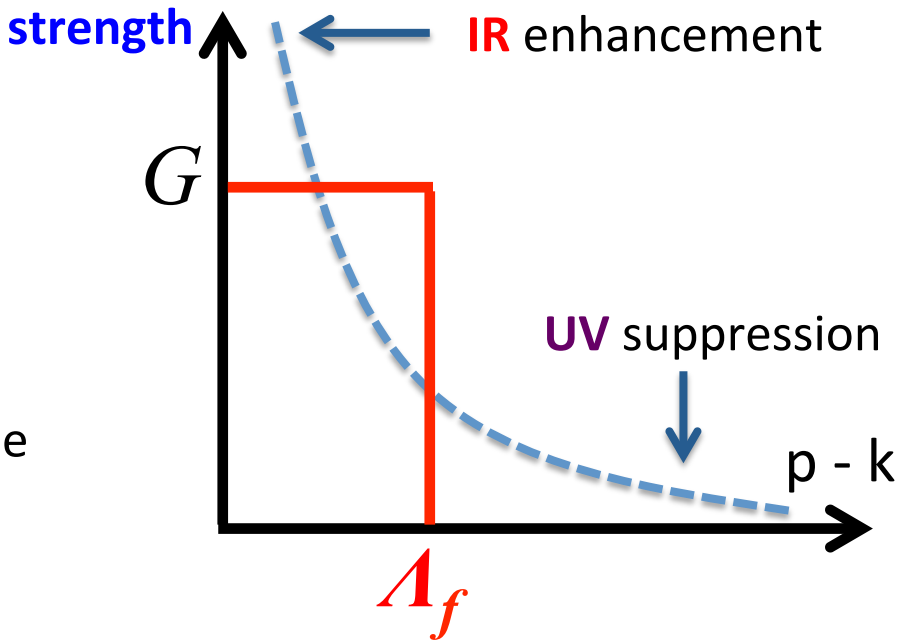


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$$\frac{G}{N_c}\int dx^0 \int_{q,p,k} \left(\bar{\psi}(\vec{p}+\vec{q})\psi(\vec{p})\right)\left(\bar{\psi}(\vec{k})\psi(\vec{k}+\vec{q})\right)\theta_{p,k}$$

A crude model with asymptotic freedom





$$\theta_{p,k} \equiv \theta\left(\Lambda_f^2 - (\vec{p} - \vec{k})^2\right)$$

- ex) **Scalar** - **Scalar** channel

$$\frac{G}{N_c}\int dx^0 \int_{q,p,k} \left(\bar{\psi}(\vec{p}+\vec{q})\psi(\vec{p})\right)\left(\bar{\psi}(\vec{k})\psi(\vec{k}+\vec{q})\right)\theta_{p,k}$$

Comparison with other form factor models

	Typical model	Ours
function of :	**quark mom.**	**mom. transfer**
Strength at large μ :	weaken	unchange (at large Nc)

