Thermalization in collisions of large nuclei at high energies

Aleksi Kurkela
with Guy Moore

Arxiv:
- 1107.5050 — thermalization in generic setup, complete treatment of plasma instabilities
- 1108.4684 — specialized to heavy ion collisions

869 order of magnitude estimates in total

What?

Hydrodynamics provides a fantastic description of HIC, but:

- assumes local thermal equilibrium
  - needs thermalization time τ_0 as input
- Why is $\tau_0 \lesssim 0.2 \ldots 1.2\text{fm/c}$???
- What happens before? Maybe observable in experiment

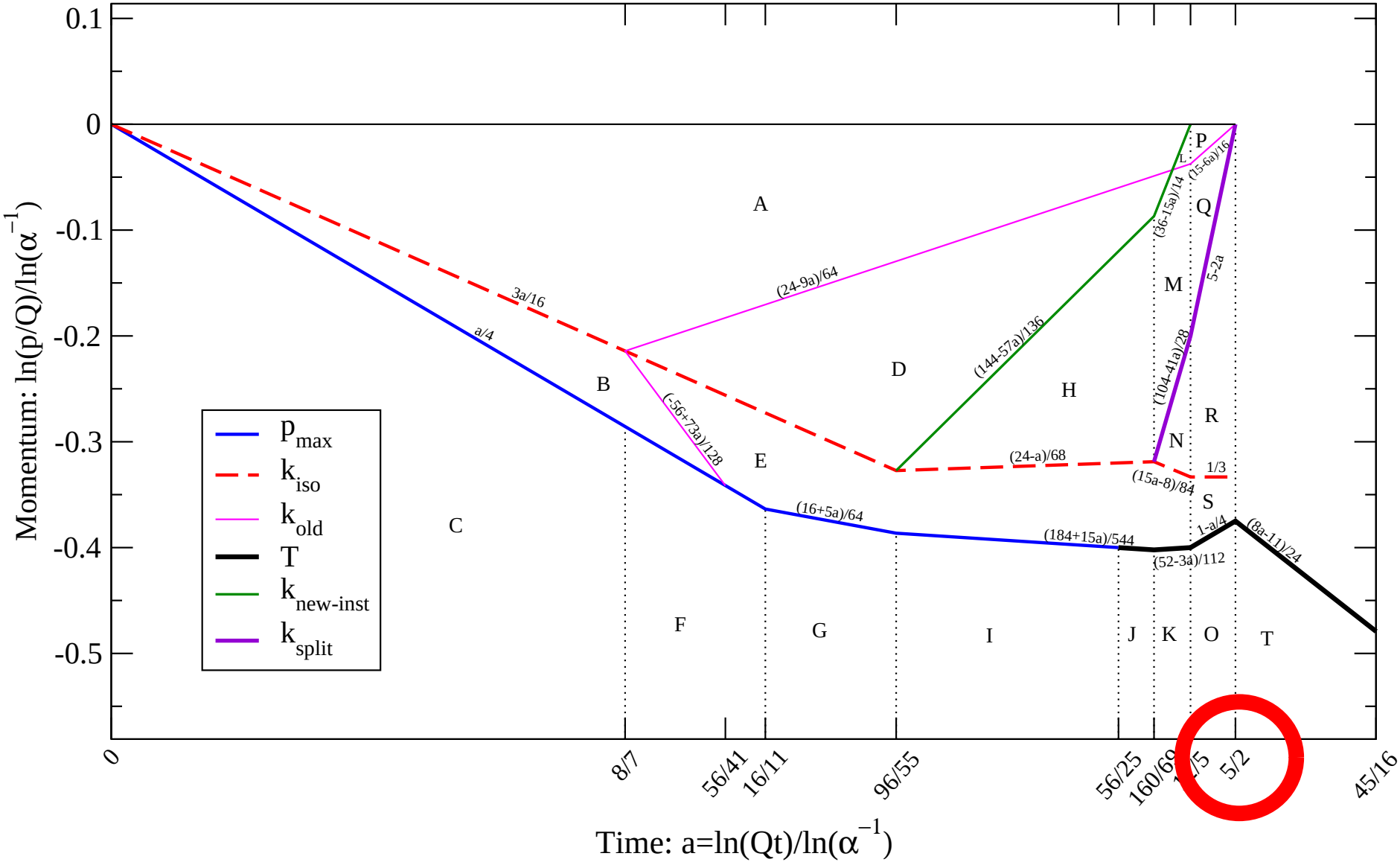
How?

- Extremely big and at extremely high energy
  - Saturation scale $Q_s \gg \Lambda_{QCD}$
  - weak coupling: $\alpha_s(Q_s) \ll 1$
- Identifying relevant physics

New:

- Complete parametric treatment of plasma instabilities
- Instability driven bottom-up thermalization: $\tau_0 \sim \alpha^{-5/2} Q_s^{-1}$

Instability driven bottom-up thermalization



Momentum scales vs. time scales

Initial condition: $t \sim Q_s^{-1}$

At weak coupling, well understood: Color Glass Condensate

- Characteristic, *saturation* scale: Q_s
- Highly occupied: $f(p < Q_s) \sim 1/\alpha$

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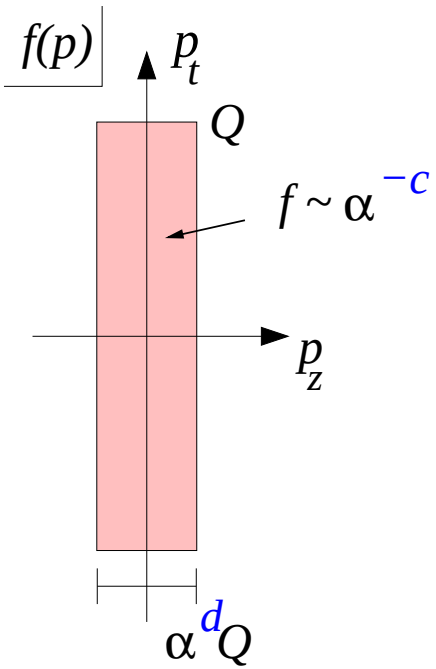
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- Initial energy density: $\varepsilon(t \sim Q_s^{-1}) \sim \alpha^{-1} Q_s^4$

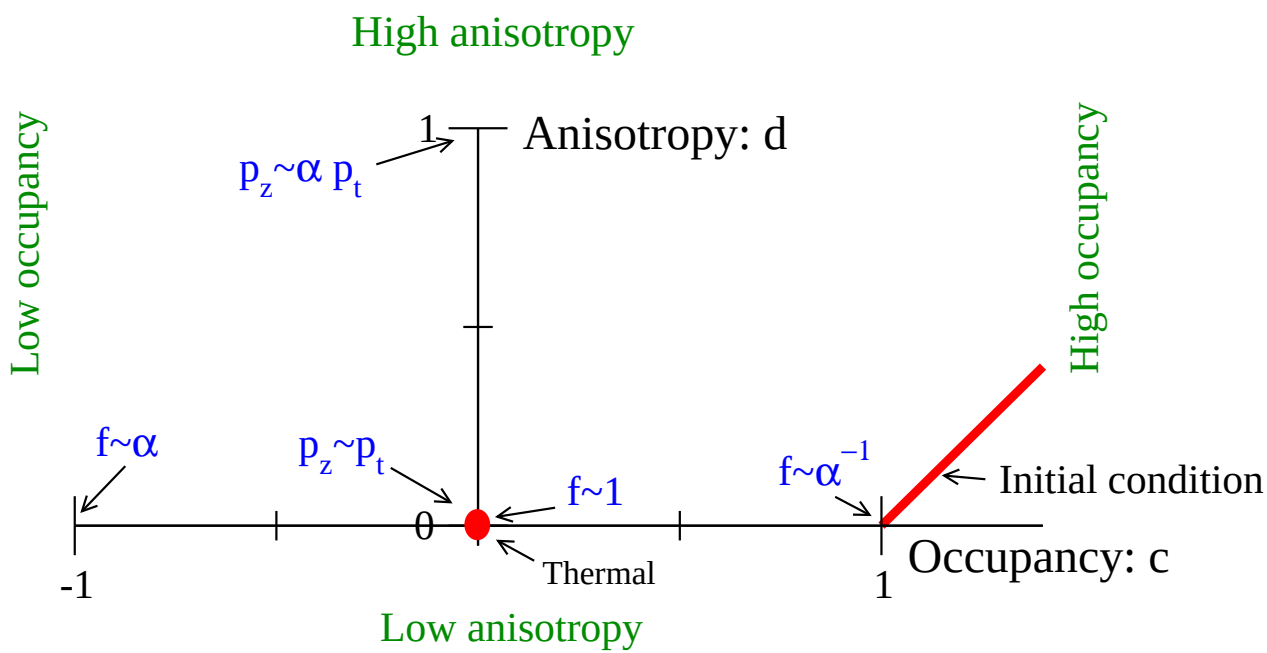
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$$f_{gluon}(p) \sim \underbrace{\alpha^{-c}}_{\text{occupancy}} \theta(Q_s - p)\theta(\underbrace{\alpha^{d}}_{\text{anisotropy}} Q_s - p_z); \quad \delta \equiv \alpha^d$$





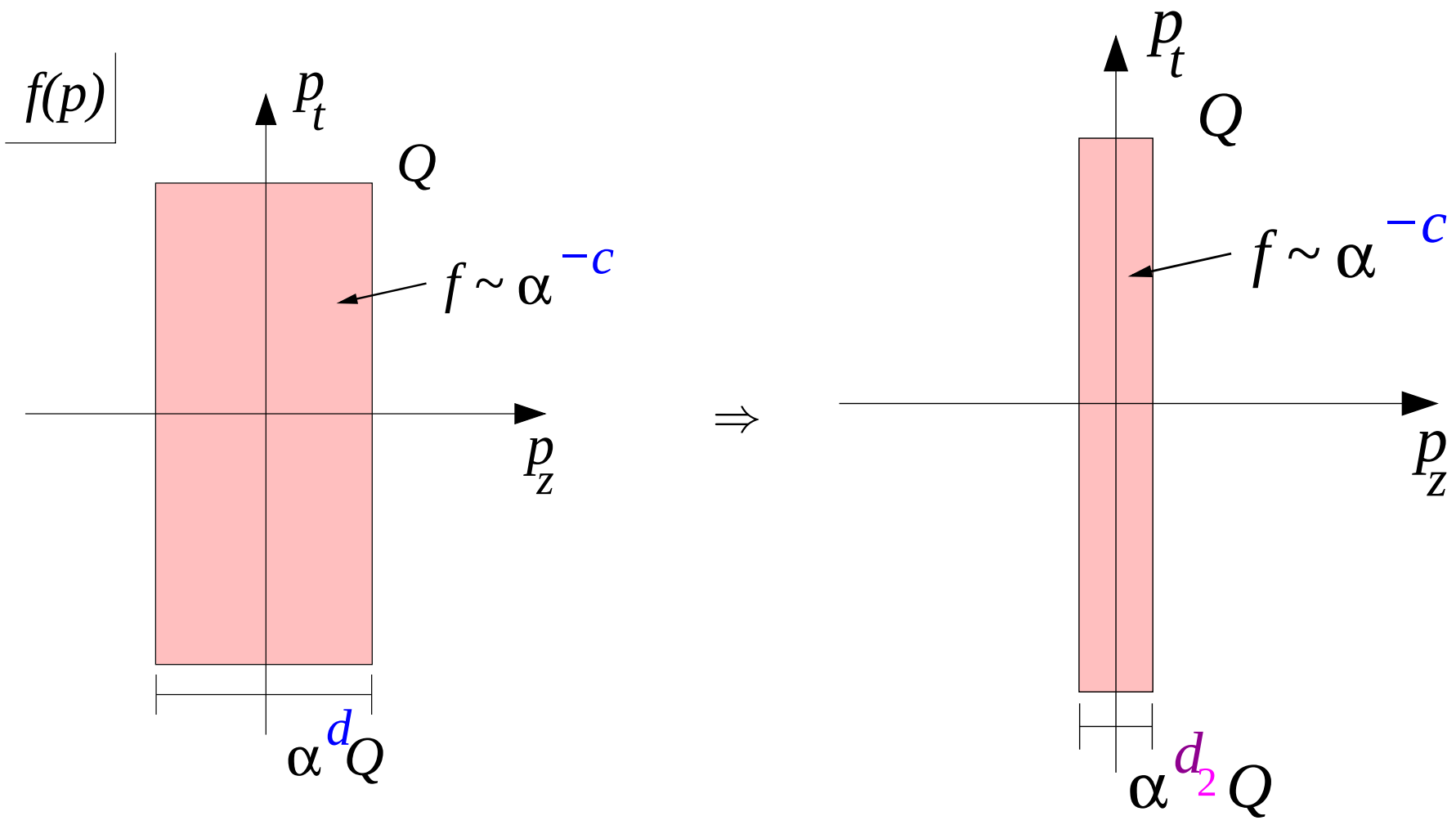
Dynamics

- Longitudinal expansion
- Elastic scattering
- Inelastic scattering
- Plasma instabilities

Longitudinal expansion

Spatial expansion translates into redshift in $p_z \sim \alpha^d Q_s$

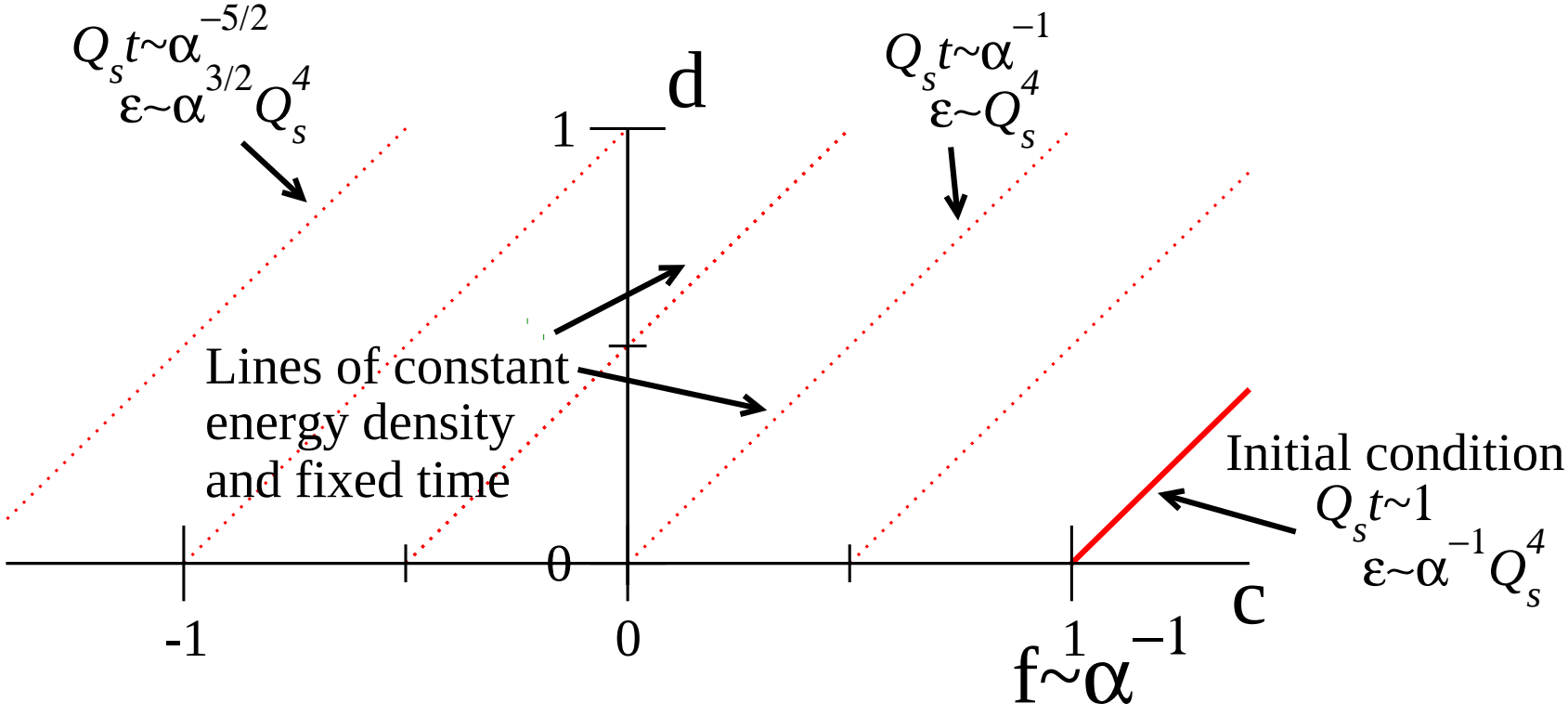
- Reduces $p_z = \alpha^d Q$, Q_s stays constant
- Reduces energy: $\varepsilon(t) \sim \alpha^{-1} Q_s^4/(Q_s t)$



