Non-perturbative investigation of conformality in SU(2) gauge theory with two adjoint fermions

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From QCD to Walking

The running of the coupling in QCD is determined by the β-function

$$\beta(\mu) = \mu \frac{dg}{d\mu} = -b_0 g^3 - b_1 g^5 + \dots \ ,$$

with

$$b_0 = \frac{1}{(4\pi)^2}\left(\frac{11}{3}N - \frac{4}{3}T_R N_f\right)$$

$$b_1 = \frac{1}{(4\pi)^4}\left(\frac{34}{3}N^2 - \frac{20}{3}NT_R N_f - 4\frac{N^2-1}{d_R}T_R^2 N_f\right)$$

For conformal field theories the coupling is constant and the β-function is zero

At points for which $\beta(g) = 0$ the coupling is constant (infrared fixed point)

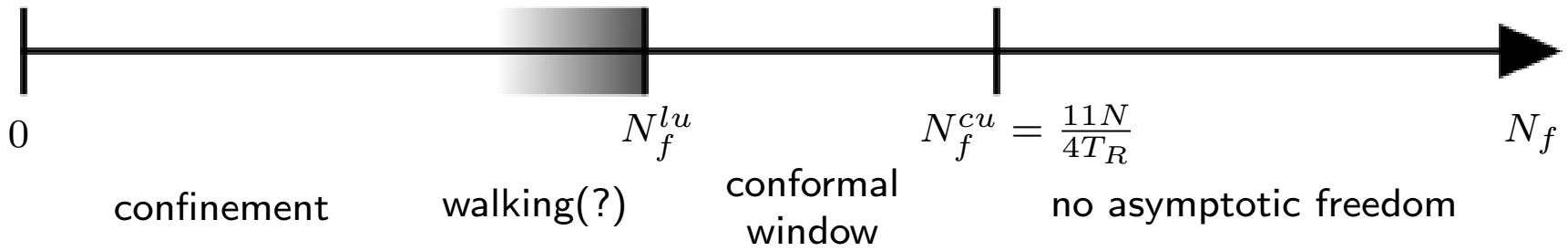
Near zeros of the β-function the coupling walks