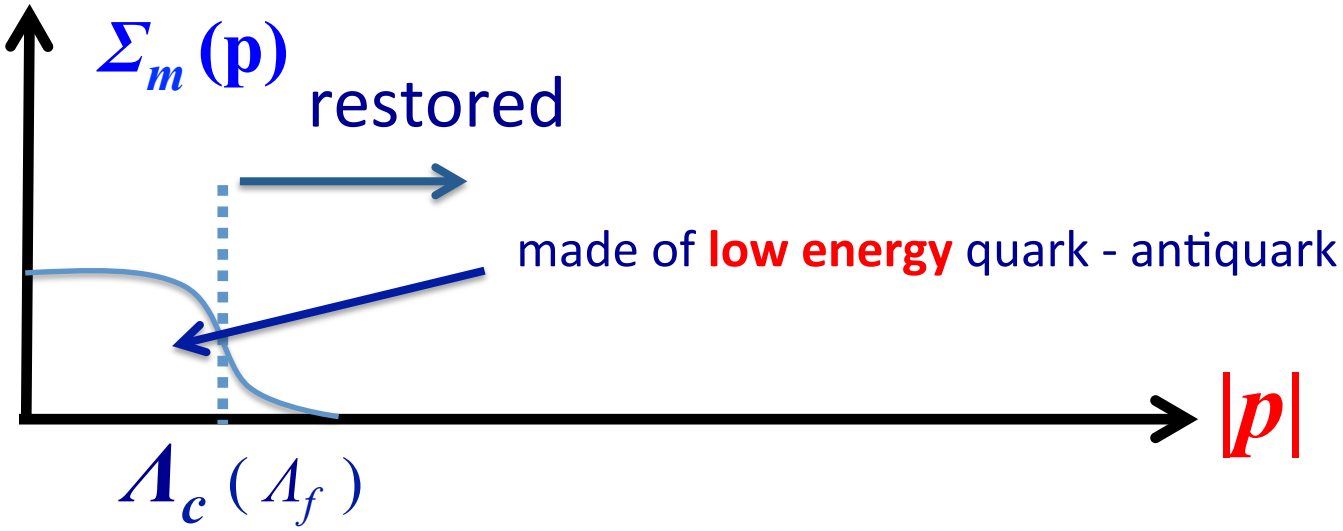
- As far as we estimate **overall** size of free energy,
  two pictures would not differ so much, because:

  Hard quarks ⟶ ***Typical*** *int. : Hard*
  (dominant in free energy)

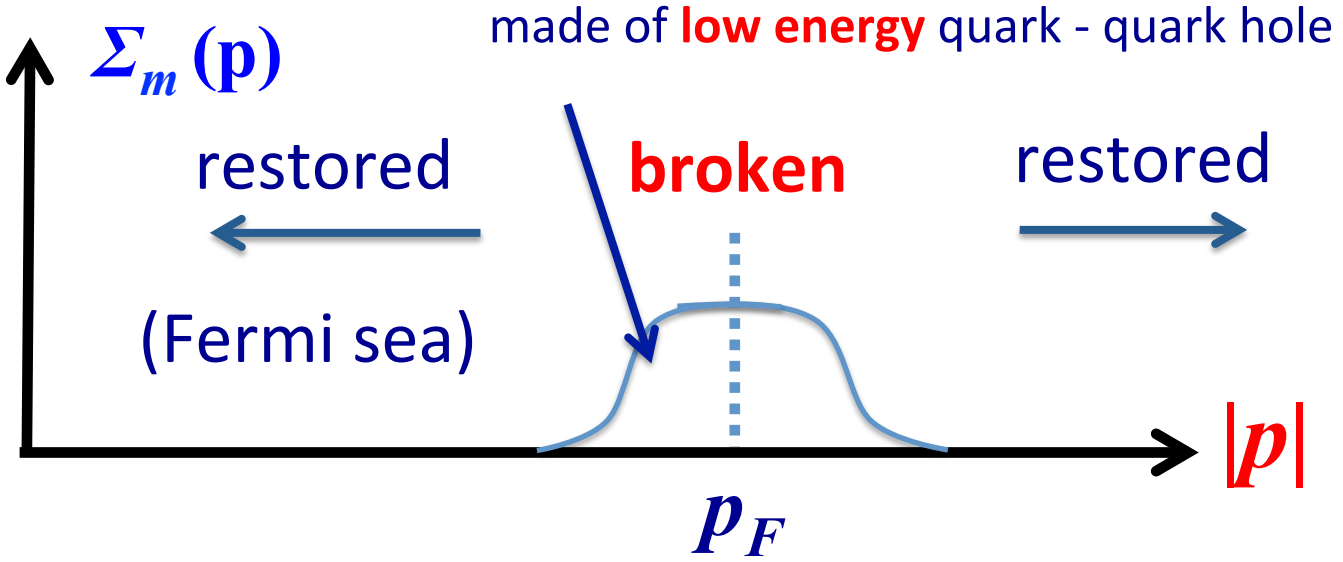
- **However,** if we compare **energy difference b.t.w. phases**,
  typical part **largely** **cancel out**,
  so we **must** distinguish these two pictures.