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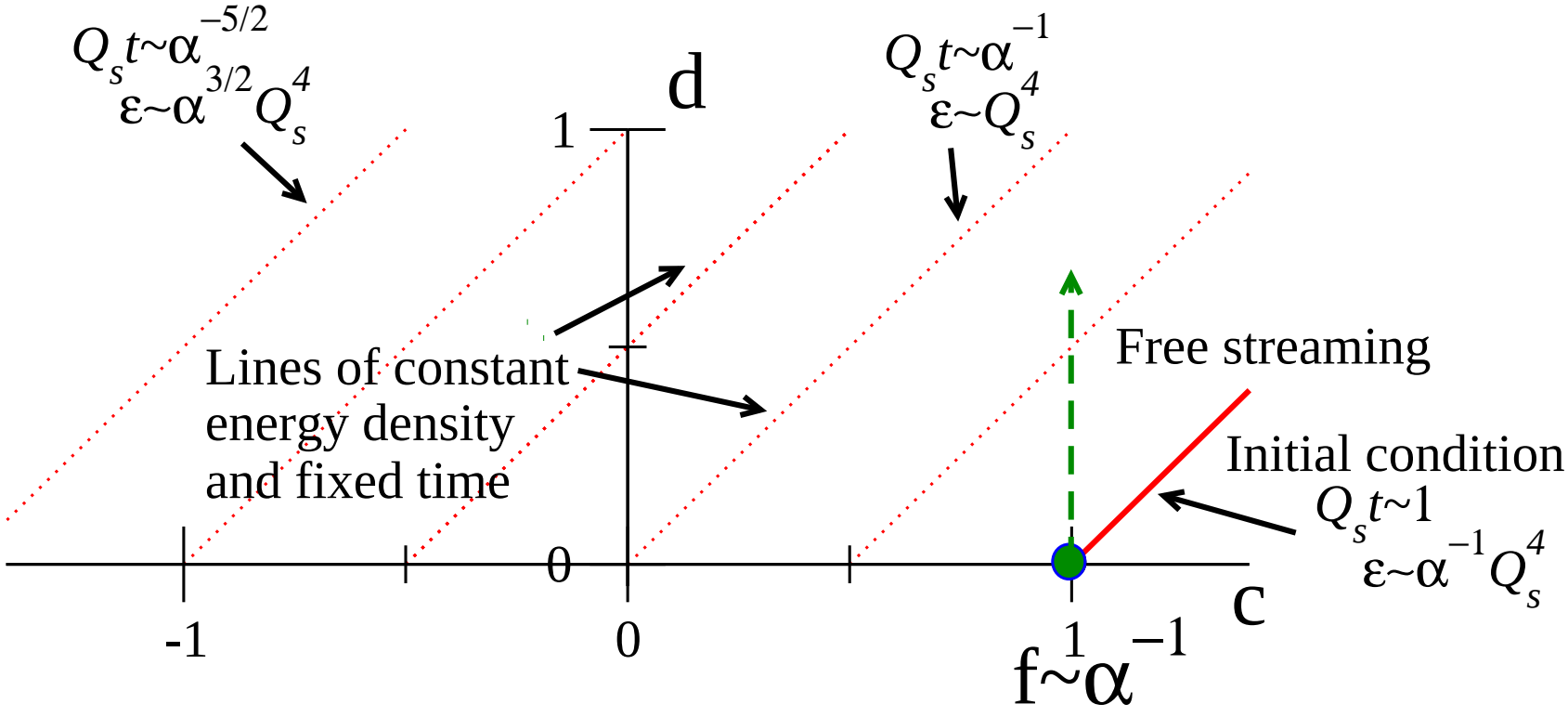
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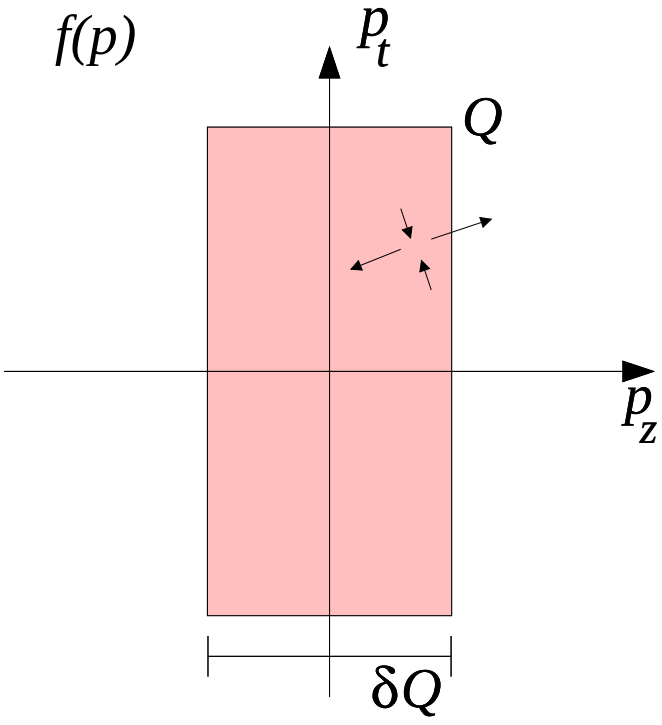
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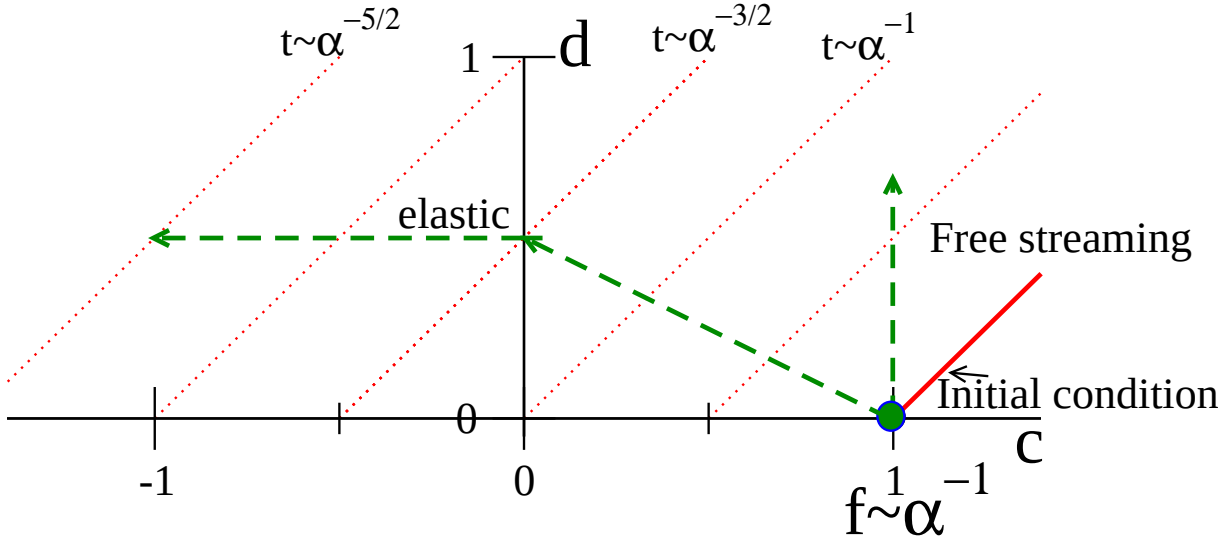
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Scattering: Elastic

Elastic scattering makes the distribution fluffier $(\delta \equiv \alpha^d)$

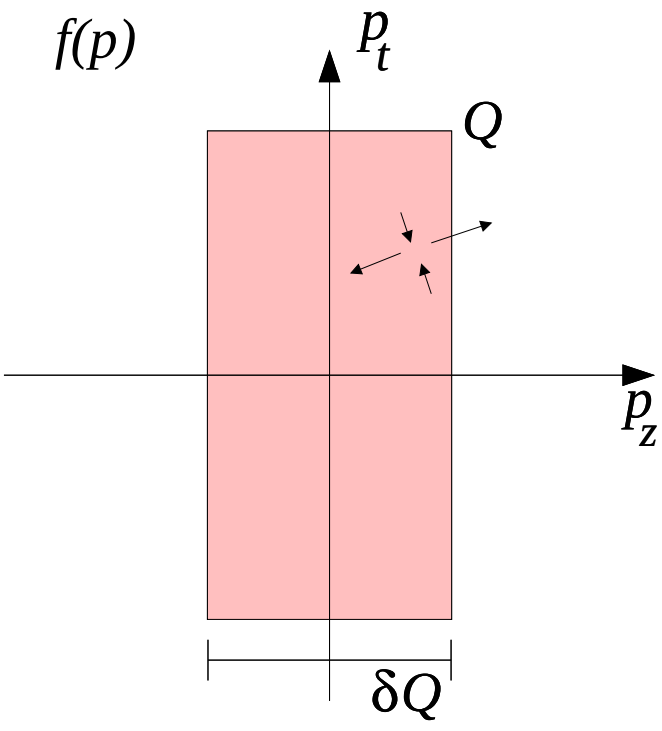


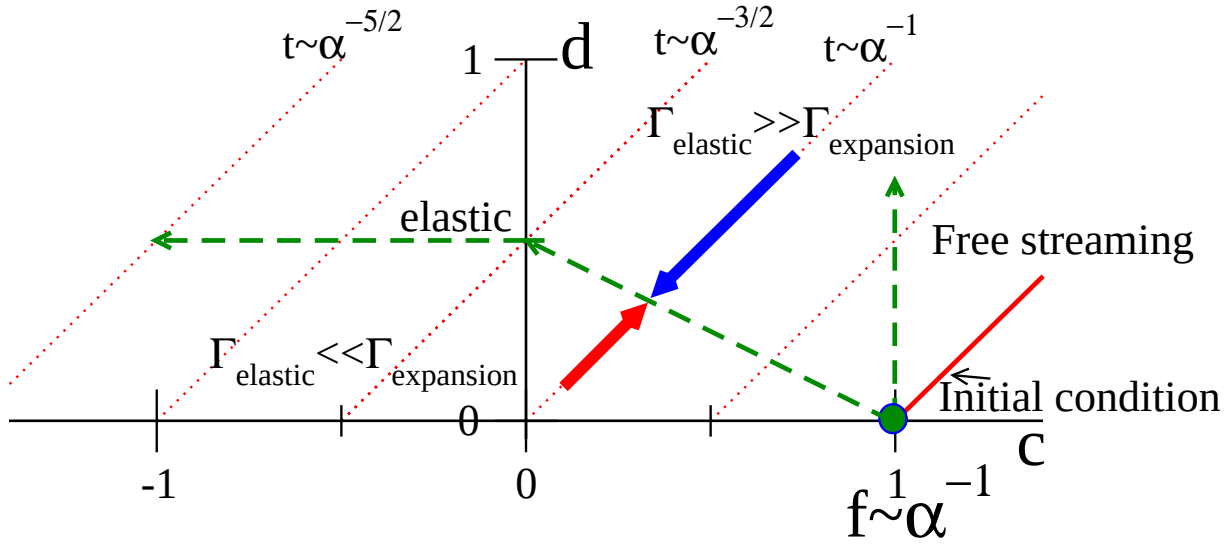


- Along the attractive solution scattering and expansion compete
- Eventually system becomes underoccupied
- At late times: Fixed anisotropy, dilute away

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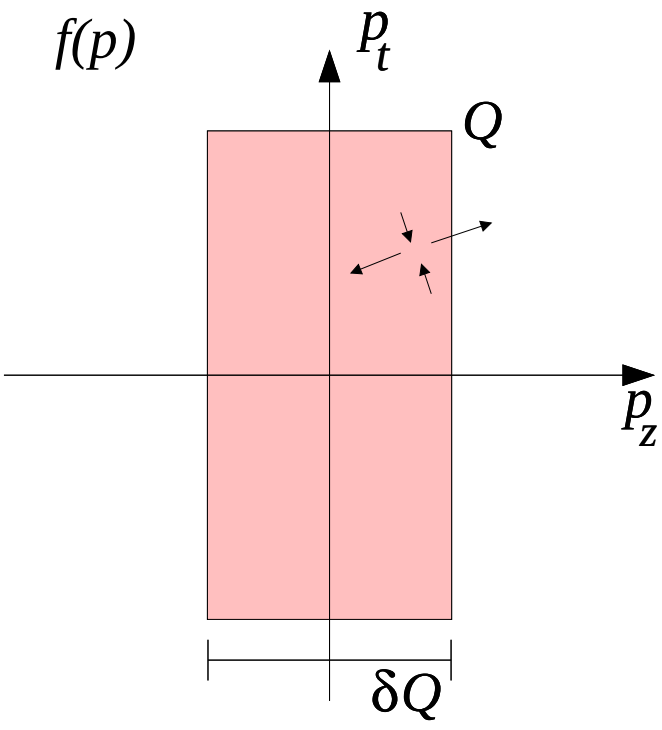


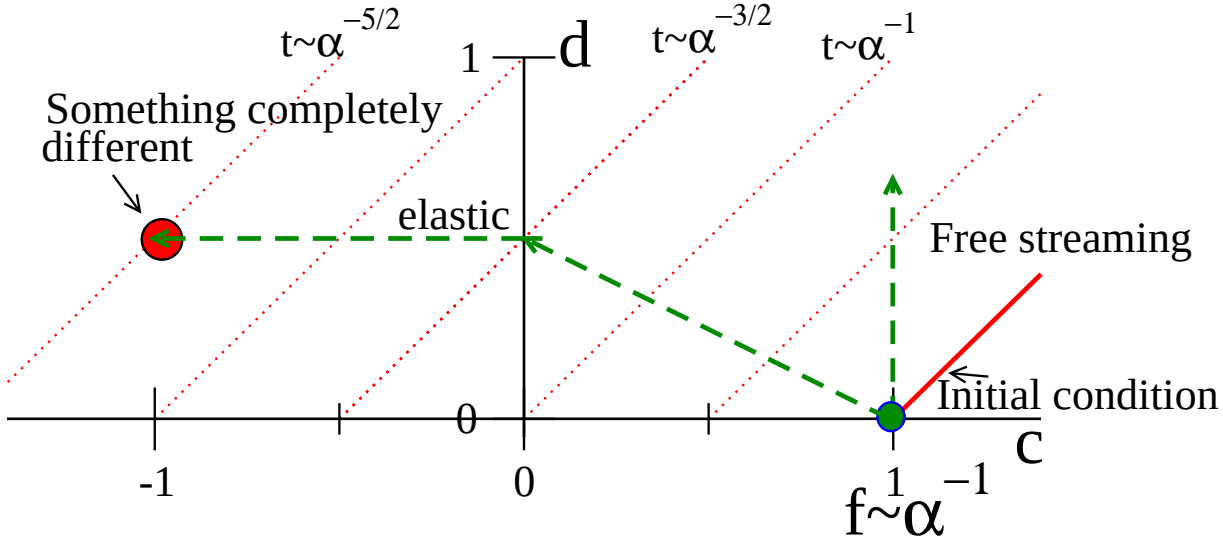
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Elastic scattering not enough for thermalization!