Walking and Conformal window

$$\beta(\mu) = \mu \frac{dg}{d\mu} = -b_0 g^3 - b_1 g^5 + \dots$$

$$b_0 = \frac{1}{(4\pi)^2}\left(\frac{11}{3}N - \frac{4}{3}T_R N_f\right)$$

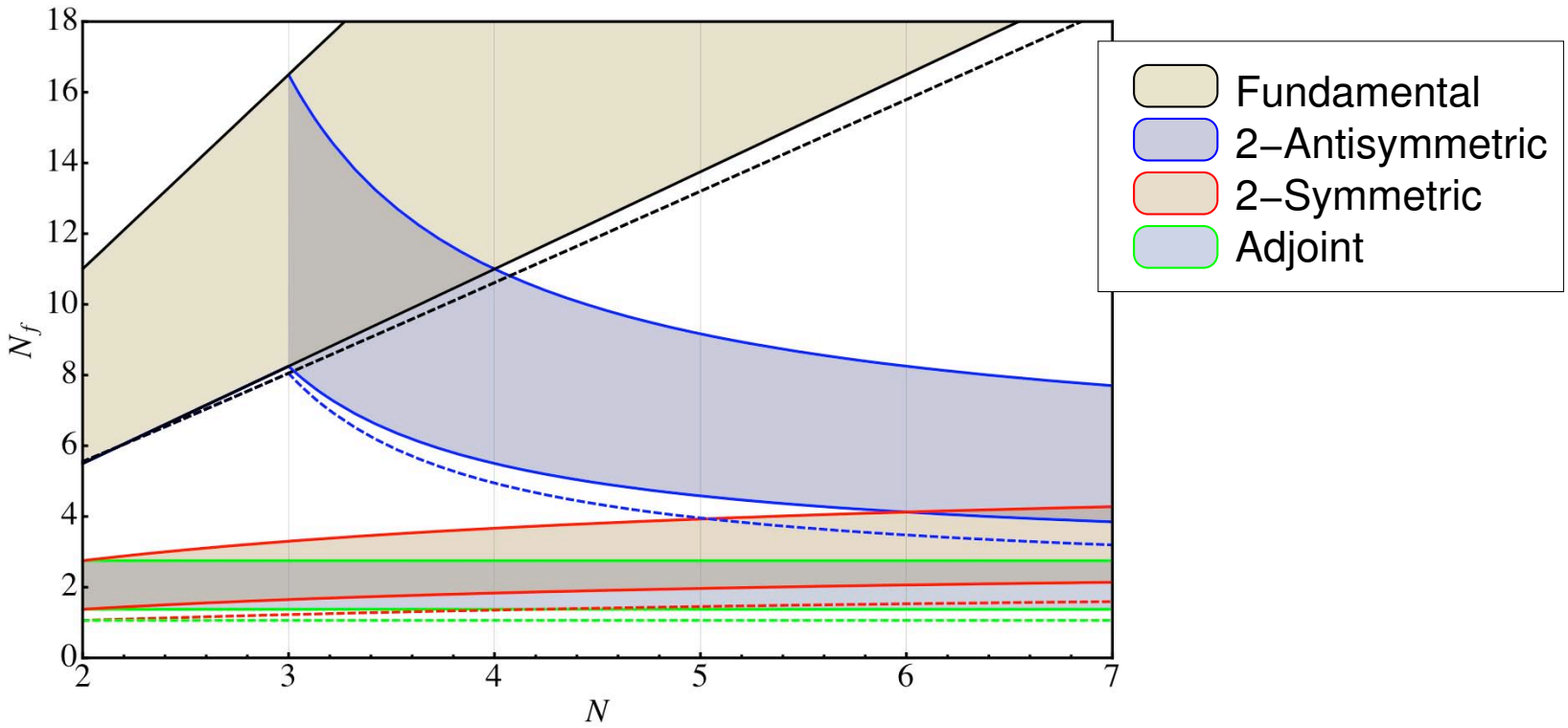
$$b_1 = \frac{1}{(4\pi)^4}\left(\frac{34}{3}N^2 - \frac{20}{3}NT_R N_f - 4\frac{N^2-1}{d_R}T_R^2 N_f\right)$$



Banks-Zaks (perturbative) fixed point:

$$g_*^2 \simeq -\frac{b_0}{b_1} << 1$$

Higher Representations Fermions



Dietrich and Sannino: Phys. Rev. D **75** (2007) 085018 [arXiv:hep-ph/0611341].

Fermions in higher representations have a lower contribution to the S-parameter, the minimal being given by SU(2) with two flavours of adjoint (symmetric) fermions

(Minimal Walking Technicolor)

Deforming the IR-conformal theory with a small mass

$$C(t,g,m,\mu) = \int d^3x \ \langle \Phi_R(t,\mathbf{x})\Phi_R(0)\rangle(g,m,\mu)$$

Weinberg-Callan-Symanzik equation.

$$\left\{ t\frac{\partial}{\partial t} + \beta(g)\frac{\partial}{\partial g} - [1+\gamma(g)]\, m\frac{\partial}{\partial m} + 2\left[d_\Phi - \gamma_\Phi(g)\right] \right\} C(t,g,m,\mu) = 0$$

$$\mu\frac{dg}{d\mu} = \beta(g)$$

$$\frac{\mu}{m}\frac{dm}{d\mu} = -\gamma(g)$$

Deforming the IR-conformal theory with a small mass

$$C(t,g,m,\mu) = \int d^3x \, \langle \Phi_R(t,\mathbf{x}) \Phi_R(0) \rangle (g,m,\mu)$$

Weinberg-Callan-Symanzik equation. Close to the fixed point...

$$\left\{ t\frac{\partial}{\partial t} + \beta(g)\frac{\partial}{\partial g} - [1+\gamma(g)]\, m\frac{\partial}{\partial m} + 2\left[d_\Phi - \gamma_\Phi(g)\right] \right\} C(t,g,m,\mu) = 0 + \text{corrections}$$

$$\mu\frac{dg}{d\mu} = \beta(g)$$

$$\frac{\mu}{m}\frac{dm}{d\mu} = -\gamma(g)$$

Deforming the IR-conformal theory with a small mass

$$C(t,g,m,\mu) = \int d^3x \ \langle \Phi_R(t,\mathbf{x})\Phi_R(0)\rangle(g,m,\mu)$$

Weinberg-Callan-Symanzik equation.

$$\left\{ t\frac{\partial}{\partial t} - [1+\gamma]\, m\frac{\partial}{\partial m} + 2\left[d_\Phi - \gamma_\Phi\right] \right\} C(t,g,m,\mu) = 0$$