Relevant domain of Non-pert. effects

Vac.

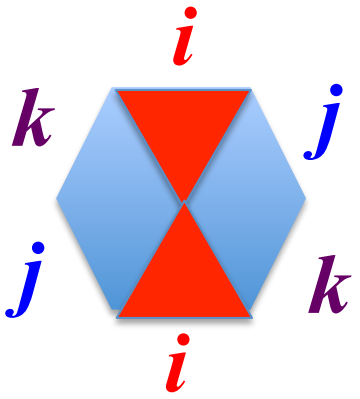


Finite Density



Picking out one patch Lagrangian

ψ_i : momentum belonging to ***i*** -th patch



- **Kin. terms:** trivial to decompose

$$\mathcal{L}^{kin} \rightarrow \sum_i \bar{\psi}_i i\partial\!\!\!/ \psi_i \equiv \sum_i \mathcal{L}_i^{kin}$$

- **Int. terms:** Different patches can couple

$$\frac{G}{N_c} \sum_{i,j,k,l} \left((\bar{\psi}_i \psi_j)(\bar{\psi}_k \psi_l) + (\bar{\psi}_i i\gamma_5 \psi_j)(\bar{\psi}_k i\gamma_5 \psi_l) \right)$$

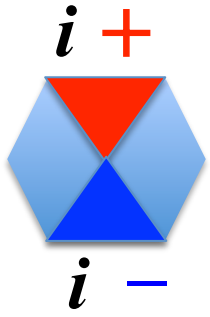
All fermions belong to the ***i*** -th patch — **Patch - Patch int.**

$$\mathcal{L} = \sum_i \mathcal{L}_i^{1patch} + \Delta\mathcal{L}$$

Dominant terms in One Patch, 1

"(1+1) D" "**chirality**" in ***i -th*** patch

$$\Gamma_{i5} \equiv \gamma_0 \gamma_{i\|} \qquad \psi_{i\pm} \equiv \frac{1 \pm \Gamma_{i5}}{2} \psi_i$$



eigenvalue: **Moving direction**

- **Fact :** "Chiral" **Non** - sym. terms → suppressed by $1/Q$

ex) free theory

- **Longitudinal Kin. (Sym.)**

$$\psi^\dagger_{i+} i(\partial_0 - \partial_{i\|}) \psi_{i+}$$

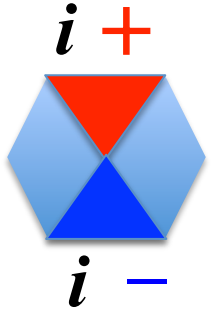
- **Transverse Kin. (Non-Sym.)**

$$\bar{\psi}_{i+} i \not{\partial}_\perp \psi_{i-}$$

Dominant terms in One Patch, 1

"(1+1) D" "**chirality**" in ***i -th*** patch

$$\Gamma_{i5} \equiv \gamma_0 \gamma_{i\parallel} \qquad \psi_{i\pm} \equiv \frac{1 \pm \Gamma_{i5}}{2} \psi_i$$



eigenvalue: **Moving direction**

▪ **Fact :** "Chiral" **Non** - sym. terms → suppressed by $1/Q$

ex) free theory

▪ **Longitudinal Kin. (Sym.)**

$$\psi^\dagger_{i+} i(\partial_0 - \partial_{i\parallel}) \psi_{i+}$$

▪ **Transverse Kin. (Non-Sym.)**

$$\bar{\psi}_{i+} i \not{\partial}_\perp \psi_{i-}$$

excitation energy

$$\epsilon^{\text{free}}(\delta\vec{p}) = |\delta p_\parallel| + \frac{\delta p_\parallel^2 + p_\perp^2}{2Q} + \cdots$$

momentum **measured from Fermi surface**

Dominant terms in One Patch, 2

$$(\bar{\psi}\psi)^2$$

"Chiral" sym. part

$$\frac{1}{2}\left((\bar{\psi}\psi)^2+(\bar{\psi}i\Gamma_5\psi)^2\right)$$

IR dominant

(must be **resummed** → **MF**)