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Scattering: Inelastic

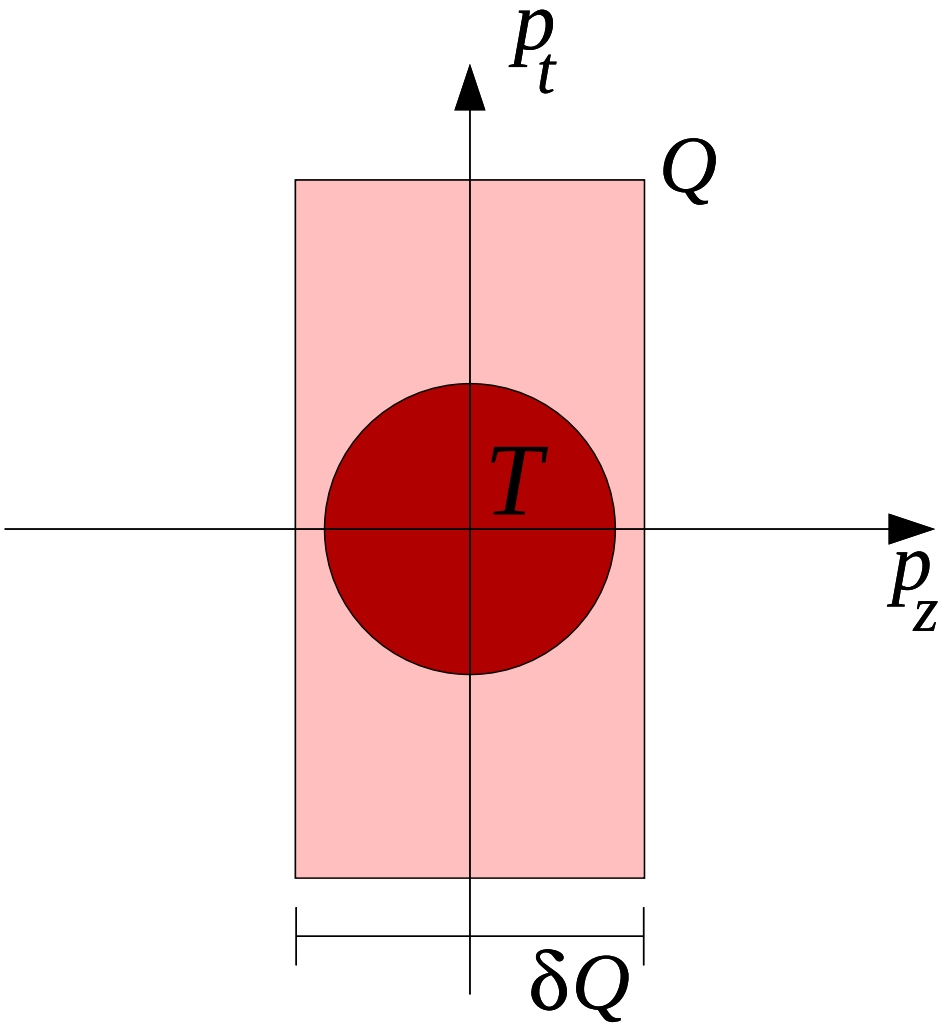
Inelastic scattering plays two significant roles (Baier, Mueller, Schiff & Son 2000)

1. soft splitting: creation of a soft thermal bath
2. hard splitting: breaking of the hard particles

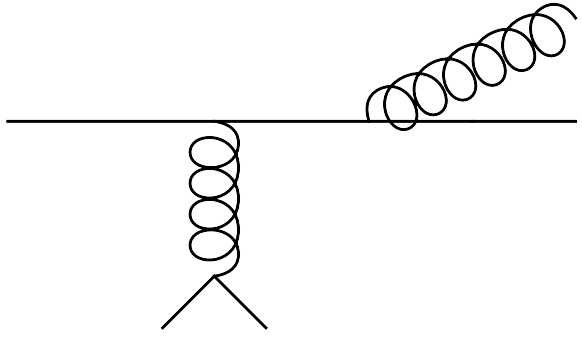
Scattering: Inelastic

Baier, Mueller, Schiff & Son 2000

Soft splitting: Creation of soft thermal bath



- Soft modes quick to emit
  $n_s \sim \alpha n_{\rm col}$



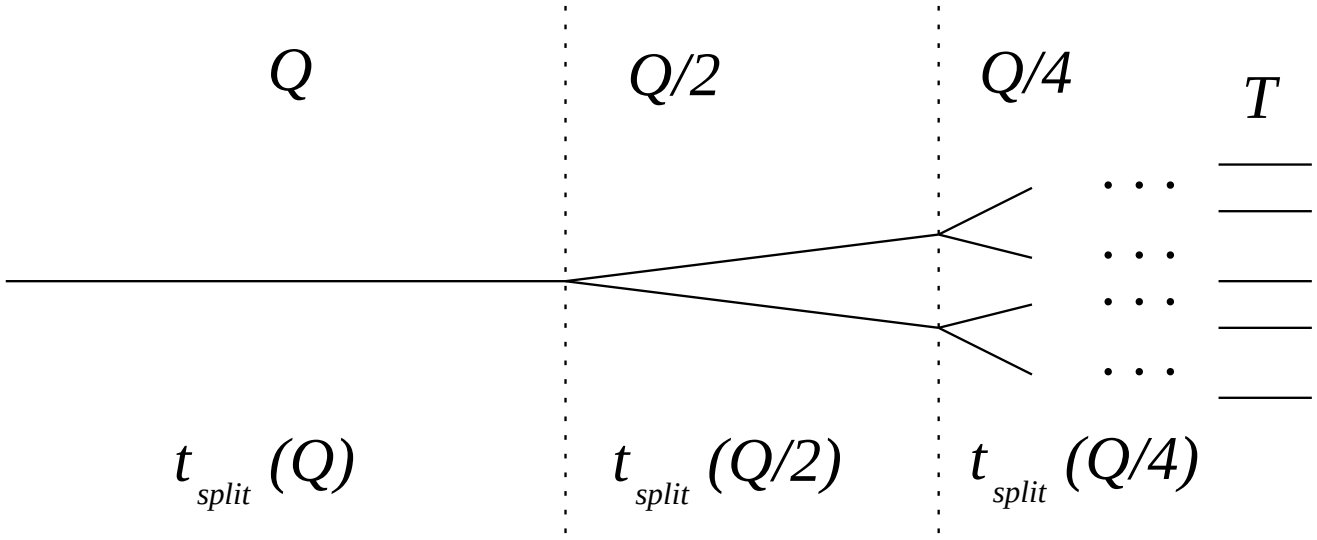
- Low p: easy to bend
  $\Rightarrow$thermalize quickly
- *Can dominate dynamics!*
  (i.e. scattering, screening, . . .)

$\Rightarrow$ Few energetic "jets" propagating in thermal bath

Scattering: Inelastic

Baier, Mueller, Schiff & Son 2000

Hard splitting: Q_s modes break before they bend!



- In vacuum: on-shell particles, no splitting
- In medium: Particles receive small kicks frequently
  - For stochastic uncorrelated kicks: Brownian motion in p-space

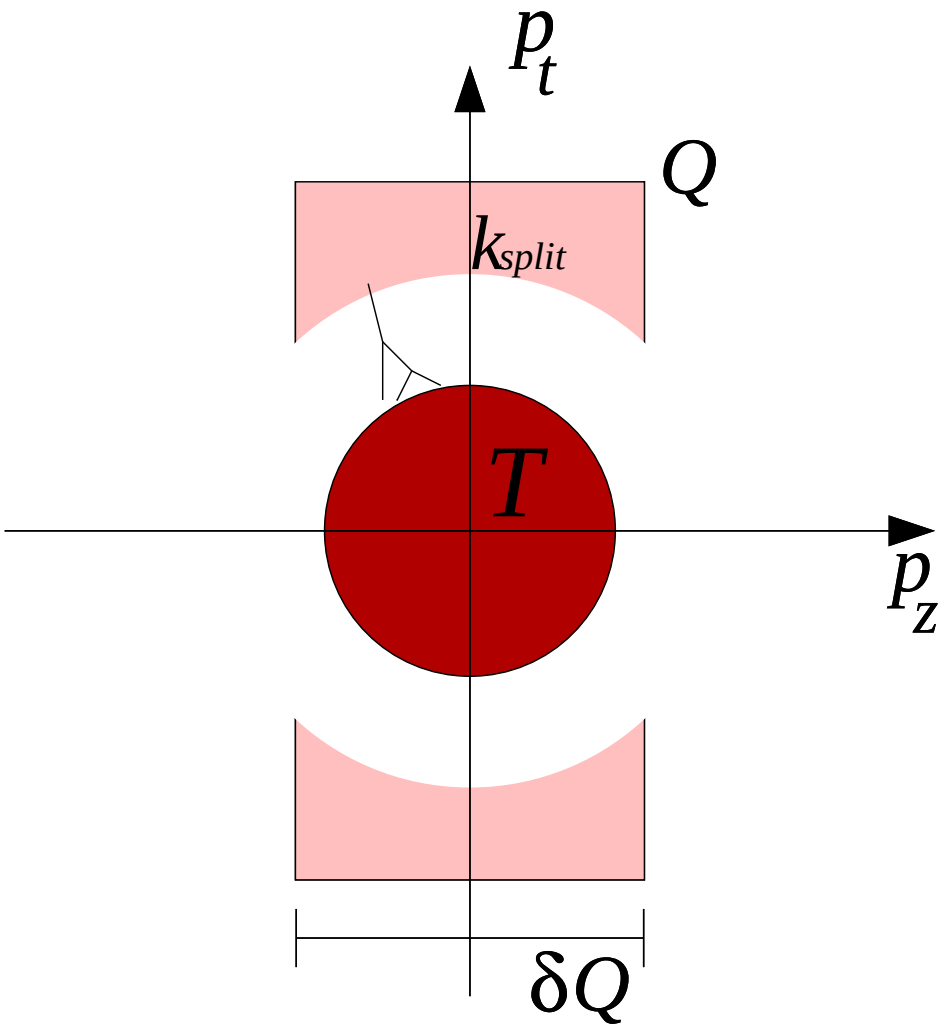
$$\Delta p_\perp^2 \sim \hat{q} t, \qquad t_{\rm split}(k) \sim l_{\rm stop}(k) \sim \alpha^{-1} \underbrace{\sqrt{k/\hat{q}}}_{t_{\rm form}} \qquad \text{(LPM)}$$

  - $\hat{q}$ describes how the medium wiggles a hard parton

Bottom-Up

Baier, Mueller, Schiff & Son 2000

Thermal bath eats the hard particles away:



- Scales below $k_{\rm split}$ have cascaded down to T-bath

$$t_{\rm split}(k_{\rm split}) \sim t \Rightarrow k_{\rm split} \sim \alpha^2 \hat{q} t^2$$

- "Falling" particles heat up the thermal bath

$$T^4 \sim k_{\rm split} \int d^3 p f(p)$$

- Thermalization when Q_s gets eaten

$$k_{\rm split} \sim Q_s$$

Needs $\hat{q}$ as an input

Old bottom up:

Baier, Mueller, Schiff & Son 2000

$\hat{q}$ dominated by elastic scattering with thermal bath??

BMSS assumed: $\hat{q} \sim \hat{q}_{\rm elastic} \sim \alpha^2 T^3$

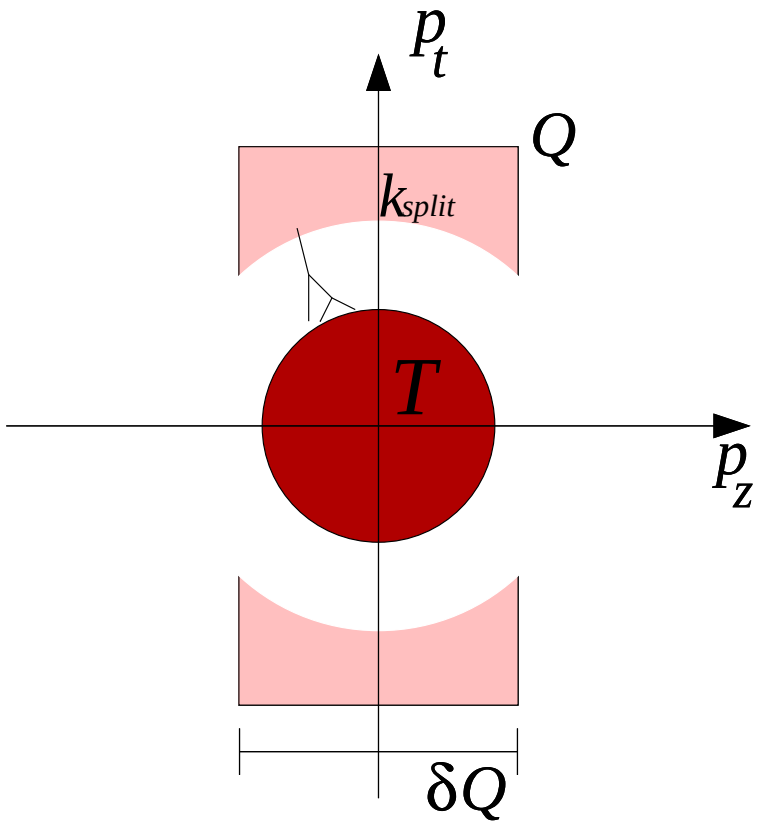
Solve self-consistently:

$$\begin{cases} k_{\rm split} & \sim & \alpha^2 \hat{q} t^2 \\ T^4 & \sim & k_{\rm split}(Q_s^3/(Q_s t)) \\ \hat{q} & \sim & \alpha^2 T^3 \end{cases}$$

for:

$$\Rightarrow \begin{cases} T & \sim & \alpha^3 Q_s (Q_s t) \\ k_{\rm split} & \sim & \alpha^{13} (Q_s t)^5 Q \\ \tau_0 & \sim & \alpha^{-13/5} Q_s^{-1} \end{cases}$$

But: Is this all there is?