Solution of the Weinberg-Callan-Symanzik equation.

$$C(t,g,m,\mu) \simeq b^{2(d_\Phi-\gamma_\Phi)} C(bt, g_*, b^{-(1+\gamma)}m, \mu) =$$

$$\simeq \mu^{2d_\Phi} \left(\frac{m}{\mu}\right)^{2\frac{d_\Phi-\gamma_\Phi}{1+\gamma}} F\left(tm^{\frac{1}{1+\gamma}}, \mu\right)$$

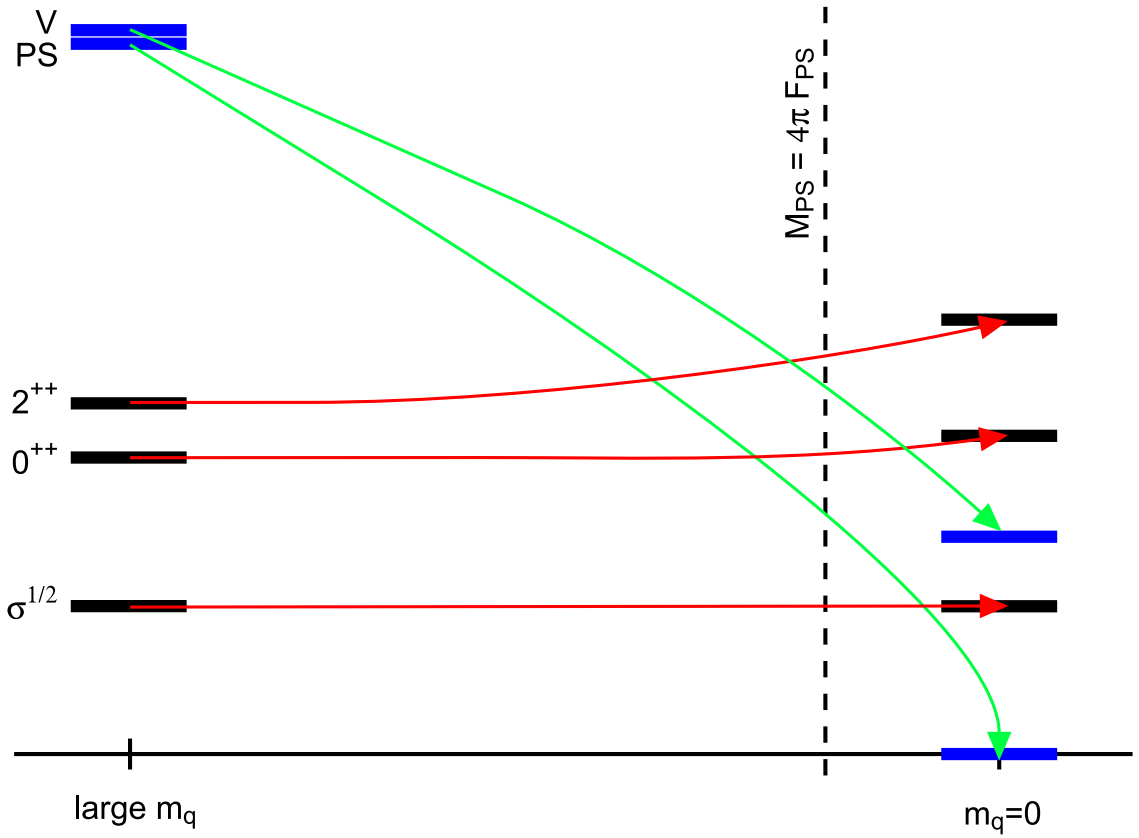
The mass term breaks the asymptotic scale invariance. A mass gap is expected to be generated.

$$C(t,g,m,\mu) \simeq A\exp\left(-M_\Phi t\right)$$

$$M_\Phi = a_\Phi \mu \left(\frac{m}{\mu}\right)^{\frac{1}{1+\gamma}} \qquad m \to 0$$

Some cartoons

Confinement and χSB



large fermion mass
(pure YM + NR fermions)