Non - sym. part

$$\frac{1}{2}\left((\bar{\psi}\psi)^2-(\bar{\psi}i\Gamma_5\psi)^2\right)$$

$1/Q$ suppressed

(can be treated in **Pert**.)

Dominant terms in One Patch, 2

"Chiral" sym. part ← $(\bar{\psi}\psi)^2$ → Non - sym. part

$$\frac{1}{2}\left((\bar{\psi}\psi)^2+(\bar{\psi}i\Gamma_5\psi)^2\right)$$

IR dominant

(must be **resummed** → **MF**)

$$\frac{1}{2}\left((\bar{\psi}\psi)^2-(\bar{\psi}i\Gamma_5\psi)^2\right)$$

$1/Q$ suppressed

(can be treated in **Pert**.)

- **IR** dominant : **Unperturbed Lagrangian**

  Longitudinal Kin. + "Chiral" sym. 4-Fermi int.

  → Gap eq. can be reduced to (1+1) D
  (P_T - factorization)

- **IR** suppressed : **Perturbation**

  Transverse Kin. + Non - sym. 4-Fermi int.

Quick Summary of 1-Patch results

At leading order of Λ_{QCD}/μ

- **Integral** eqs. such as Schwinger-Dyson, Bethe-Salpeter, can be reduced from (**2**+1) D to (**1**+1) D. cf) *kT factorization*
- **Chiral Spirals** emerge, generating **large** quark mass gap. (even larger than vac. mass gap)
- **Quark num**. is **spatially uniform**. (in contrast to chiral density)

Pert. corrections

- **Quark num**. **oscillation**.
- **CSs** : **Plane wave** → **Solitonic**

} approach to **Baryonic Crystals**

Chiral Spirals (CSs)

- One can find (**1**+1) D **solution** for the gap equation.
  (**except** boundaries of patches)
- The size of mass gap is $\sim \Lambda_f$, if we choose $G \sim 1/\Lambda_f$.
- The form of chiral condensates: **Spirals**

$$\langle \bar{\psi}_{i+}\psi_{i-}\rangle = \Delta\, e^{-2iQx_{i\|}} \;\;\&\;\; \langle \bar{\psi}_{i-}\psi_{i+}\rangle = \Delta\, e^{2iQx_{i\|}}$$

$$\langle \bar{\psi}_i\psi_i\rangle = \Delta\cos 2Qx_{i\|} \;\;\&\;\; \langle \bar{\psi}_i\gamma_0\gamma_{i\|}\psi_i\rangle = \Delta\sin 2Qx_{i\|}$$

- The subdominant terms can be computed
  systematically as $1/Q$ *or* Θ expansion .