$\hat{q}$ dominated (always!) by

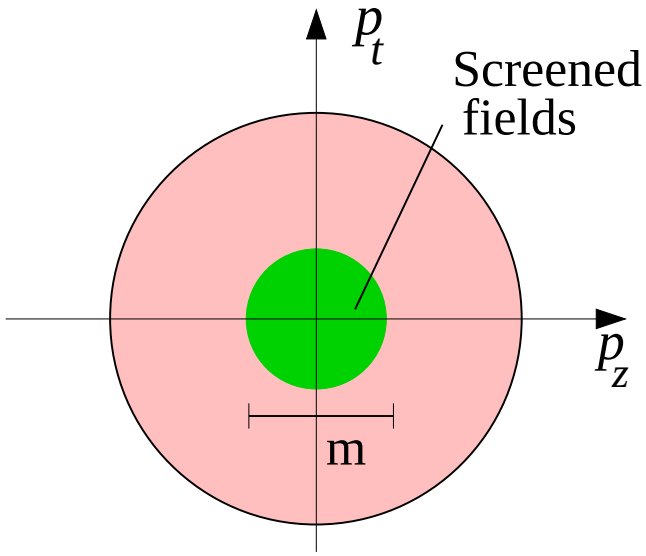
plasma instabilities

Plasma instabilities:

Isotropic distributions:

- Screening stabilizes soft E-fields

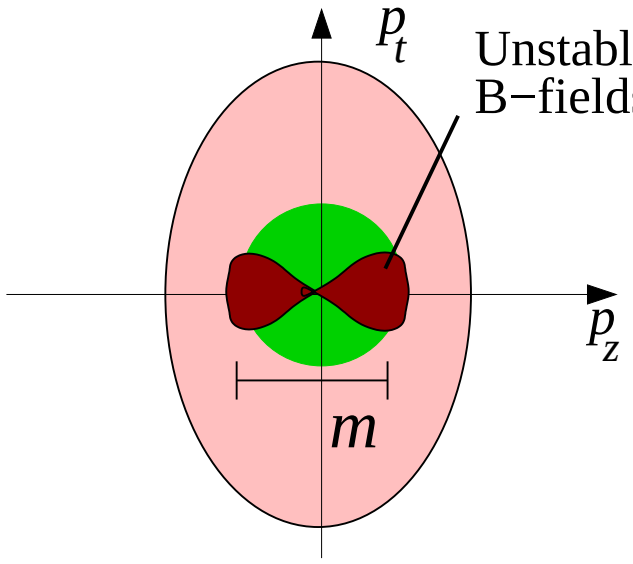
$$\omega_{pl}^2 \sim m^2 \sim \alpha \int d^3p \frac{f(p)}{p}$$



- B-fields induces a rotation on $f(p) \rightarrow$ No screening for static B-fields

Anisotropic distributions:

- B-field induces non-trivial rotation:
  - Some B-fields stabilized
  - ...others destabilized: Plasma-unstable modes

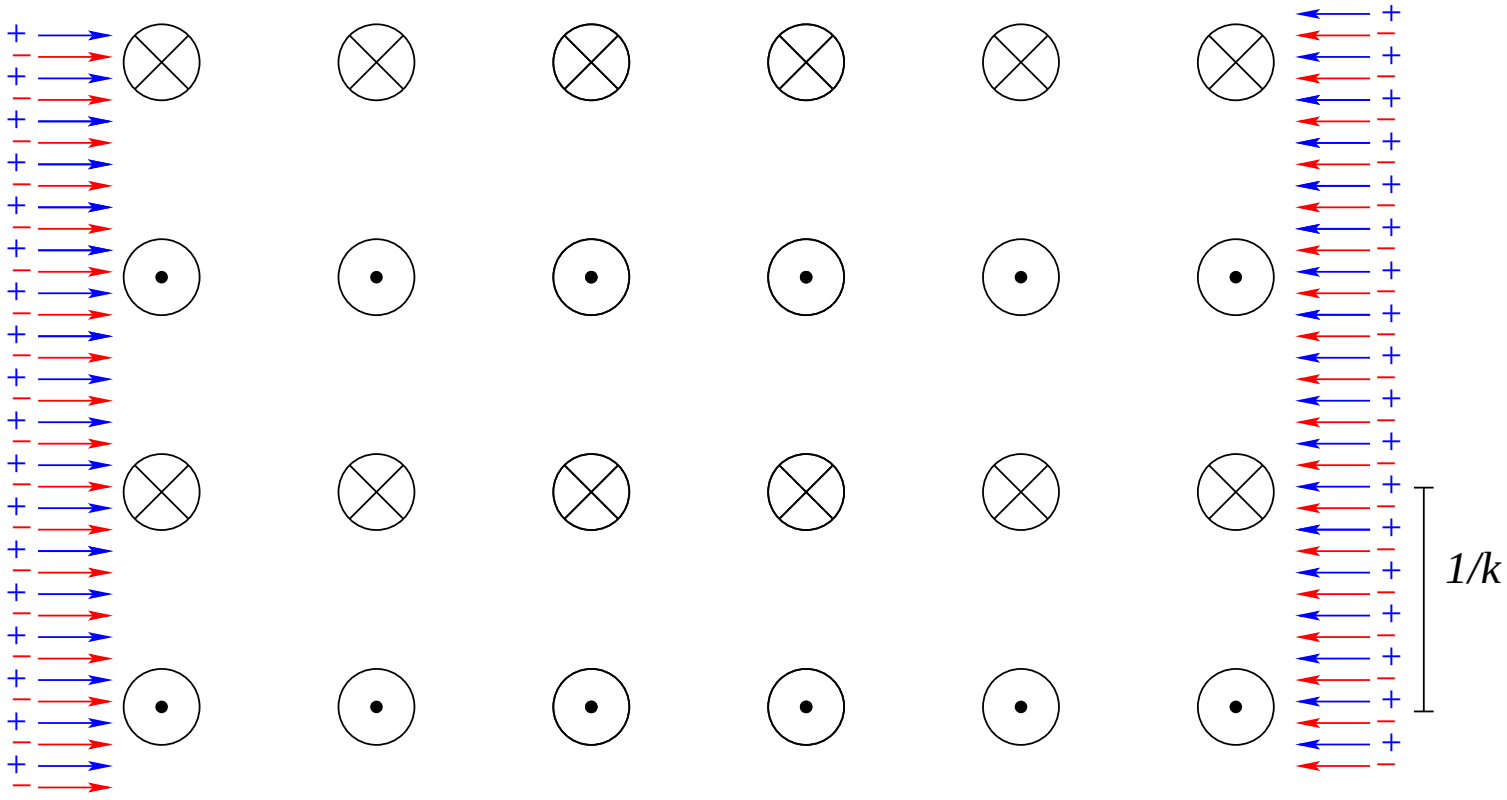


- Unstable modes grow exponentially...

Plasma instabilities: Idea

Mrowczynski

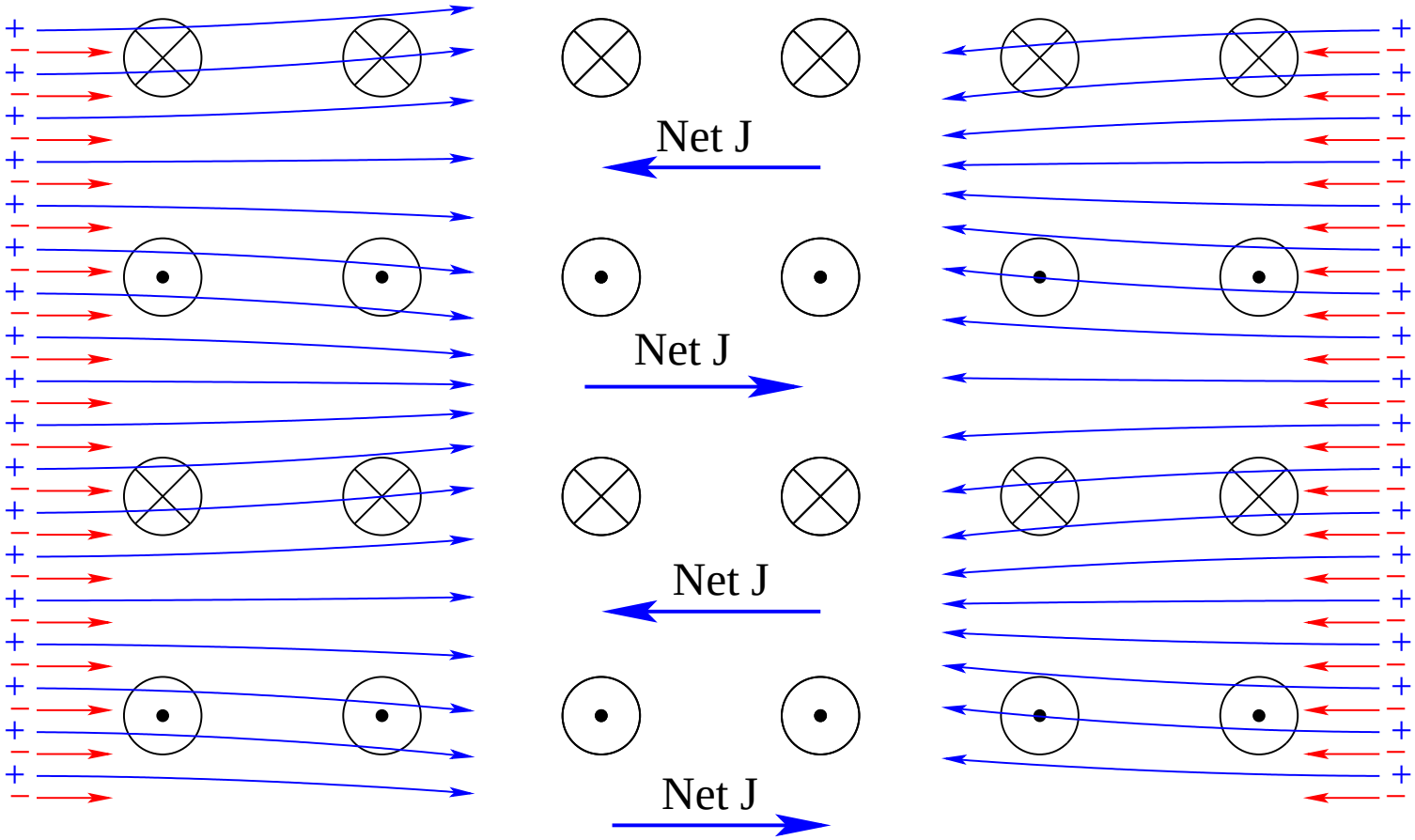
Exponential growth of (chromo)-magnetic fields in anisotropic plasmas



How do particles deflect?

Plasma instabilities: Idea

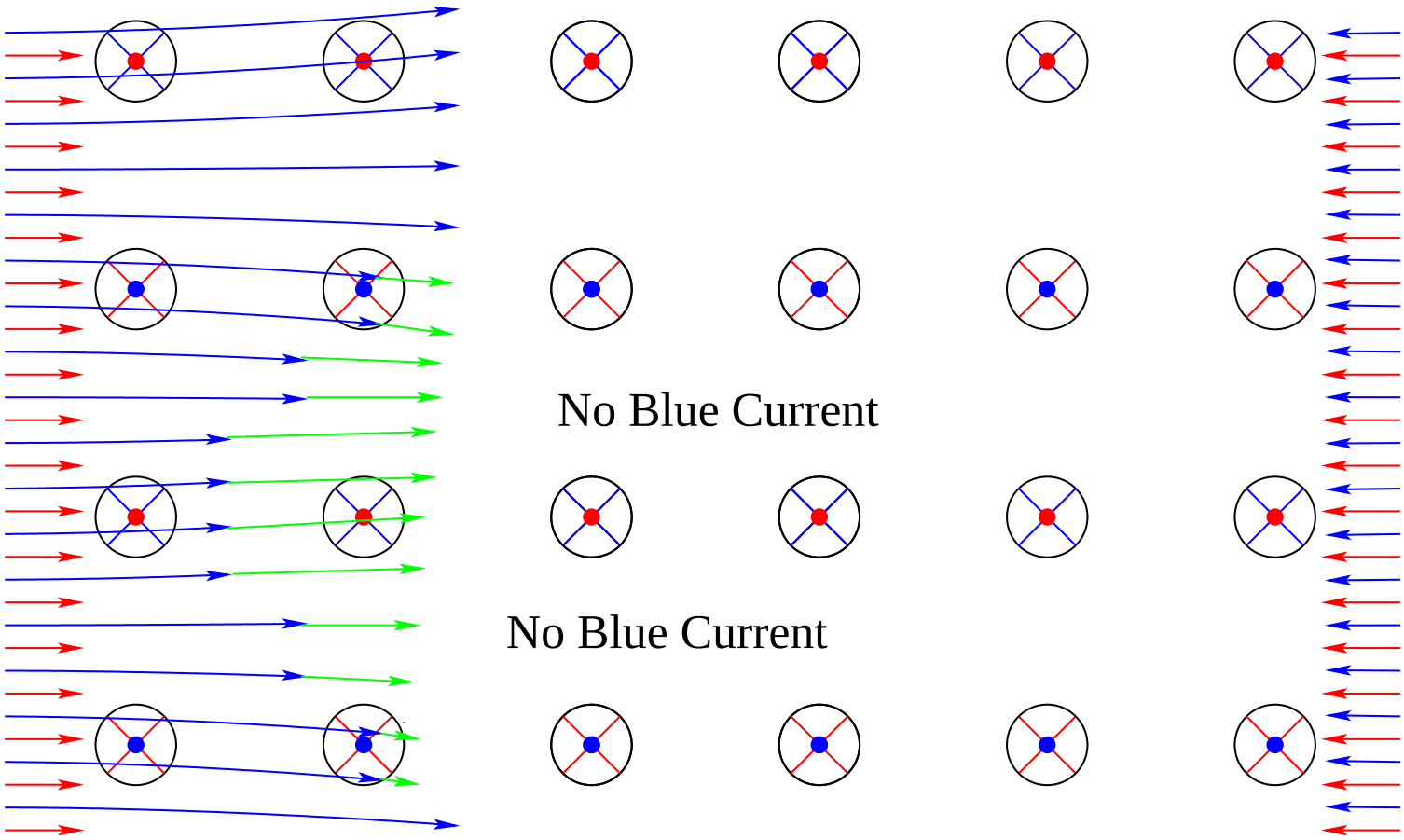
Exponential growth of (chromo)-magnetic fields in anisotropic plasmas



Induced current feeds the magnetic field

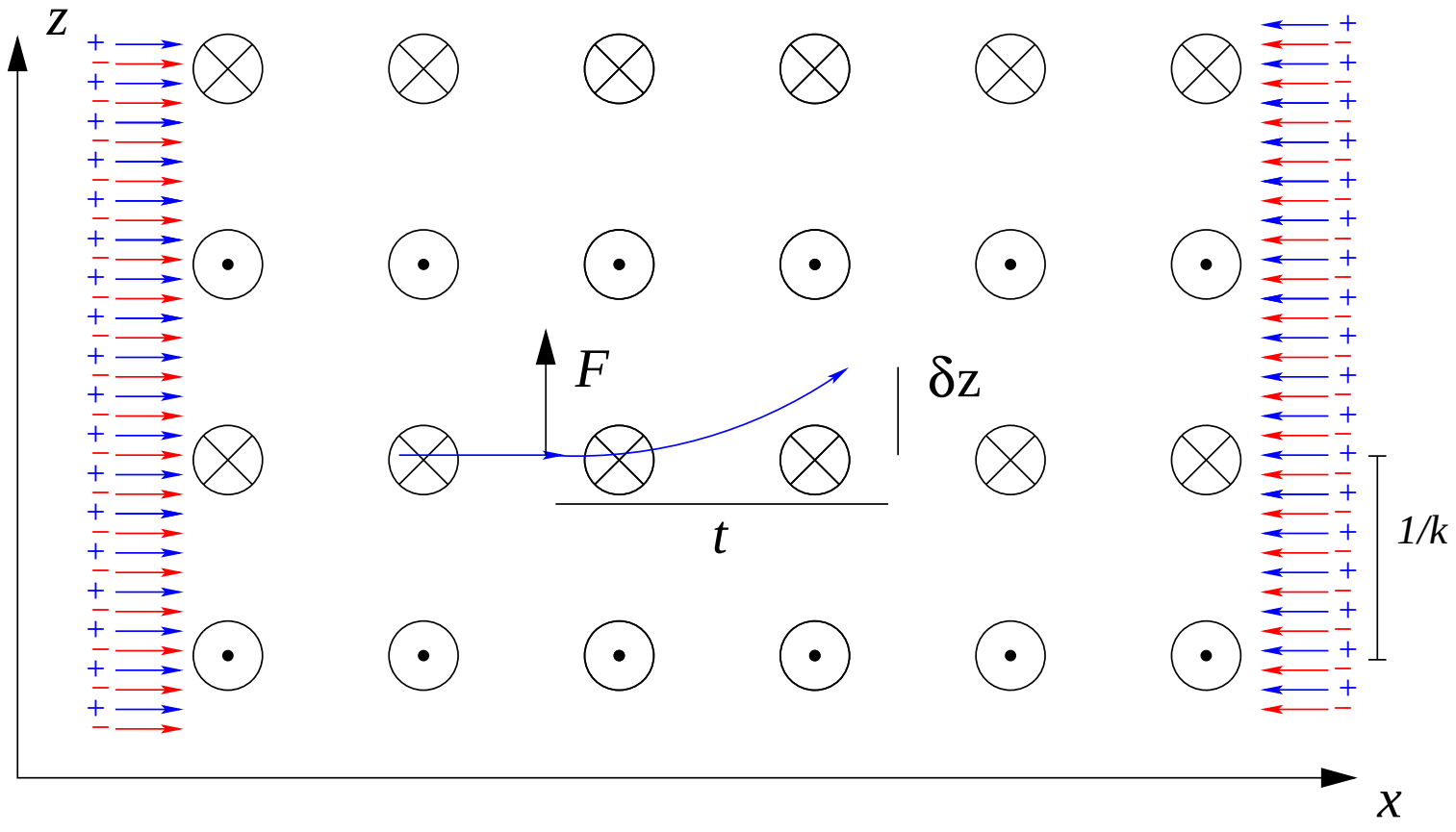
Plasma instabilities: Idea

When chromo-magnetic field becomes strong enough to mix colors, currents no longer "feed" the magnetic field. Growth is cut off.



