Shear viscosity in a superfluid cold Fermi gas at unitarity

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M. Mannarelli, C.M., L. Tolos, arXiv:1201.4006

Outline

- The system under study
- Some basic facts about superfluid systems: U(1) SSB and the NGB
- Shear viscosity due to superfluid phonons

Fermions at unitarity

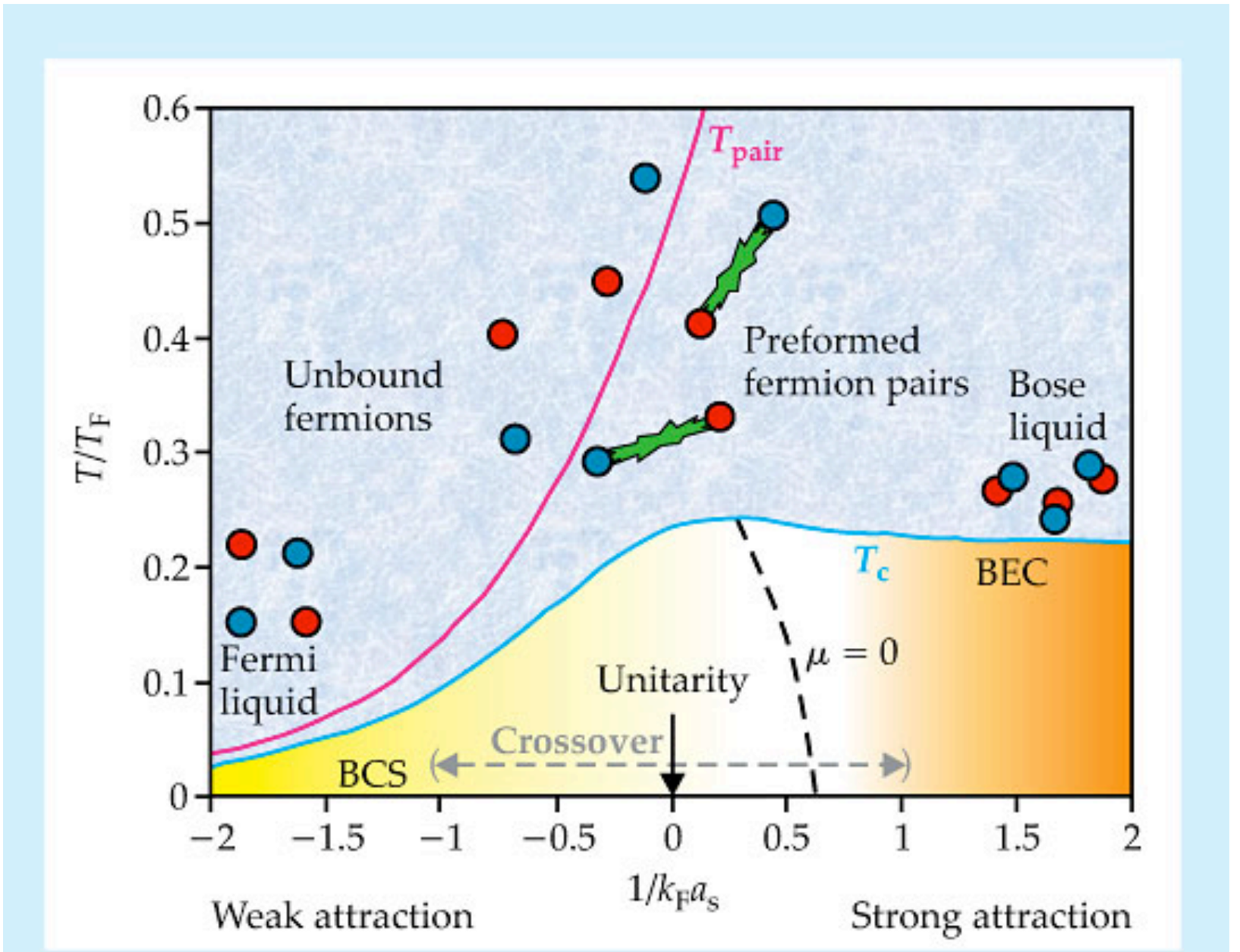
- Non-relativistic fermions at finite density, subject to two body (zero range) contact interactions

- s-wave amplitude saturates the unitarity limit

$$f_0(k) = \frac{1}{ik - \frac{1}{a_0} + \frac{1}{2} r_0 k^2}$$

$$|f_0|^2 \leq \frac{1}{k^2} \qquad r_0 k_F^2 \ll 1 \,, \qquad \frac{1}{k_F a} \to 0$$

At unitarity the system is scale invariant!