Consequences of form factor. 2

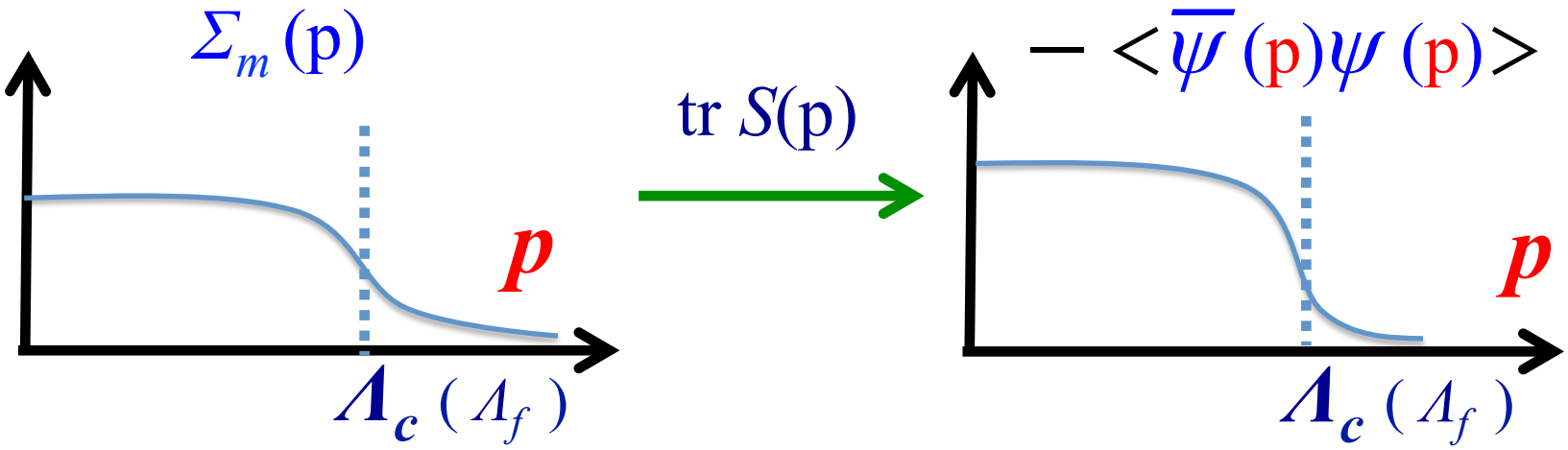
Dominant contributions to condensates : Low **energy** modes

(for vacuum)

When $p \longrightarrow \infty$:

$$\Sigma_m(\vec{p}) = \int \frac{d\vec{k}}{(2\pi)^2} \frac{\Sigma_m(\vec{k})}{2\epsilon(\vec{k})} \theta_{p,k}$$

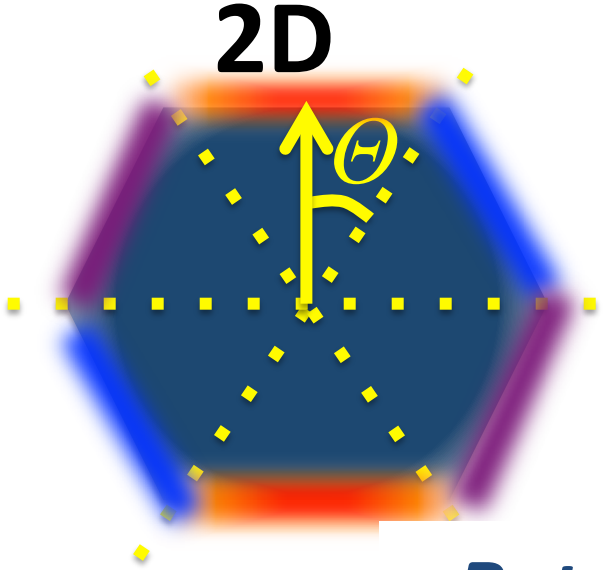
- k must also go to ∞, so $\varepsilon(k) \longrightarrow \infty$.
- Phase space is **finite** : Nothing compensates denominator.



Remark)

- finite density: Low **energy** modes appear **near the Fermi surface**.