Plasma instabilities: Slightly more quantitive

- Lorentz force: $F \sim gB$
- Displacement: $\delta z \sim gBt^2/p$



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- Current:

$$J \sim \underbrace{g}_{\textit{charge}} \underbrace{\int d^3p f(p)}_{\#\text{ of part.s}} \underbrace{[\delta z\, k]}_{\text{disp. fraction}} \sim kBt^2\, \alpha \underbrace{\int \frac{d^3p}{p} f(p)}_{m^2} \sim kBm^2t^2$$

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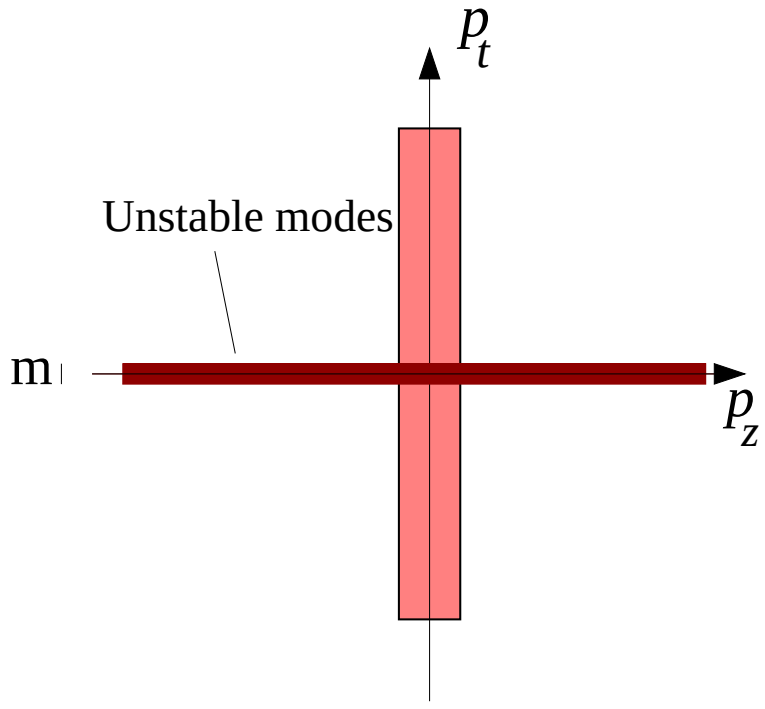
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- Competes in Maxwell's equation with $\nabla \times B \sim k\,B$

$$\boxed{\Rightarrow \text{Exponential growth if } m^2t^2 > 1}$$

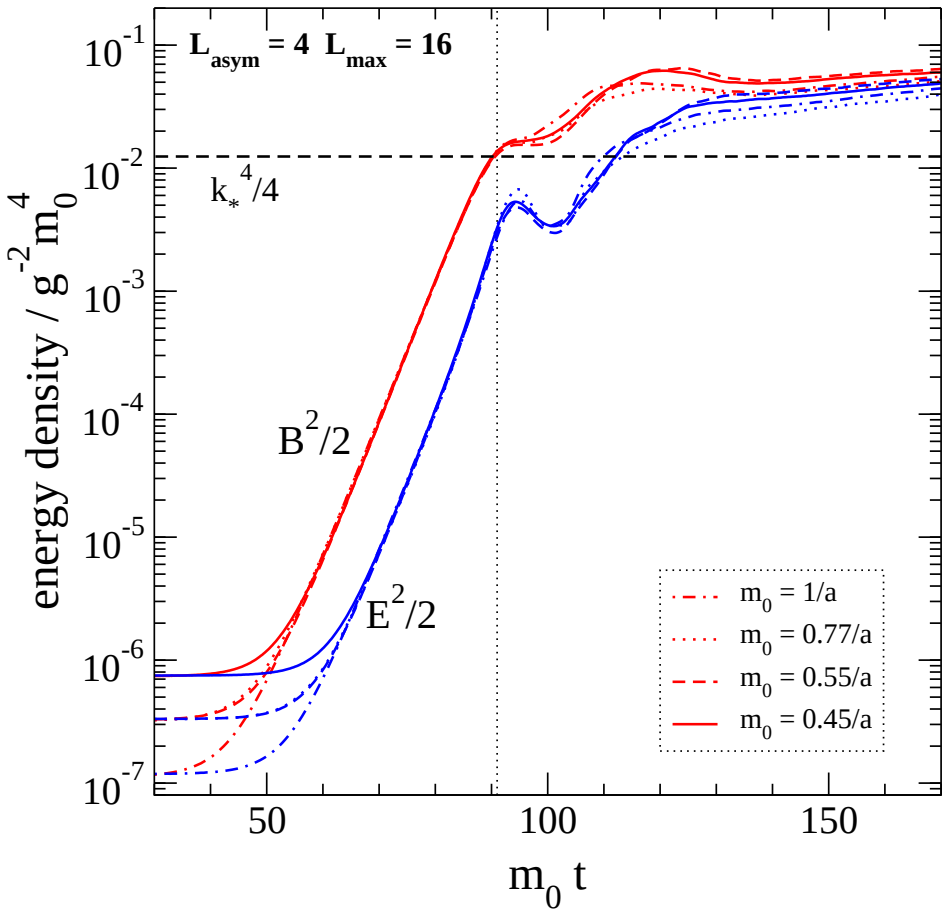
$$\Rightarrow \begin{cases} k_z^{inst} \sim any \\ k_x^{inst} < m \end{cases}$$



Saturation of the instabilities.

Saturation when fields become strongly non-linear

$$D_\mu = k_{\rm inst} + igA_\mu \Rightarrow \begin{cases} A \sim k_{\rm inst}/g \\ f(k_{\rm inst}) \sim 1/\alpha \end{cases}$$

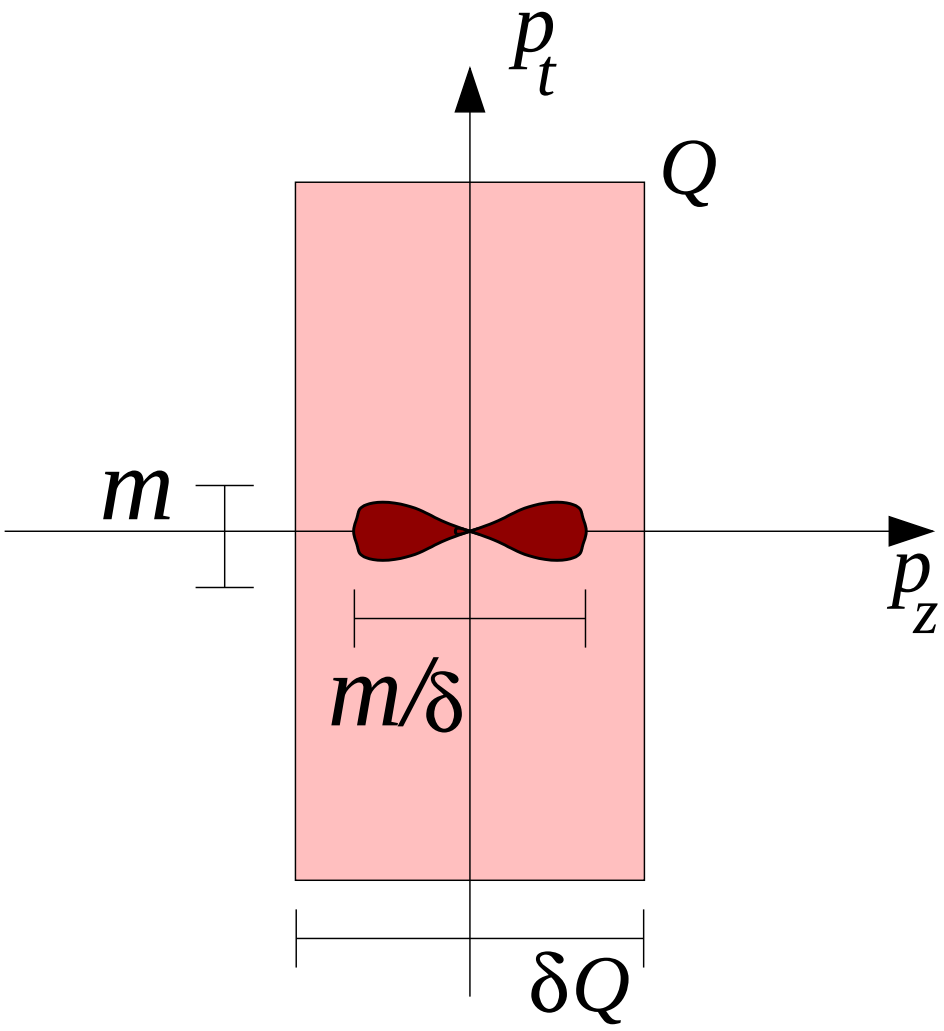


Bödeker and Rummukainen (0705.0180) and many others. . .

Plasma instabilites: More complicated distribtutions

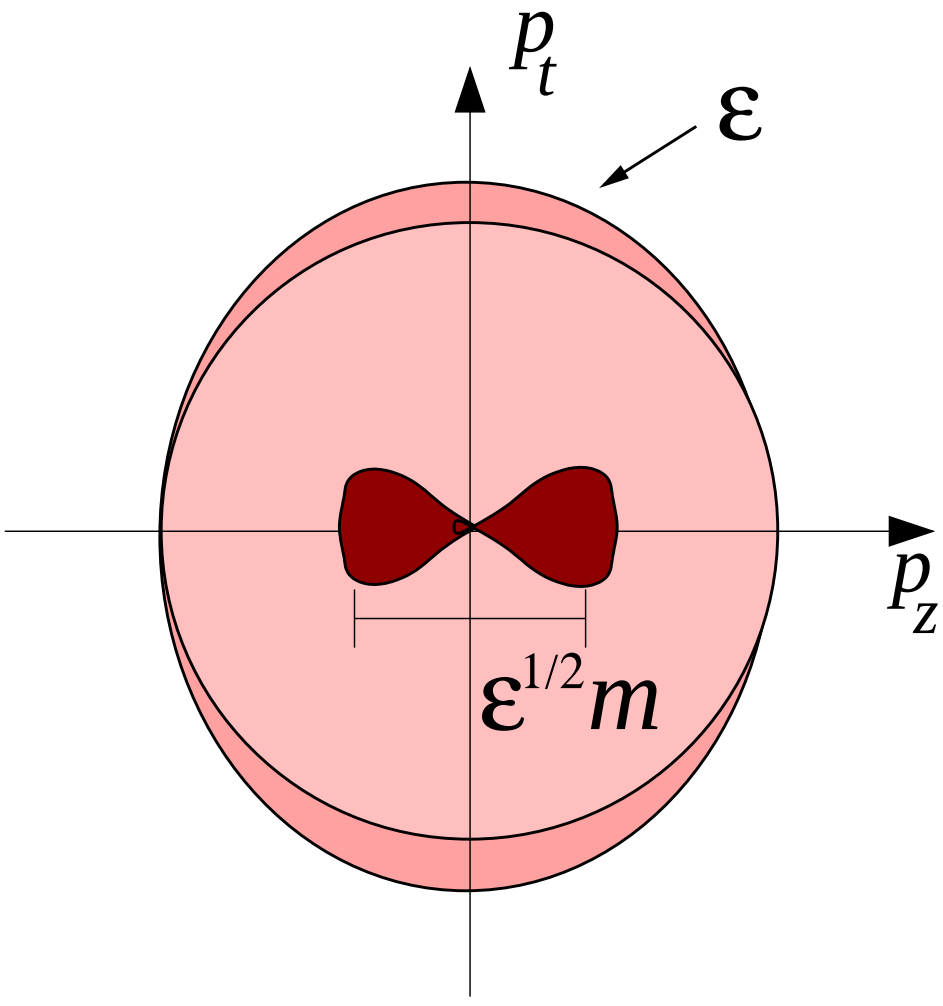
Strong anisotropy:

$$f(\vec{p}) \sim \alpha^{-c}\Theta(Q_s - p)\Theta(\delta Q_s - p_z)$$



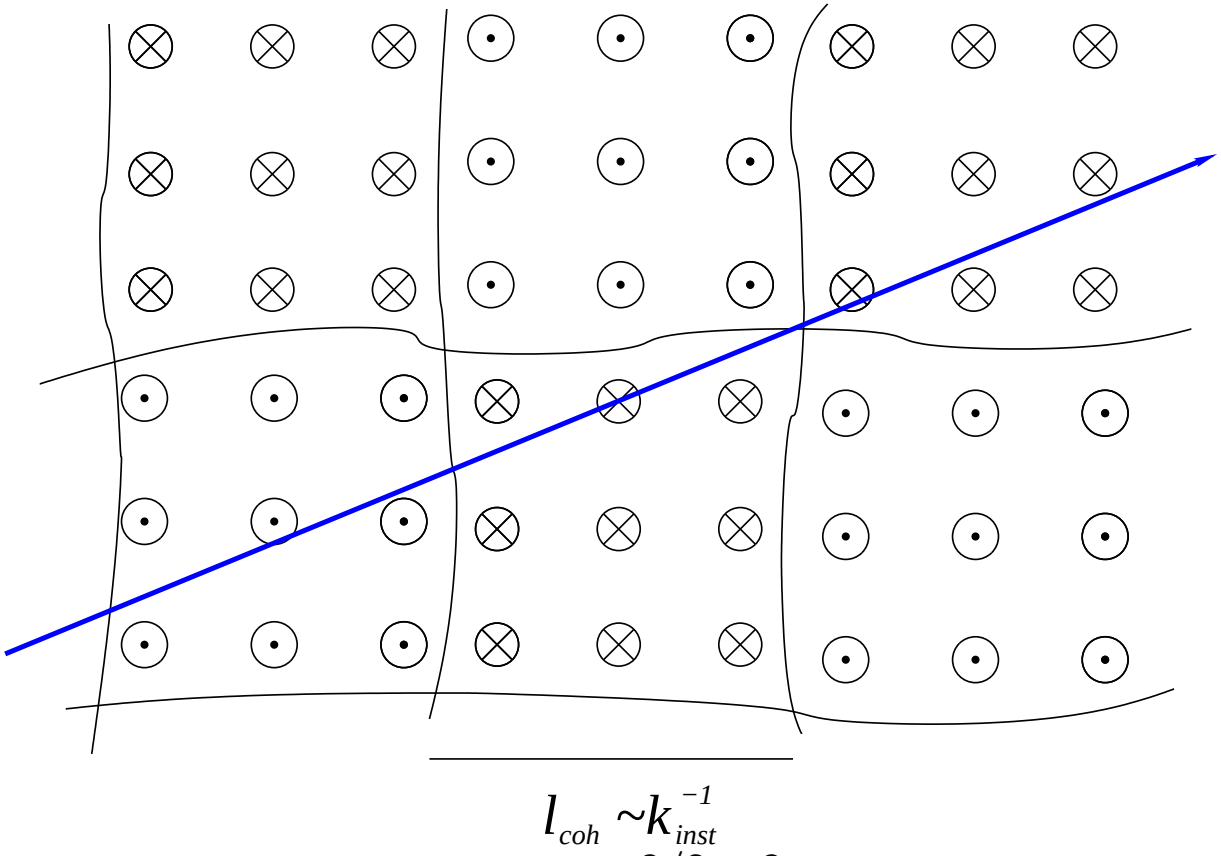
Weak anisotropy:

$$f(\vec{p}) \sim f_0(|\vec{p}|)(1 + \epsilon F(\vec{p}))$$



Plasma instabilites: Momentum diffusion $\hat{q}$

Hard parton traveling through magnetic fields receives coherent kicks from patches of same-sign magnetic fields



