Heavy quark kinetic and chemical equilibration

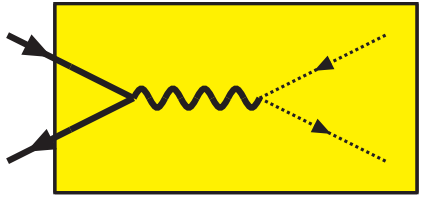
Mikko Laine

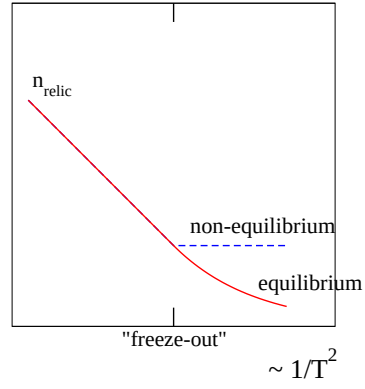
(University of Bern, Switzerland)

Origin of Weakly Interacting Massive Dark Matter

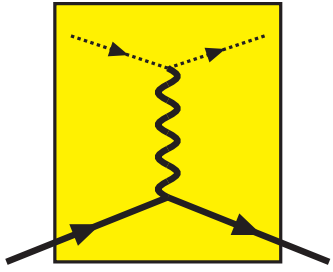
The system thermalizes after inflation, but then

(i) chemically decouples at $T \sim 10$ GeV.





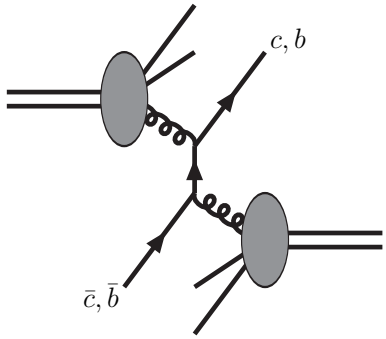
(ii) kinetically decouples at $T \sim 30$ MeV.



Velocity distribution freezes out as $\exp(-\frac{Mv^2}{2T_{\text{kin}}})$, fixing the distance scale for "large scale structure".

Could something similar happen with heavy quarks in QCD?

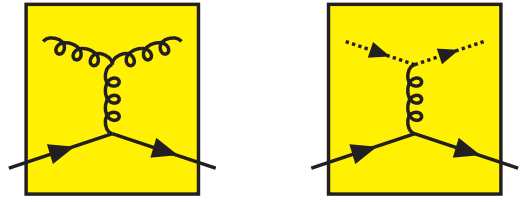
Initial state is out-of-equilibrium, with an overabundance of heavy quarks with hard momenta:



Could these chemically and kinetically equilibrate, losing memory of the initial state in the process, and then decouple again?

$\Rightarrow$ Compute equilibration rates and compare with expansion rate!

Kinetic equilibration in perturbation theory



$$\Gamma_{\rm kin} = \frac{g^2 C_{\rm F} m_{\rm D}^2}{12\pi M}\left(\ln\frac{2T}{m_{\rm D}} + \frac{1}{2} - \gamma_{\rm E} + \frac{\zeta'(2)}{\zeta(2)} + \frac{N_{\rm f}\ln 2}{2N_{\rm c}+N_{\rm f}}\right) \,.$$

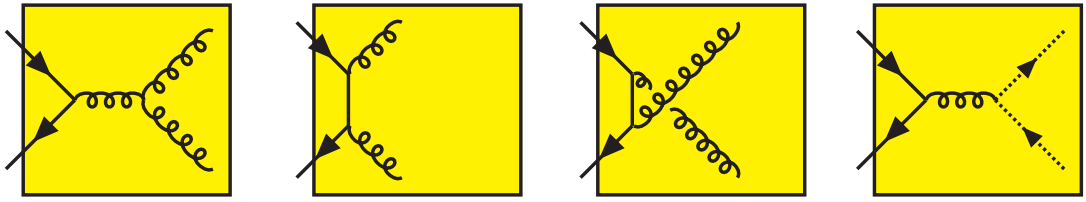
Svetitsky PRD 37(88)2484; Braaten Thoma PRD44(91)2625; Moore Teaney hep-ph/0412346.

The rate is a matching coefficient for effective classical dynamics:

$$\dot{p}_i = -\Gamma_{\rm kin}\, p_i + \xi_i \;, \quad \langle\!\langle \xi_i(t)\, \xi_j(t')\rangle\!\rangle = \kappa\, \delta_{ij}\, \delta(t-t') \;.$$

⇑	⇑	⇑
single quark	non-eq. ⇒ pheno	equilibrium ⇒ theory

Chemical equilibration in perturbation theory



$$\Gamma_{\text{chem}} = \frac{g^4 C_F}{8\pi M^2}\left(N_{\text{f}} + 2C_F - \frac{N_{\text{c}}}{2}\right)\left(\frac{TM}{2\pi}\right)^{\frac{3}{2}} e^{-M/T} \; .$$

Biró Zimányi PLB 113(82)6; Rafelski Müller PRL 48(82)1066; Matsui et al PRD 34(86)783.