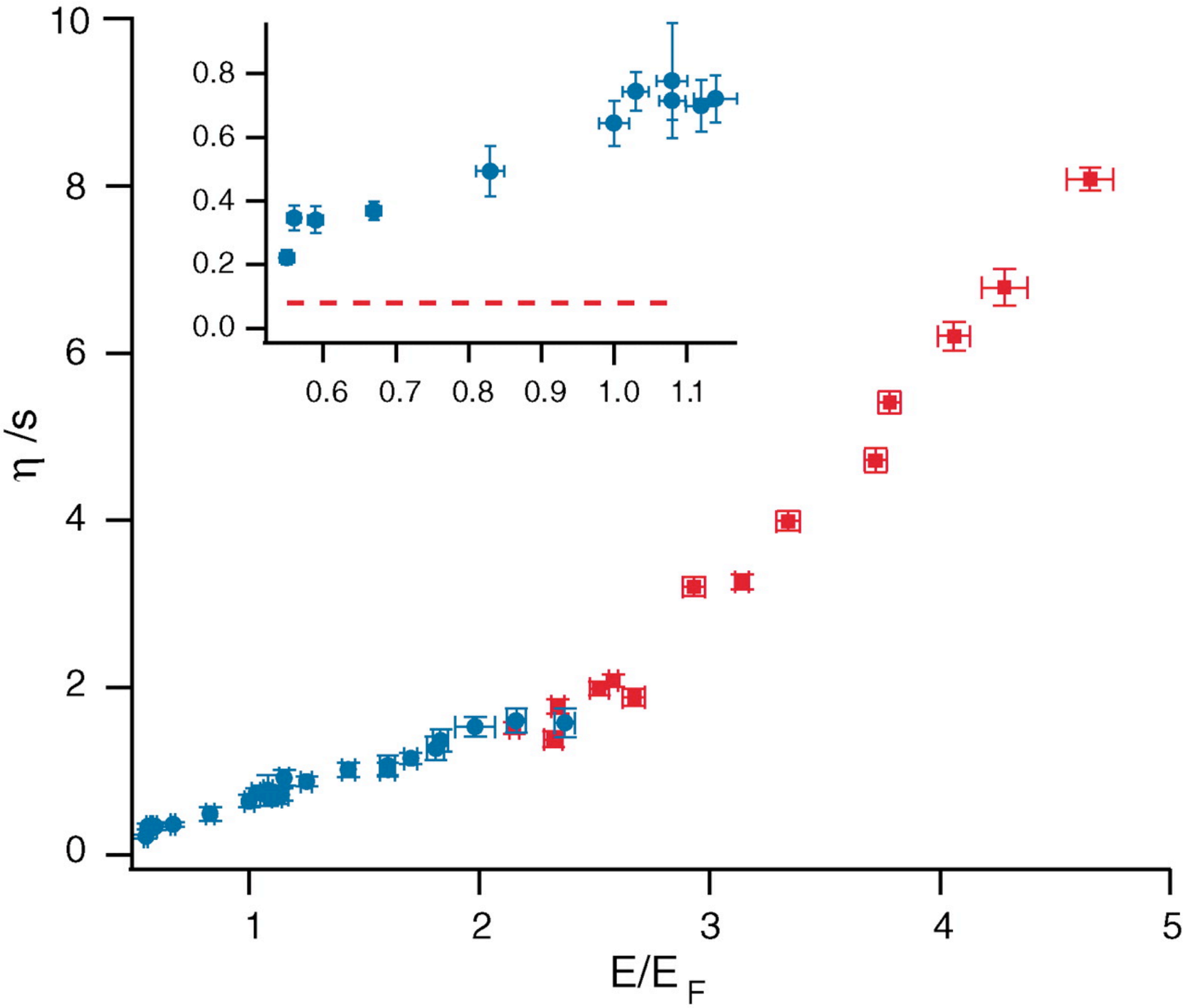


from: Sa de Melo, Phys.Today (Oct. 2008)

Experimental success

- Ultracold: Fermi T in the microKelvin range
- Feshbach resonance techniques allow to tune $k_F a$ at will, and reach the unitarity limit
- We are particularly interested in the measure of transport coefficients at unitarity

Fig. 3 Estimated ratio of the shear viscosity to the entropy density.