Our goal



- **Expressing the energy density as a function of theta, then determine the best shape.**

Rotating Arm Approximations

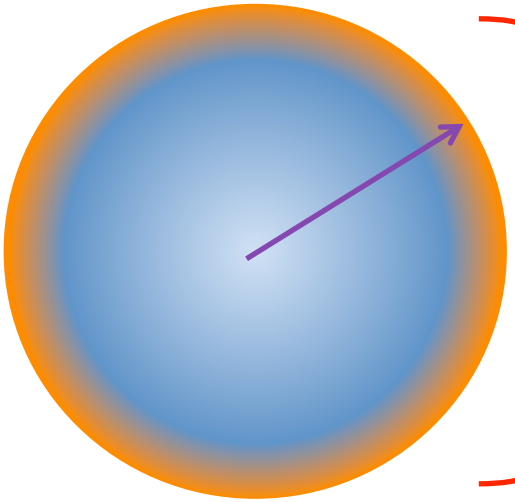
- **Simple Interaction with asym. free nature**
- **Large Nc** (**MF** treatments & ignore **screening**)
- **Large density** (Λ_{QCD} / p_F expansion, T=0)

- **Although not very realistic for dense QCD, but:**

→ **Simple *analytic* insights**

Quarkyonic Matter: Basic picture

McLerran-Pisarski (2007)



Bulk properties:

(EOS, Pressure, etc.)

Weakly int. quarks

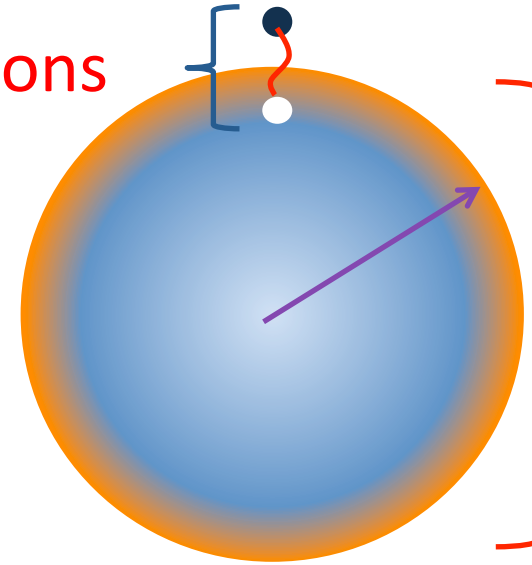
Pauli-blocking.

forming color singlet B.G.

Quarkyonic Matter: Basic picture

McLerran-Pisarski (2007)

hadronic excitations



Transport properties

Phase structures
(condensation)

etc.

Bulk properties:
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Quarkyonic Matter: Basic picture

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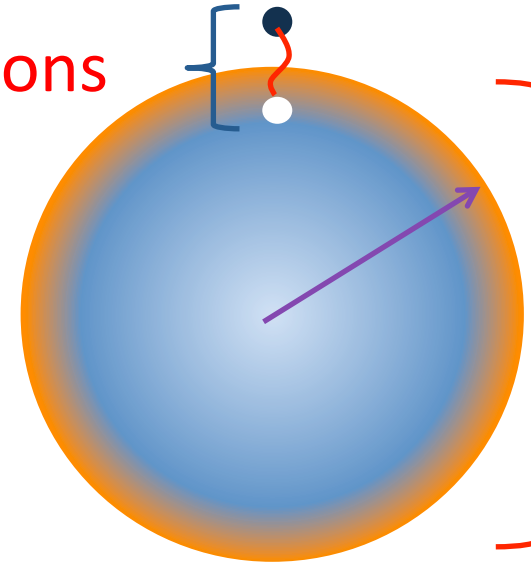
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forming color singlet B.G.