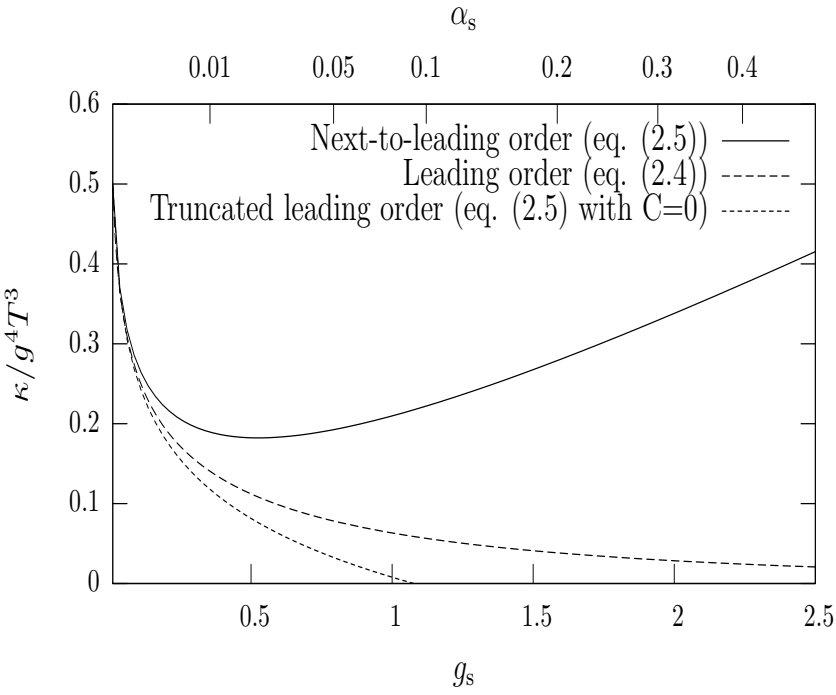
The rate is a matching coefficient for effective classical dynamics:

$$\delta\dot{n} = -\Gamma_{\text{chem}}\,\delta n + \xi \; , \quad \langle\langle \xi(t)\,\xi(t')\rangle\rangle \; = \; \Omega\;\delta(t-t') \; .$$

$\Uparrow$ N/V $N\to\infty$ — $\Uparrow$ non-eq. $\Rightarrow$ pheno — $\Uparrow$ equilibrium $\Rightarrow$ theory — $\Uparrow$ $\sim e^{-2M/T}$

There is the usual problem of slow convergence.

Kinetic equilibration at NLO: Caron-Huot Moore 0801.2173



Big effects in an interesting direction

$\Rightarrow$

a non-perturbative study is needed.

Kinetic equilibration non-perturbatively

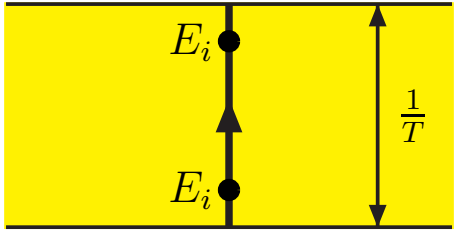
Casalderrey et al hep-ph/0605199;
Caron-Huot et al 0901.1195.

Match equilibrium domain of $\boxed{\dot{p}_i = -\Gamma_{\rm kin}\, p_i + \xi_i}$ to QCD.

Operator: $v_i \to \hat{J}_i \equiv \hat{\bar{\psi}}\gamma_i\hat{\psi}$; "force": $\dot{p}_i \to M\,\dfrac{\mathrm{d}\hat{J}_i}{\mathrm{d}t}$.

Normalization through: $\chi_f \equiv \displaystyle\int \mathrm{d}^3\mathbf{x}\,\langle \hat{J}_0(t,\mathbf{x})\,\hat{J}_0(0,\mathbf{0})\rangle$.

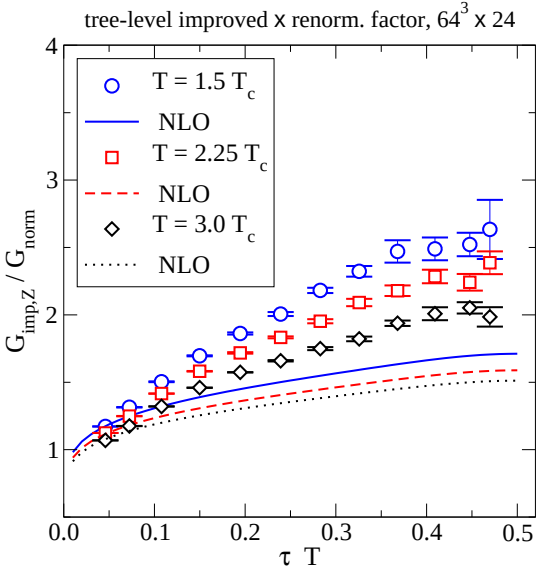
Taking subsequently the heavy-quark limit ($M \gg \pi T$) removes fermions and yields a purely gluonic 2-point correlator:



$$\frac{\Big\langle \mathrm{Re\,Tr}\Big[U_{\beta;\tau}\, gE_i(\tau)\, U_{\tau;0}\, gE_i(0)\Big]\Big\rangle}{\Big\langle \mathrm{Re\,Tr}[U_{\beta;0}]\Big\rangle}\,.$$