- $\Delta p_{\rm kick} \sim g\, B\, l_{\rm coh}$
- $\hat{q}\, t \sim \underbrace{N_{\rm kick}}_{t/l_{\rm coh}} (\Delta p_{\rm kick})^2$

$$\hat{q} \sim \alpha\, l_{\rm coh}\, B^2$$

Weak anisotropy: $\hat{q}_\epsilon \sim \epsilon^{3/2} m^3$

Strong anisotropy: $\hat{q}_\delta \sim m^3/\delta^2$

The new bottom-up

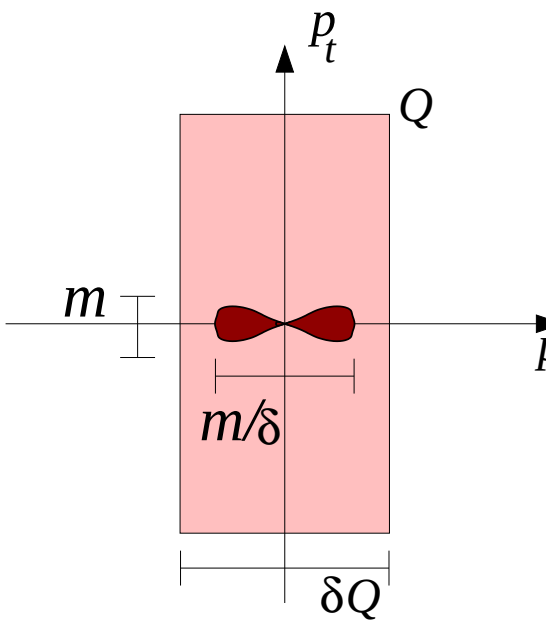
Important physics:

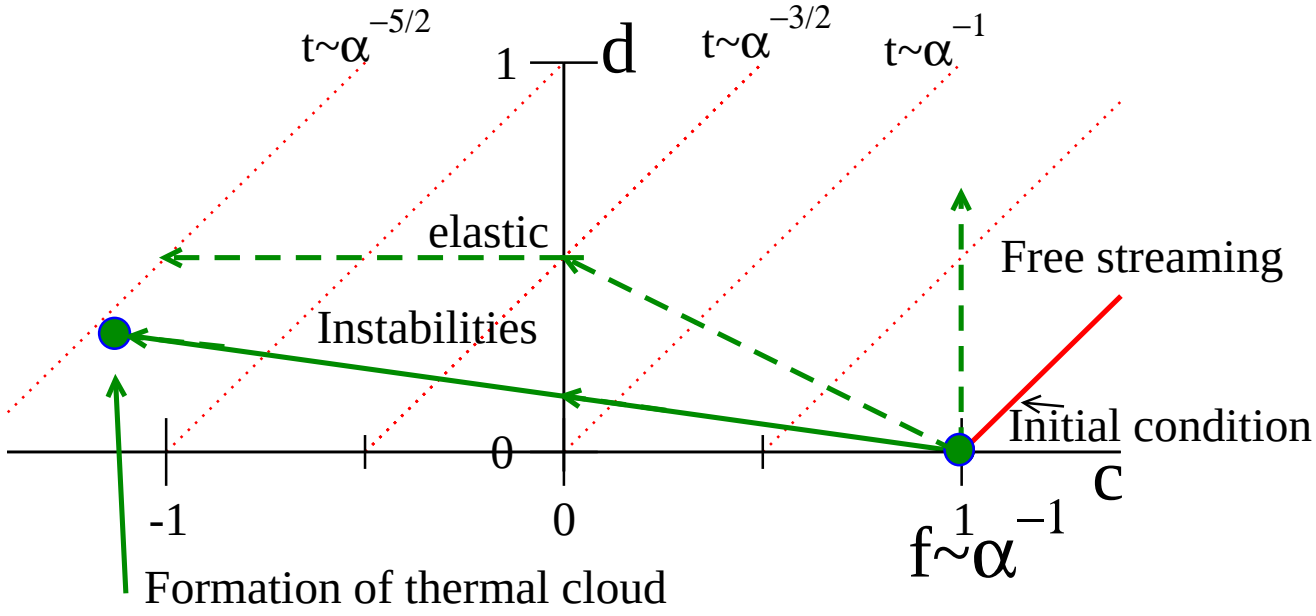
1. soft splitting: creation of a soft thermal bath
2. hard splitting: breaking of the hard particles
3. Plasma instabilities, screening

In particular, elastic scattering irrelevant

The new bottom-up: Early stages: $Q_s t < \alpha^{-12/5}$

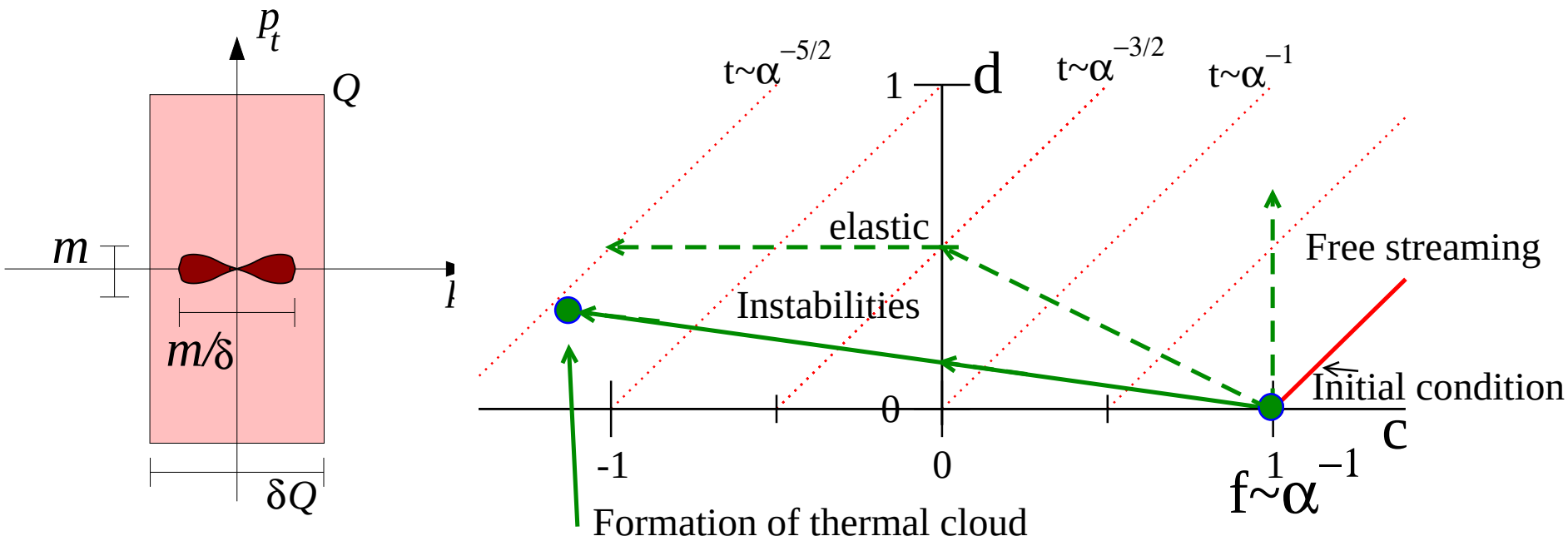
Broadening of the hard particle distribution dominated by the plasma instabilities ($\hat{q}_{el} \ll \hat{q}_{inst}$) originating from the scale Q_s





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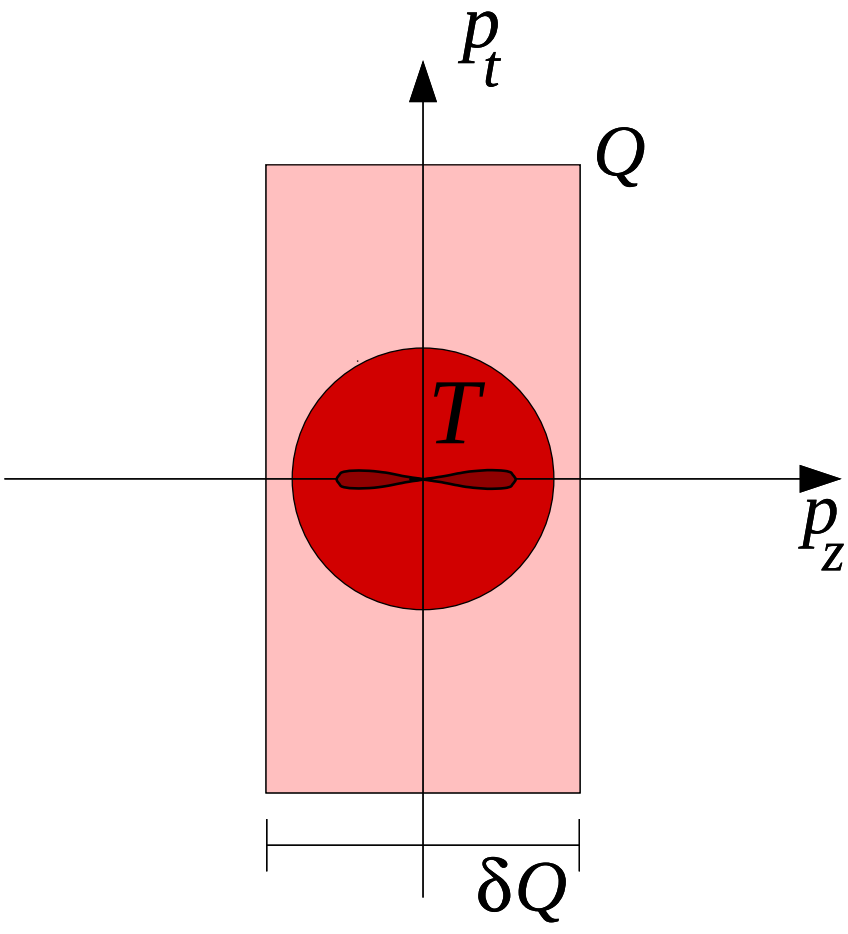
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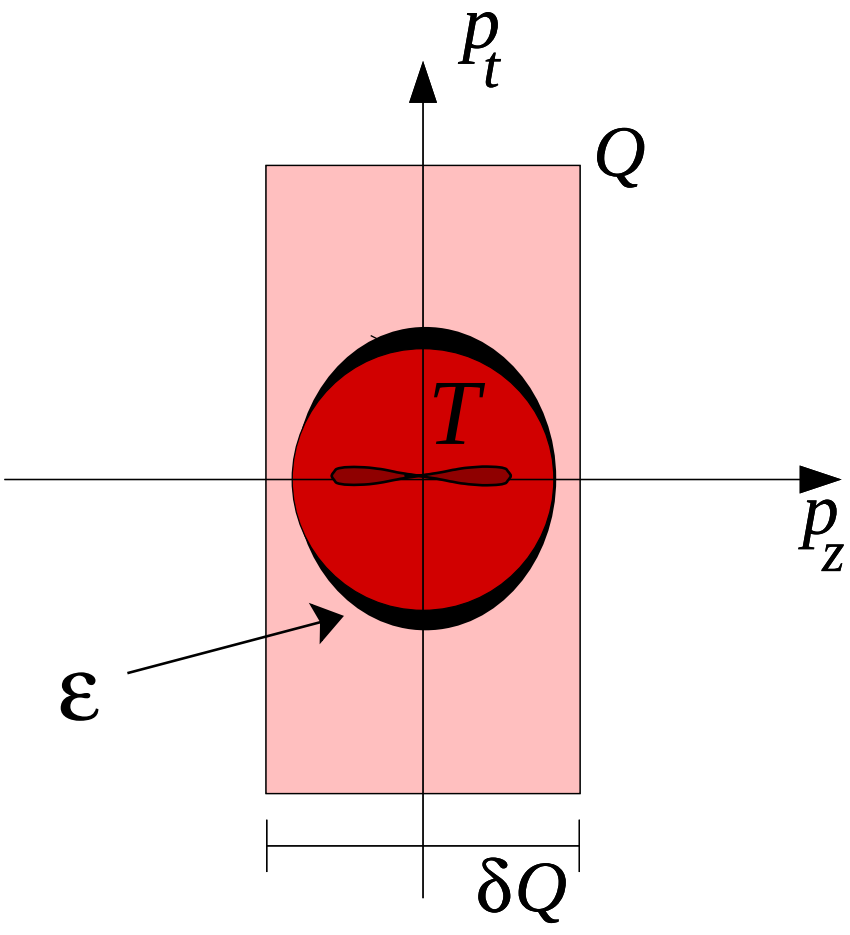
- Even the instabilities are not strong enough to thermalize when $f \gg 1$
  $\Rightarrow$ classical theory does not thermalize under longitudinal expansion!

The new bottom-up: Late stages: $\alpha^{-12/5} < Qt < \alpha^{-5/2}$:

- Splitting creates a soft T-bath

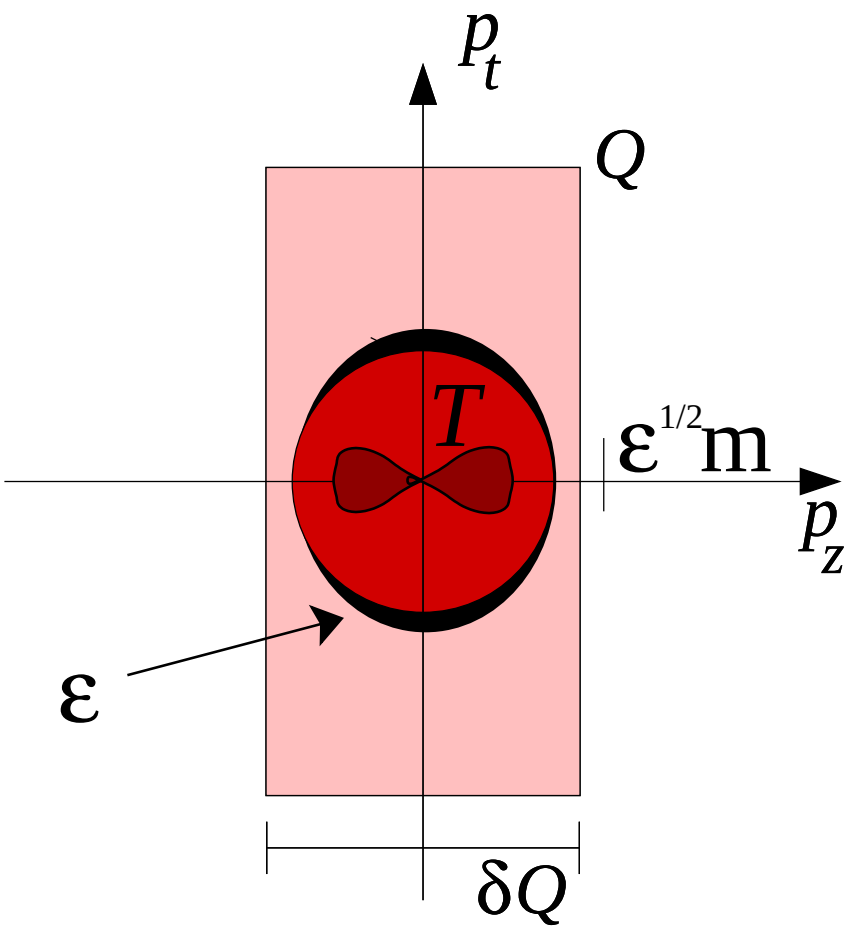


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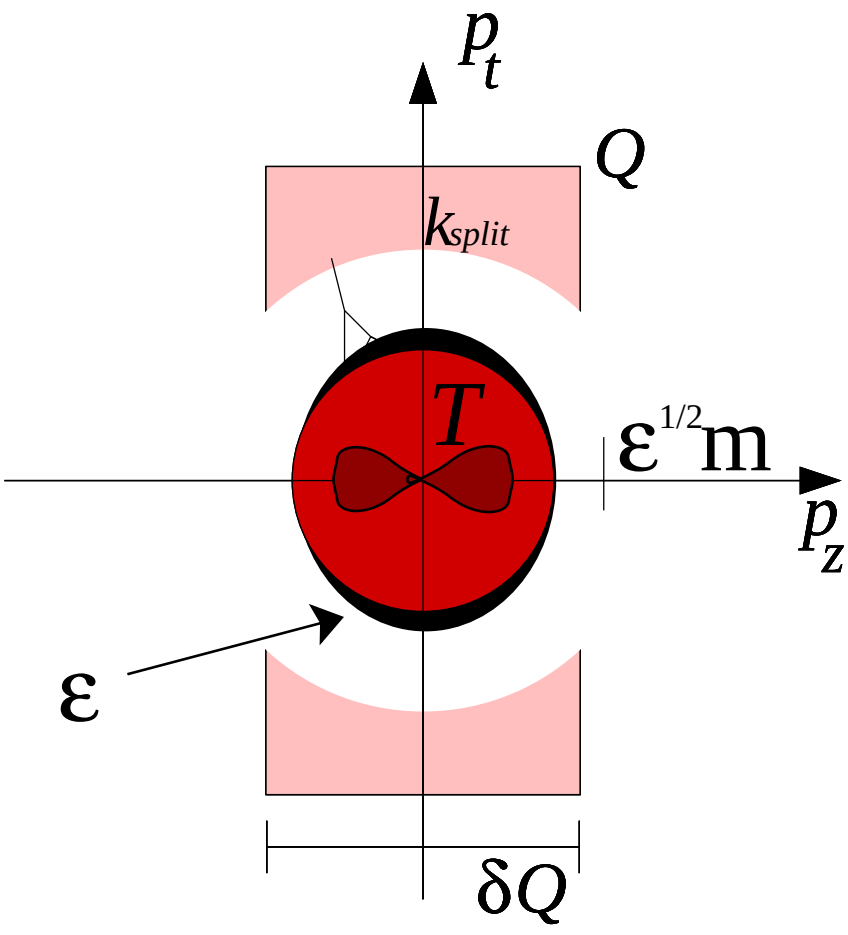
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- The expansion and anisotropic rain make T-bath anisotropic
  $\epsilon \sim t/t_{iso} \sim \hat{q}t/T^2$