C Cao et al. Science 2011;331:58-61

Science
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Superfluidity

- Quantum phenomenon associated to the appearance of a quantum condensate = SSB of a global U(1)

$$\psi = |\psi| e^{-i\varphi} \qquad \langle\psi\psi\rangle = |\langle\psi\psi\rangle| e^{-2i\varphi}$$

$$\mathbf{j} = -\frac{i}{2m}\left(\psi^\dagger \nabla\psi - \nabla\psi^\dagger \psi\right)$$

$$\mathbf{j} = \rho_s \mathbf{v_s} \ , \qquad \mathbf{v_s} = \frac{\nabla\phi}{m}$$

- Hydrodynamics complicated: two-fluid model

superfluid component: no dissipation

normal component: dissipative processes are possible

$$\rho = \rho_n + \rho_s \qquad \mathbf{j} = \rho_n \mathbf{v_n} + \rho_s \mathbf{v_s}$$

Low energy EFT for the phonons

Son and Wingate 2006

$$\mathcal{L}_{\rm LO} = P(X) \qquad X = \mu_0 - V(\mathbf{r}) - \partial_t\varphi - \frac{(\nabla\varphi)^2}{2m}$$

valid for all superfluids, with the same global symmetries

$$P = \frac{2^{5/2}}{15\pi^2\xi^{3/2}} m^{3/2}\mu_0^{5/2}$$ **EoS fermi gas at unitarity**

$$\mu_0 = \xi E_F \ , \qquad \xi \sim 0.4$$

$$
\begin{aligned}
\mathcal{L}_{\rm LO} &= \frac{1}{2}\left((\partial_t\phi)^2 - v_{\rm ph}^2(\nabla\phi)^2\right) - g\left((\partial_t\phi)^3 - 3\eta_g\,\partial_t\phi(\nabla\phi)^2\right) \\
&+ \lambda\left((\partial_t\phi)^4 - \eta_{\lambda,1}(\partial_t\phi)^2(\nabla\phi)^2 + \eta_{\lambda,2}(\nabla\phi)^4\right)
\end{aligned}
$$

$$
g = \frac{1}{6\sqrt{m\rho}\;c_s}\left(1 - 2\frac{\rho}{c_s}\frac{\partial c_s}{\partial\rho}\right)\;, \qquad \eta_g = \frac{c_s}{6\sqrt{m\rho}\;g}
$$

$$
\begin{aligned}
\lambda &= \frac{1}{24\;m\rho\;c_s^2}\left(1 - 8\frac{\rho}{c_s}\frac{\partial c_s}{\partial\rho} + 10\frac{\rho^2}{c_s^2}\left(\frac{\partial c_s}{\partial\rho}\right)^2 - 2\frac{\rho^2}{c_s}\frac{\partial^2 c_s}{\partial\rho^2}\right)\;, \\
\eta_{\lambda_2} &= \frac{c_s^2}{8\;m\rho\;\lambda}\;, \qquad \eta_{\lambda,1} = 2\frac{\eta_{\lambda,2}}{\eta_g}
\end{aligned}
$$

Escobedo and CM, 2010

Low energy EFT for the phonons

$$\mathcal{L}_{\rm NLO} = c_1 m^{1/2} \frac{(\nabla X)^2}{\sqrt{X}} + \frac{c_2}{\sqrt{m}} (\nabla^2 \varphi)^2 \sqrt{X}$$

$$E_k = c_s k (1 + \gamma k^2) \qquad \gamma = -\left(c_1 + \frac{3}{2} c_2\right) \frac{\pi^2 \sqrt{2\xi}}{k_F^2} \simeq \frac{0.18}{k_F^2}$$