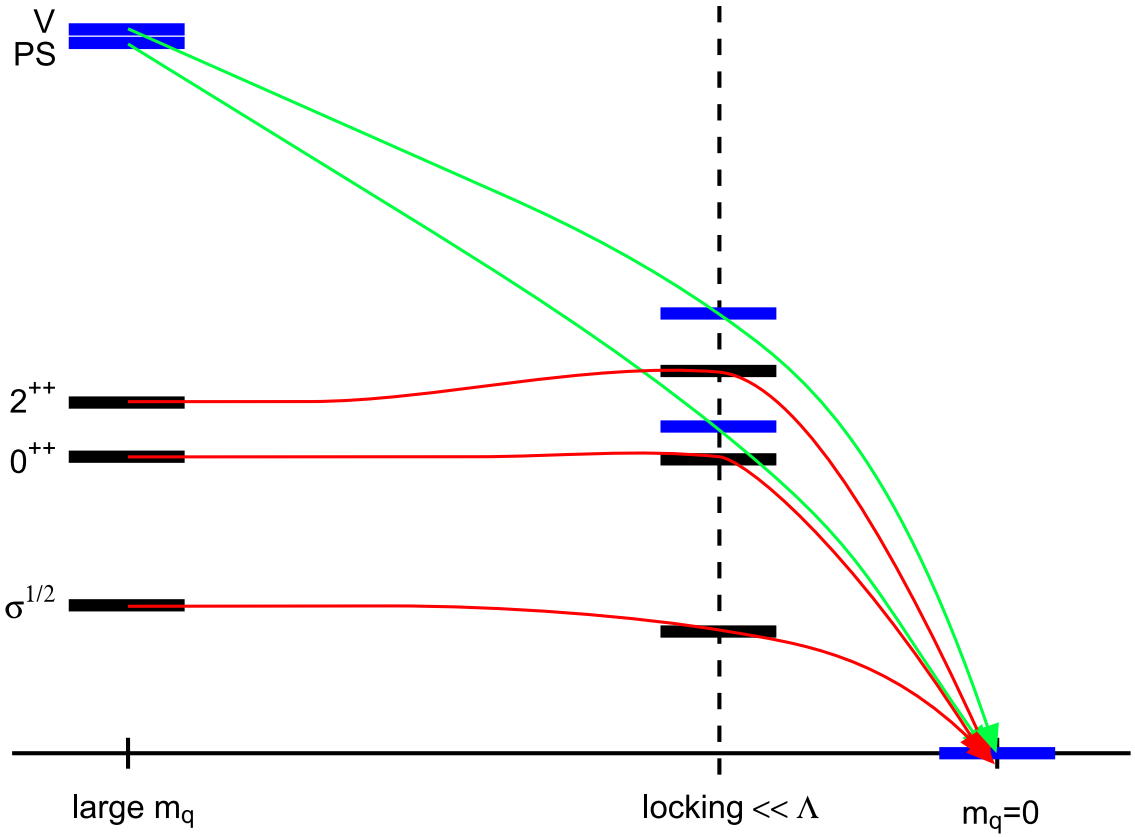
$$M_{PS} \simeq M_V \simeq 2m_q$$
$$M_V/M_{PS} \simeq 1$$
$$M_G \simeq M_G^{(YM)}$$

chiral limit (χPT)

$$M_{PS}^2 \simeq -\frac{\langle\bar{\Psi}\Psi\rangle}{F_{PS}^2} m_q$$
$$M_V \simeq M_V^{(0)}$$
$$M_G \simeq M_G^{(0)}$$

Some cartoons

IR conformality



large fermion mass
(pure YM + NR fermions)