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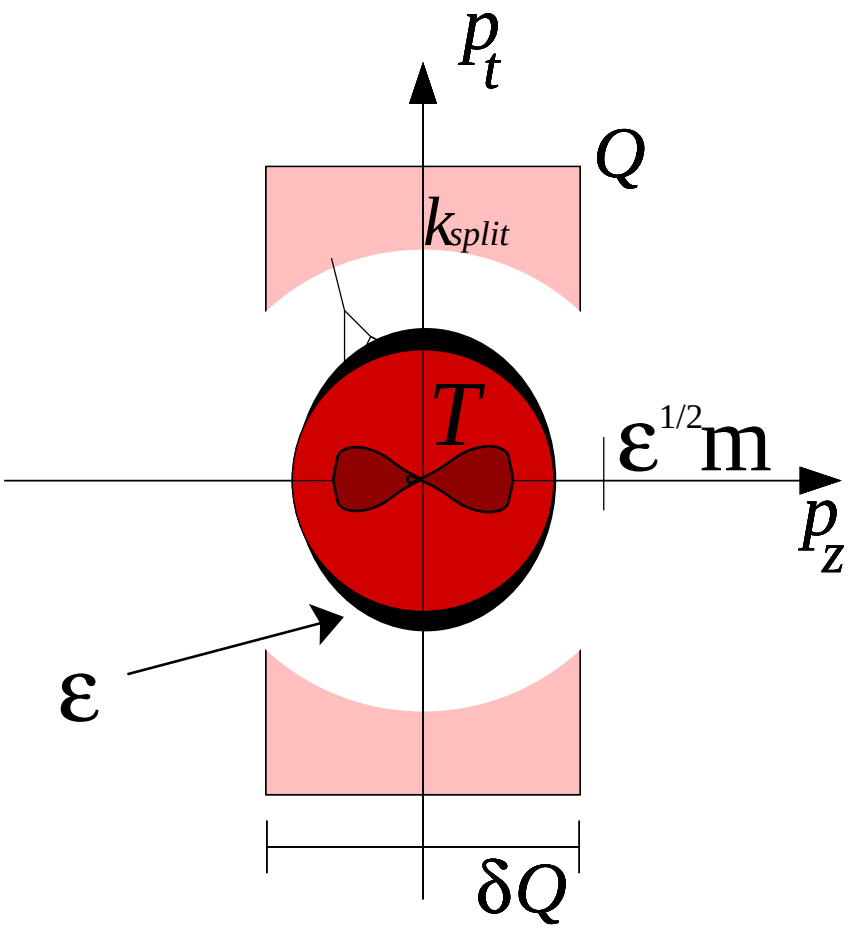
- Splitting creates a soft T-bath
- The expansion and anisotropic rain make T-bath anisotropic $\epsilon \sim t/t_{iso} \sim \hat{q}t/T^2$
- Anisotropic T-bath generates its own instabilities. Dominates $\hat{q}_\epsilon \sim \epsilon^{3/2} m^3$ for $(Qt) > \alpha^{-12/5}$

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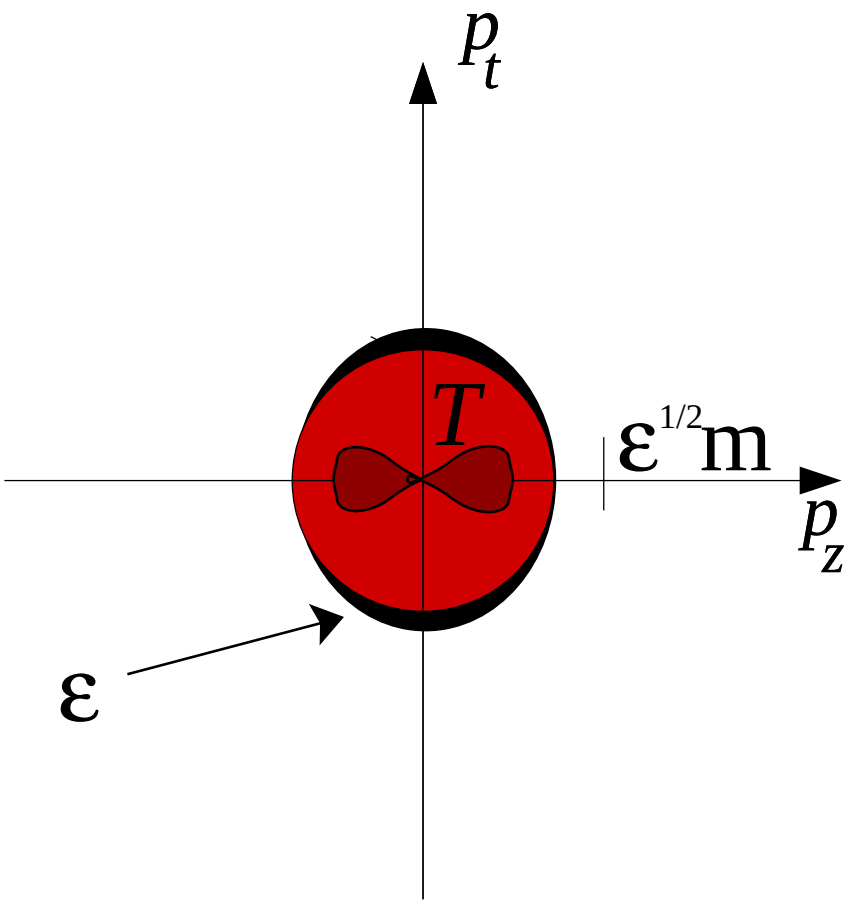
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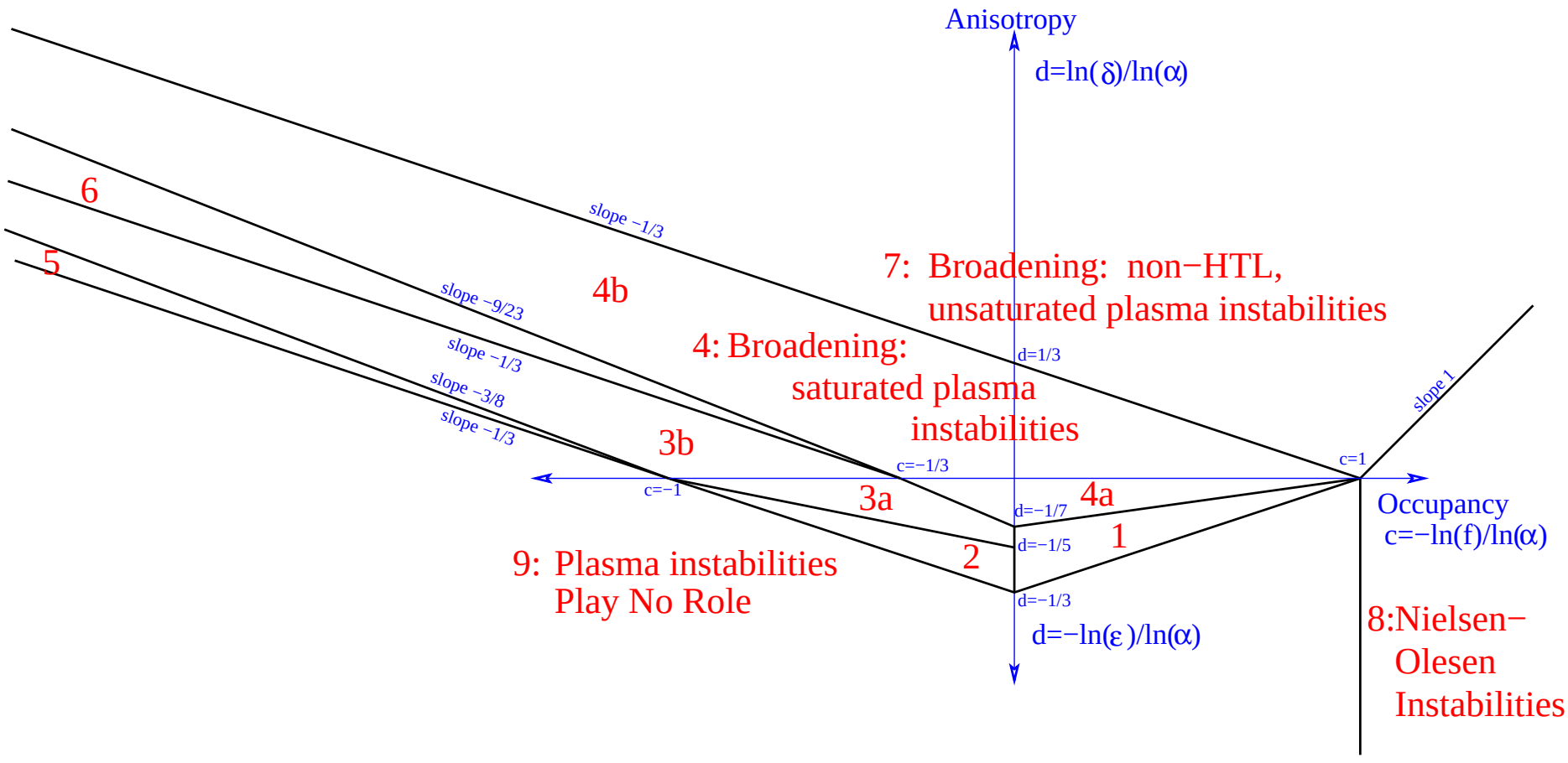
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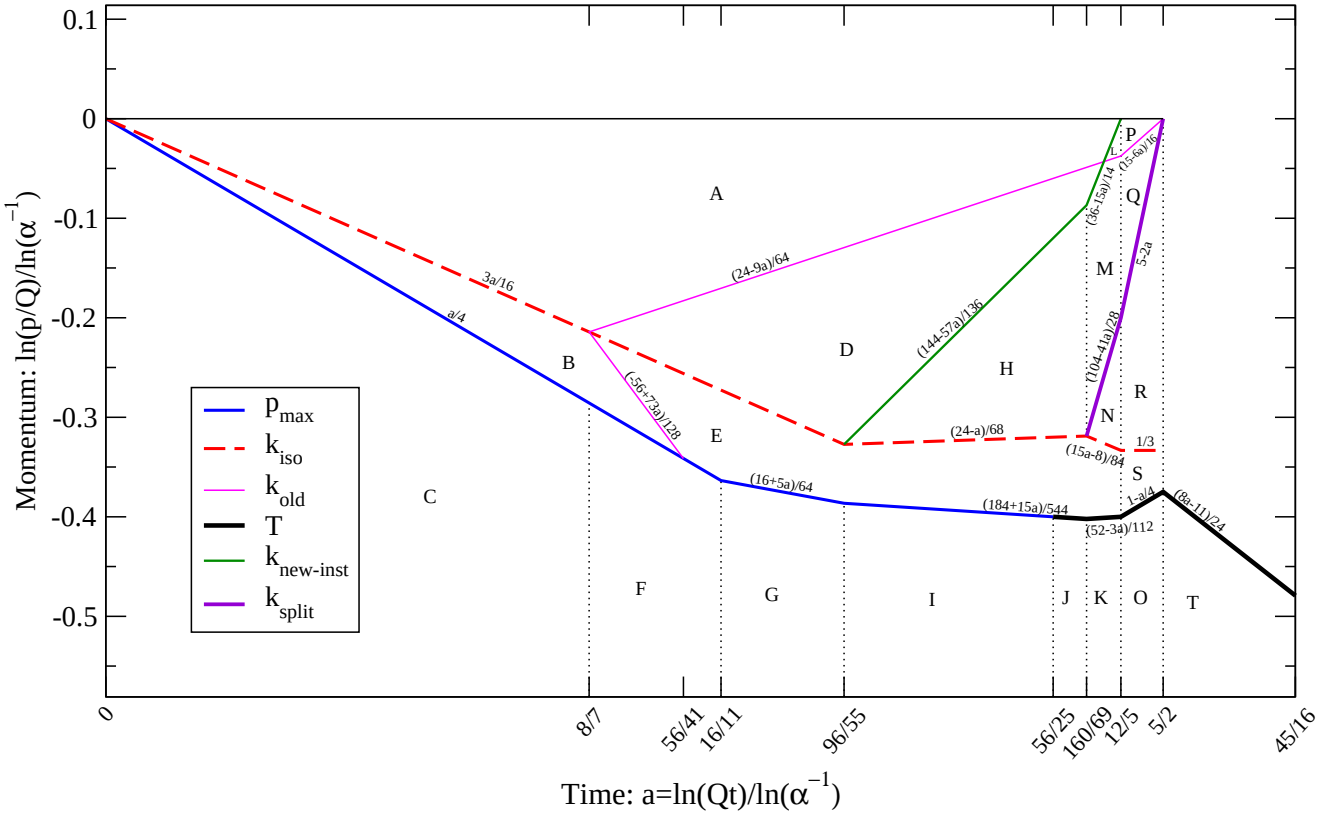
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- At $(Q_s t) \sim \alpha^{-5/2}$, $k_{\rm split} \sim Q_s$. Plasma instabilities continue to dominate until $(Q_s t) \sim \alpha^{-45/16}$.

Summary, complete catalog:



We identified the relevant physics in occupancy-anisotropy plane...

Summary, complete (parametric) description of HIC:



...applied to the case of longitudinal expansion.