Rupak and Schafer, 2009

and at higher orders

$$E_k = c_s k (1 + \psi(k))$$

$$\psi(k) = \gamma\, k^2 + \delta\, k^4 + \mathcal{O}\left(\frac{k^6}{k_F^6}\right)$$

Shear viscosity due to phonons

$\varphi\varphi \leftrightarrow \varphi\varphi$ **large angle collisions** $\eta \propto \frac{1}{T^5}$

Rupak & Schafer 2007

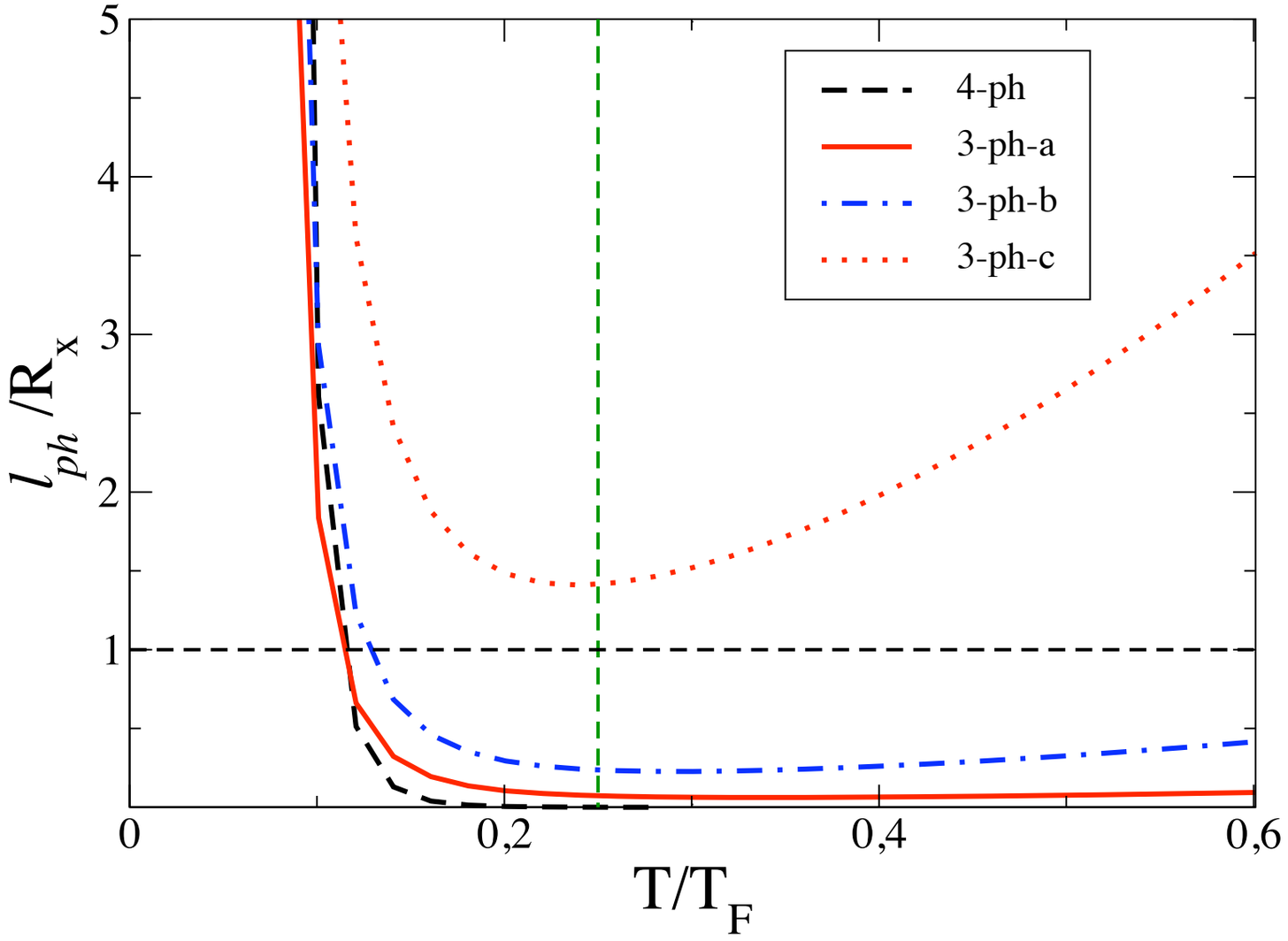
$\varphi \leftrightarrow \varphi\varphi$ **small angle collisions, only if E curves upward**