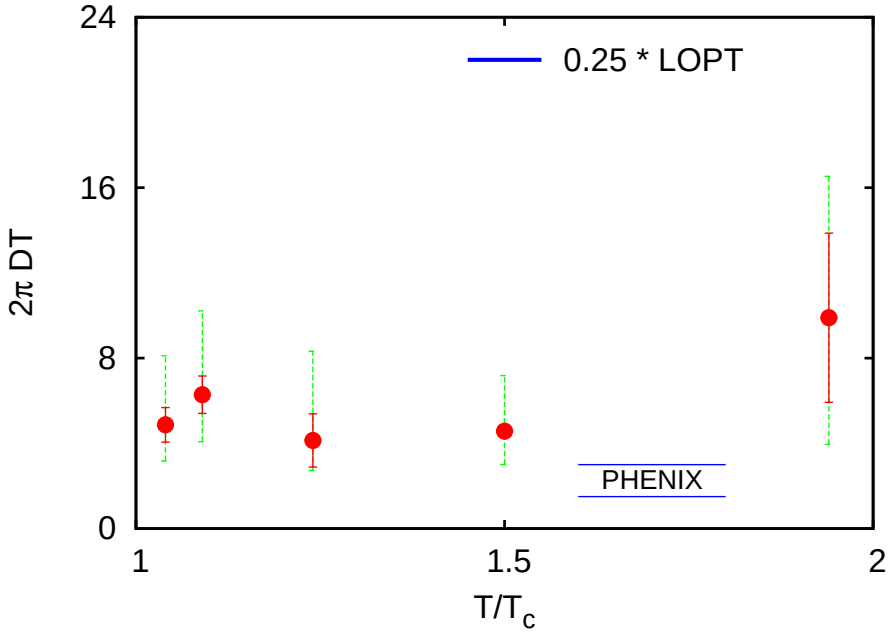
Practical measurements are under way.

With "multilevel algorithm" [Lüscher Weisz hep-lat/0108014; Meyer 0704.1801], "tree-level improvement" [Sommer hep-lat/9310022; Meyer 0904.1806], and perturbative renormalization [Eichten Hill PLB 240 (1990) 193], get results [Francis et al 1109.3941], to be compared with NLO [Burnier et al 1006.0867]:



A clear enhancement at large time separations!

Preliminary results for $D = T/(\Gamma_{\rm kin} M)$ can also be compared with phenomenology: [Banerjee et al 1109.5738]



However implementing systematic non-perturbative renormalization and analytic continuation requires further work.

Chemical equilibration non-perturbatively $\boxed{\delta\dot{n} = -\Gamma_{\rm chem}\delta n + \xi}$

Number density (particles + antiparticles) is not accessible in QCD, but the same physics is represented by energy density.

Operator: $\hat{H} \equiv \int_{\mathbf{x}} \hat{T}^{00}_{\rm f} = \int_{\mathbf{x}} \hat{\bar{\psi}}\Big(-\frac{i}{2}\gamma^j \overleftrightarrow{D}_j + M\Big)\hat{\psi}.$

"Force": $\partial_t \hat{H} = -g\int_{\mathbf{x}} \hat{\bar{\psi}}\,\gamma^j F_{0j}\hat{\psi} \;\Rightarrow\; \Omega_{\rm chem}\ .$

Normalization through: $\frac{1}{V}\langle(\delta\hat{H})^2\rangle \approx M^2\chi_f\ .$

Fluctuation-dissipation: $\Gamma_{\rm chem} = \frac{\Omega_{\rm chem}}{2M^2\chi_f}\ .$

Bödeker ML, *Heavy quark chemical equilibration rate as a transport coefficient,* 1205.4987.

Could this be measured in practice?

Force-force correlator in imaginary time:

$$\Omega(\tau) \;=\; -g^2 \int_{\mathbf{x}} \Big\langle \left[\bar{\psi}\gamma^j E_j \psi\right](\tau,\mathbf{x}) \left[\bar{\psi}\gamma^k E_k \psi\right](0,\mathbf{0}) \Big\rangle_{\mathrm{qc}} \;.$$

Good news: No constant mode expected; ...

Bad news: Strong short-distance singularity ($\sim 1/\tau^7$); ...

Other uses: Effective field theories or analogue models? (NRQCD, chiral Lagrangians, AdS/QCD, ...)

Conclusions

Kinetic equilibration rate:

Non-perturbative work under way; controlled comparisons with experiment may become possible in the bottom case.

Chemical equilibration rate:

Further theoretical work welcome; there may be links to cosmological problems.

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