Featuring:

- Obscure powers: $\alpha^{\frac{184+15a}{554}}$
- Redshifted relics
- Synchrotron absorption
- Complicated angle dependence
- . . . and many more

Conclusions:

- Thermalization occurs even for $\alpha \ll 1$

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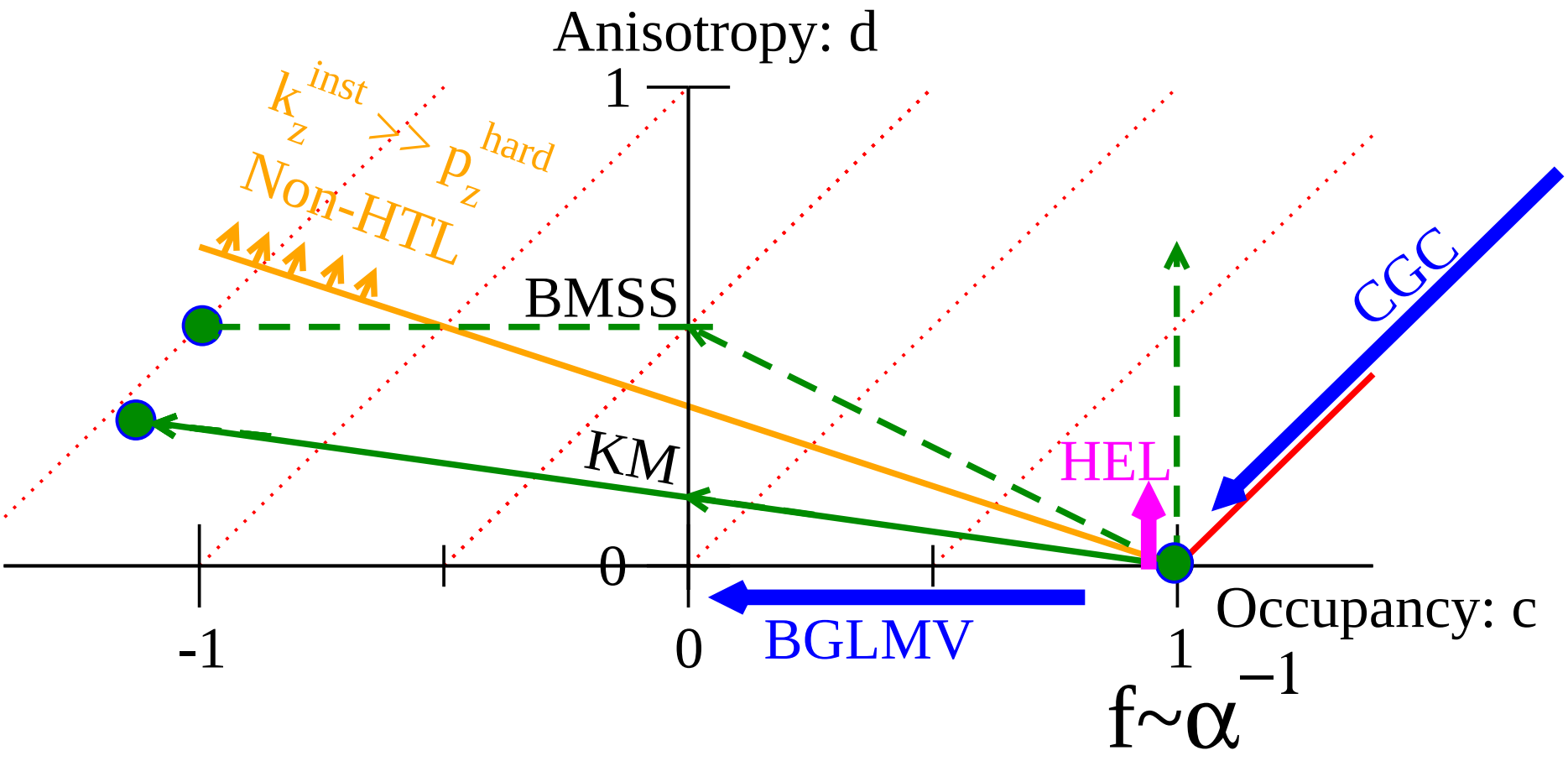
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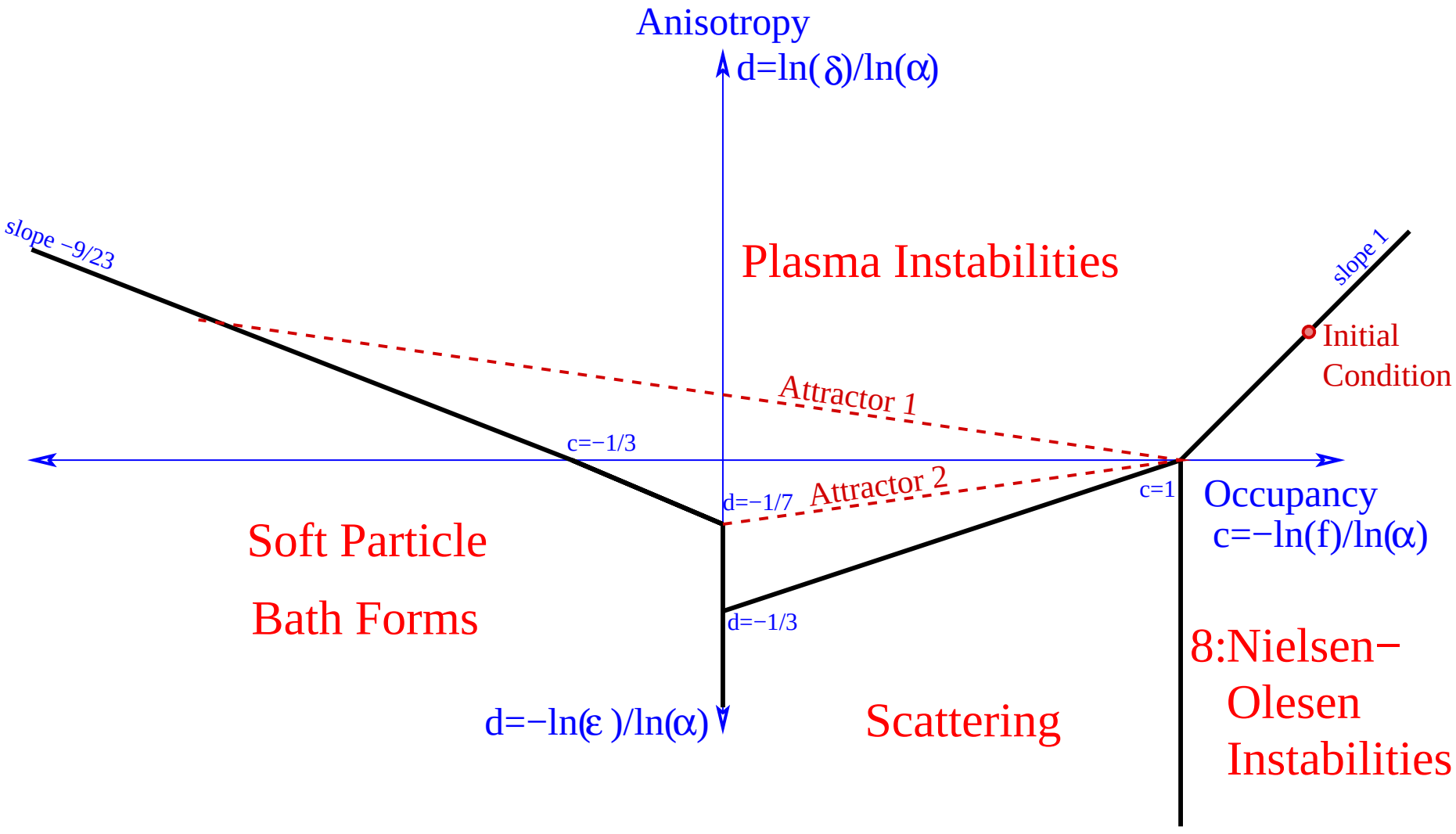
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- Apply same methods to
  - Cosmology, reheating
  - Neutron stars, anomalous viscosities

Comparison to other work



Second attractor?

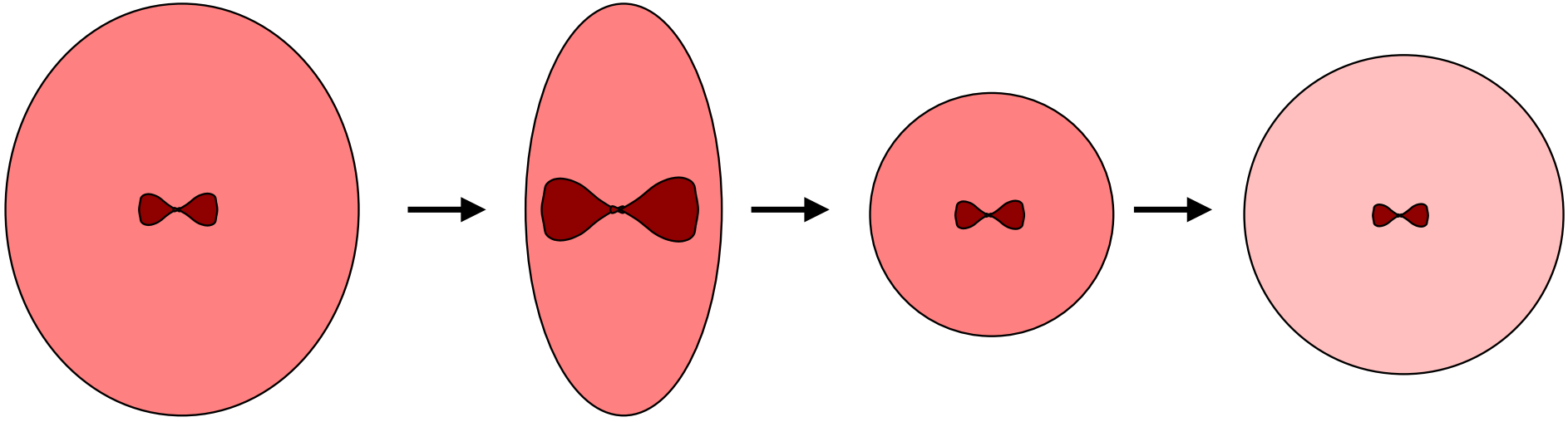


$$f(p) \sim \alpha^{-c}(1 + \alpha^{-d} F(\hat{p}))$$

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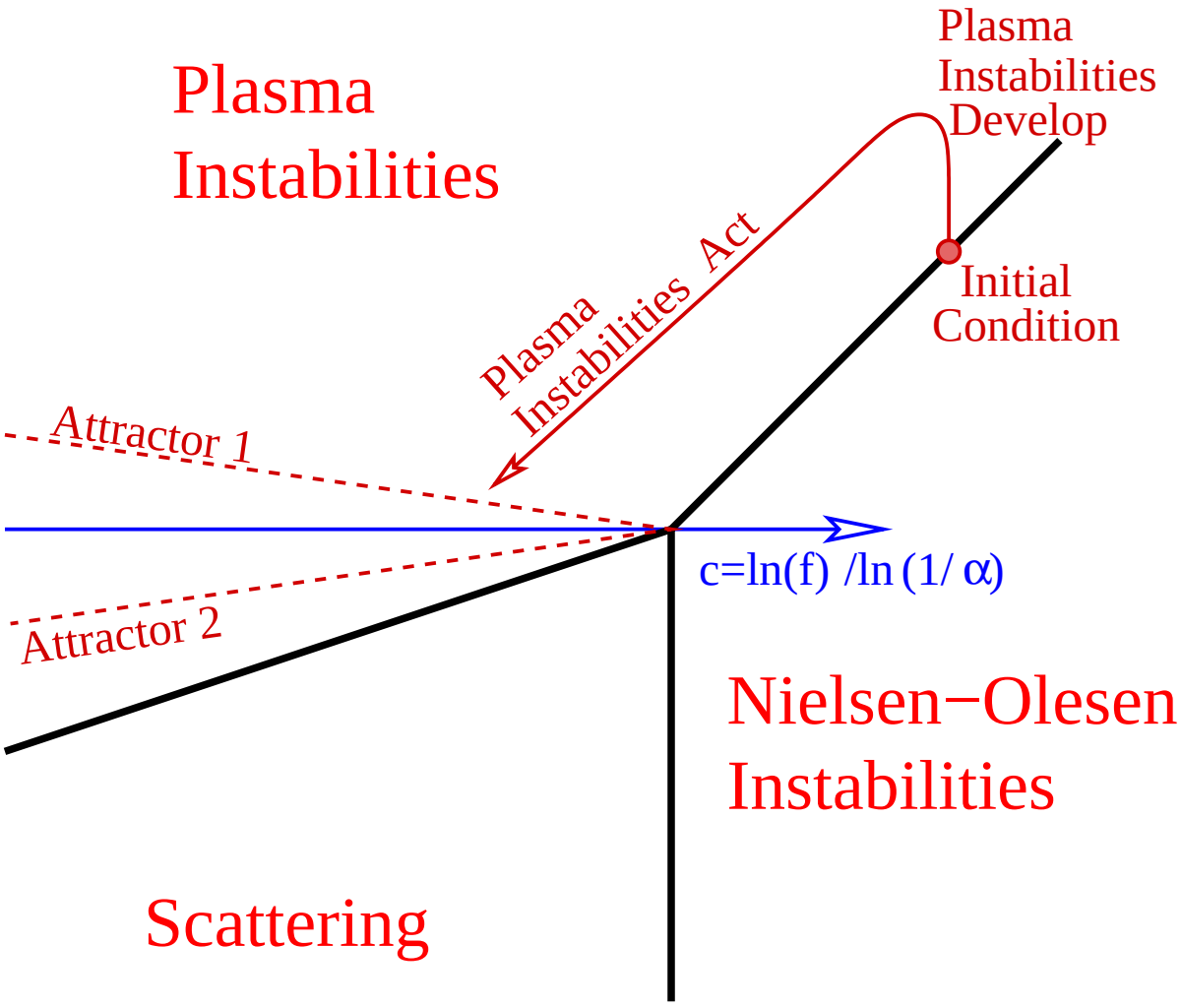
Assume nearly isotropic distribution:

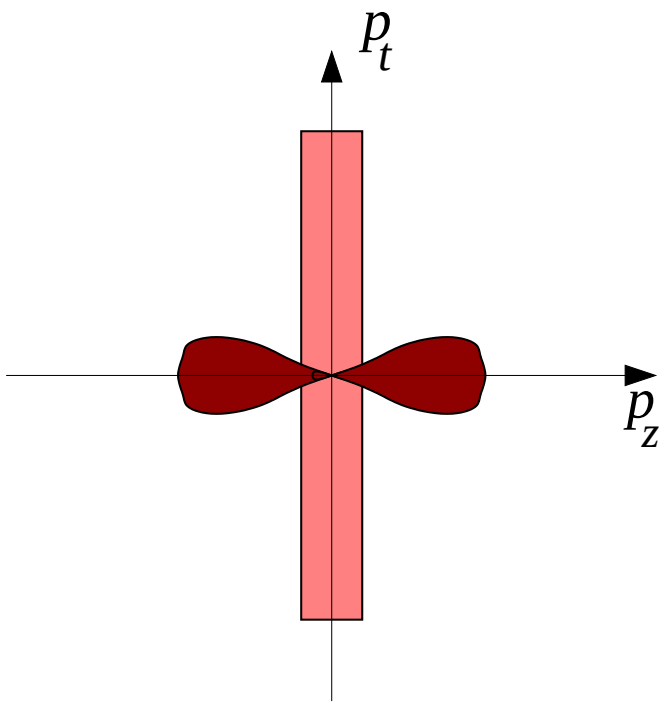
- Longitudinal expansion reduces energy: $\varepsilon \propto t^{-4/3}$
  - p_z reduces.
- Plasma-instabilities keep system nearly isotropic $\epsilon \sim t_{\rm iso}/t \sim \frac{Q^2}{\hat{q}_\epsilon t}$
  - Q reduces
- Instability induced particle joining at hard scale
  - Q increases



$$\epsilon \sim \alpha^{-d} \sim (Q_s t)^{-\frac{8}{135}}, \quad f(Q) \sim \alpha^{-1}(Q_s t)^{\frac{56}{135}}, Q \sim Q_s (Q_s t)^{-\frac{31}{135}}$$

Why do we think the first attractor is the relevant one?





- Initially, unstable modes in unpopulated part of phase space
- Growth from vacuum fluctuations, slowed by a log:
  $t_{growth} \sim Q_s^{-1} \ln^2(1/\alpha)$