$E_p = c_s p$ $\eta \sim \infty$

$E_p = c_s p(1+\gamma^2)$ $\eta \propto \frac{1}{T^5}$

$E_p = c_s p(1+\gamma p^2 - \delta p^4)$ $\eta \propto \frac{1}{T}$

Knudsen number



$$\psi_{\max} = -\frac{\gamma^2}{4\delta}$$

$$\psi_{\max} = 0.2, 0.3, 0.4$$

Hydro needs Kn << 1

For Kn > 1 phonons collide more often with the boundary

Data for the phonons of He4

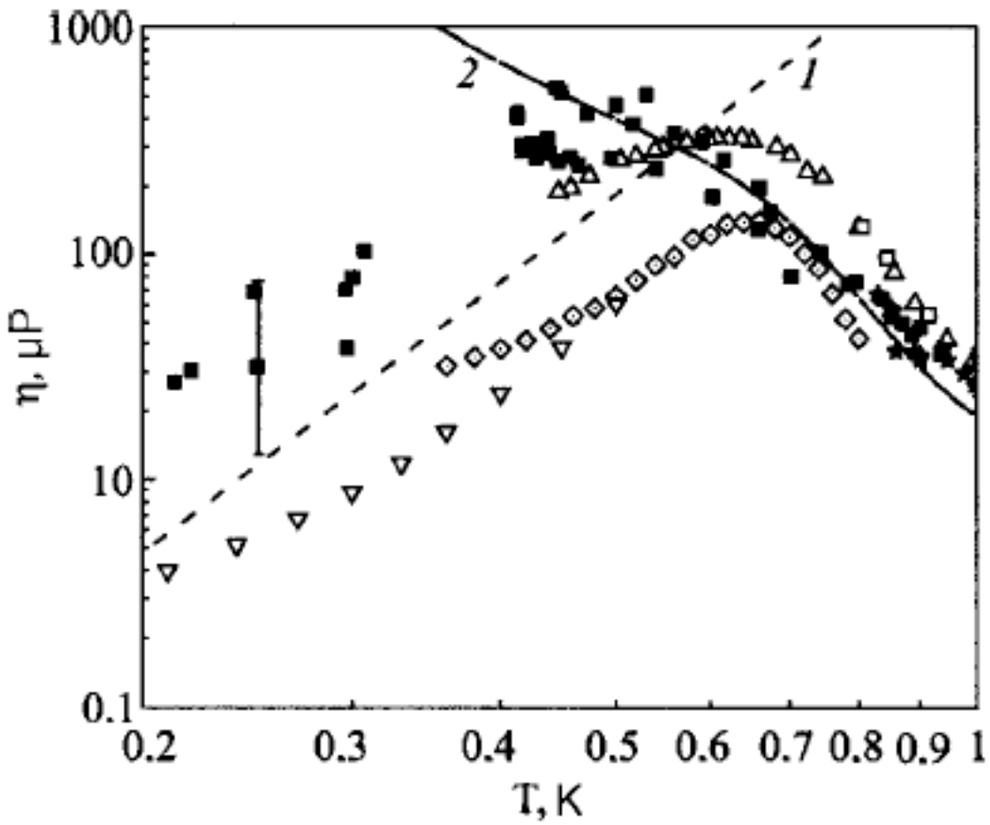


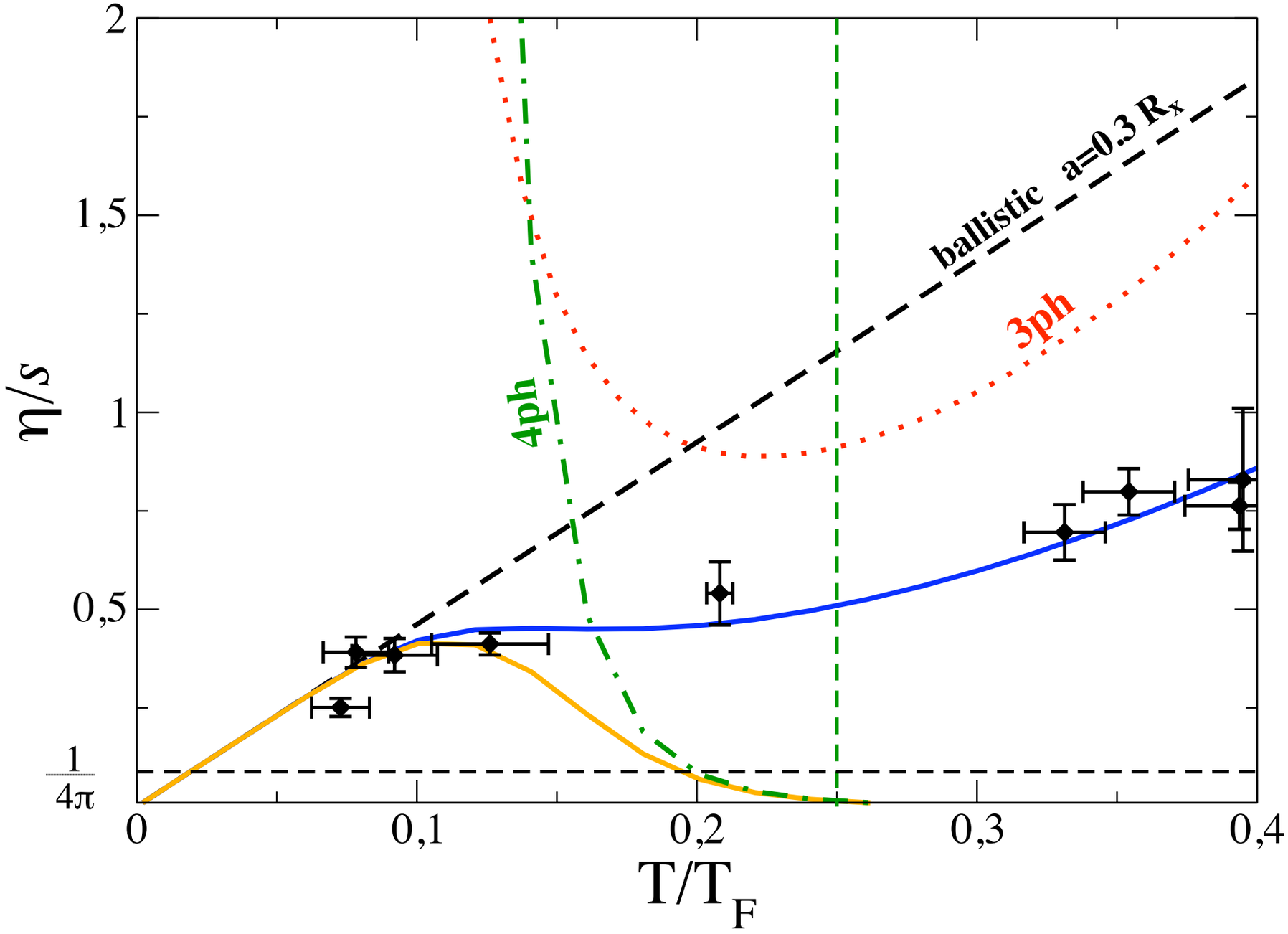
FIG. 3. Temperature dependence of the coefficient of effective viscosity near the transition from the hydrodynamic to the ballistic regime: this work (■); Ref. 11 (△); Ref. 10 (◇), (▽); Ref. 15 (☆). Line 1—calculation using Eq. (22); 2—temperature dependence of the coefficient of viscosity of the normal component.

$$\eta_{\rm bulk} = \frac{1}{5}\rho_{\rm ph} c_s l_{\rm ph}$$

$$\eta_{\rm ball} = \frac{1}{5}\rho_{\rm ph} c_s a$$

$$\eta_{\rm eff} = \left(\eta_{\rm bulk}^{-1} + \eta_{\rm ball}^{-1}\right)^{-1}$$

A. Zadorozhko et al , Low Temperature Physics 35, 100 (2009)



$\psi_{\max} = 0.3$

Conclusions

- Phonons can explain the low T values of the shear viscosity for the unitarity Fermi gas.
- We used EFT to determine the phonon interactions; an anomalous dispersion law and finite size effects are needed to explain the data