Quark Fermi sea + baryonic Fermi surface → Quark-yonic

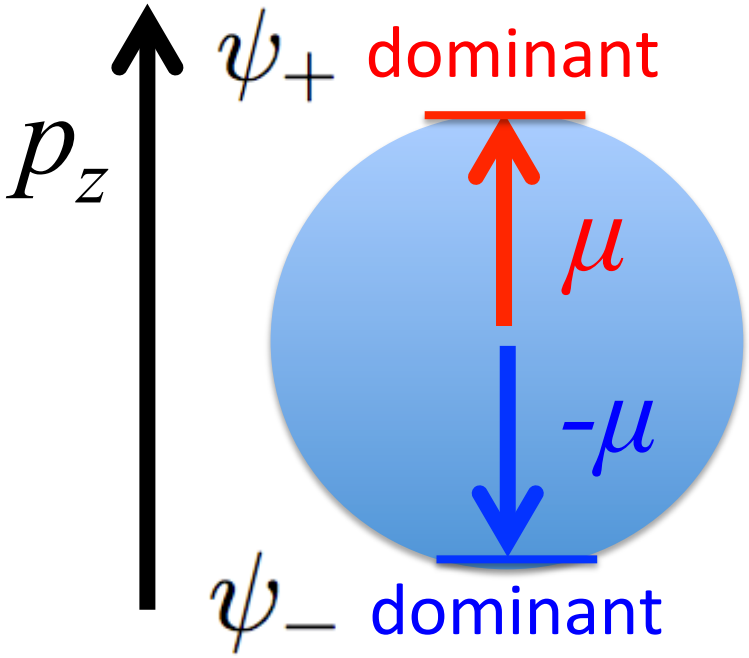
(hadronic)

Single Chiral Spiral in z-direction

- ***Projection***: $\psi_\pm = \frac{1 \pm \gamma_0\gamma_z}{2}\psi$ ← like (1+1) D chirality

- ***Kin. terms***: $\mathcal{L}_{kin} \simeq \psi_\pm^\dagger \left(\mathrm{i}(\partial_0 \mp \partial_z) + \mu \right) \psi_\pm + \psi_\pm^\dagger \frac{\partial_\perp^2}{2\mu} \psi_\mp$

(near the Fermi surface) **longitudinal** **transverse**



Remarks)

1, Relevant low E combinations

$\langle \bar{\psi}_+ \psi_- \rangle \quad \langle \bar{\psi}_- \psi_+ \rangle$

2, trans. kin. terms are suppressed.

Gap eq. → effectively (1+1) D

→ enhances quark mass gap

Quarkyonic Matter: Basic picture

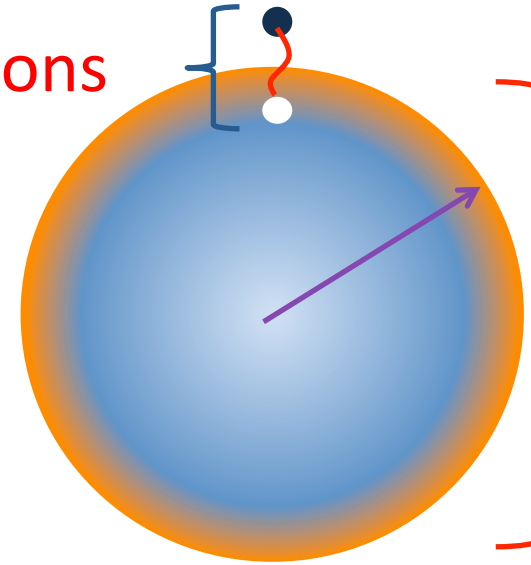
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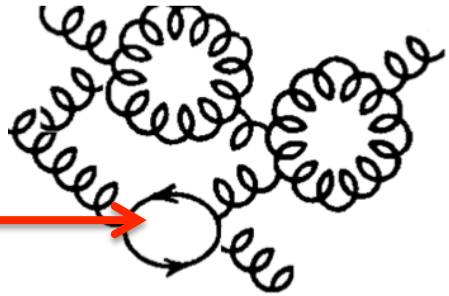
Quark Fermi sea + baryonic Fermi surface → Quarkyonic

(hadronic)

- **Gluon sector is modified when screening becomes large:**

$\mu \sim \Lambda_{QCD}$

small fraction



$\mu \sim N_c^{1/2} \Lambda_{QCD}$

large fraction