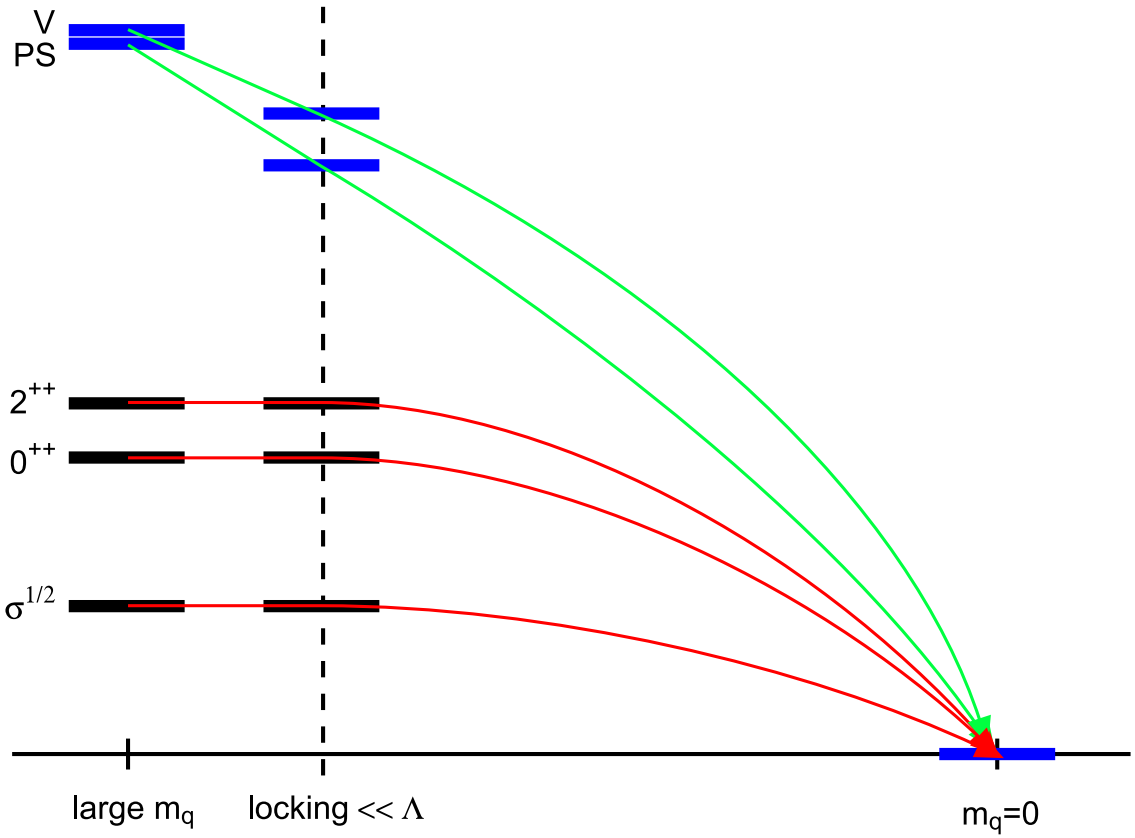
$$M_{PS} \simeq M_V \simeq 2m_q$$
$$M_V/M_{PS} \simeq 1$$
$$M_G \simeq M_G^{(YM)}$$
$$\sigma \simeq \sigma^{(YM)}$$

chiral limit (scaling region)

$$M_X \simeq a_X \mu^{1-\alpha} m_q^{\alpha}$$
$$\sigma^{1/2} \simeq a_\sigma \mu^{1-\alpha} m_q^{\alpha}$$
$$M_X/M_Y \simeq a_X/a_Y$$
$$M_X/\sigma^{1/2} \simeq a_X/a_\sigma$$

Some cartoons

IR conformality



at all scales

$$M_V/M_{PS} \simeq 1+\epsilon$$

$$M_{PS} \gg \sigma^{1/2}$$

$$M_G/\sigma^{1/2} \simeq \left[M_G/\sigma^{1/2}\right]^{(YM)}$$

Miransky's scenario for Banks-Zaks fixed point

V. A. Miransky. Dynamics in the Conformal Window in QCD like theories. hep-ph/9812350.

- Banks-Zaks fixed point $g_*^2 = -\frac{b_0}{b_1} \ll 1$.
- The fermionic mass destroys the IR fixed point. Define the fermionic pole mass M_q.

$$M_q = a_q \mu \left(\frac{m_q}{\mu}\right)^{\frac{1}{1+\gamma}} \qquad M_q \ll \Lambda$$

- In the limit $M_q \gg \Lambda$, the fermions decouple and theory is pure YM. Consider the regime $M_q \ll \Lambda$.
- The running coupling constant:

$$g(m,\mu) = \begin{cases} g(0,\mu) & \mu > M_q \\ g_* & \mu = M_q \\ \frac{1}{2b_0^{YM} \ln \frac{\mu}{\Lambda_{YM}}} & \mu \ll M_q \end{cases}$$

- Λ_{YM} is not a new scale in the theory, but is computed by requiring continuity about the energy scale $\mu \simeq M_q$.

$$\Lambda_{YM} = M_q \; e^{-\frac{1}{2b_0^{YM} g_*^2}} \ll M_q$$

Miransky's scenario for Banks-Zaks fixed point

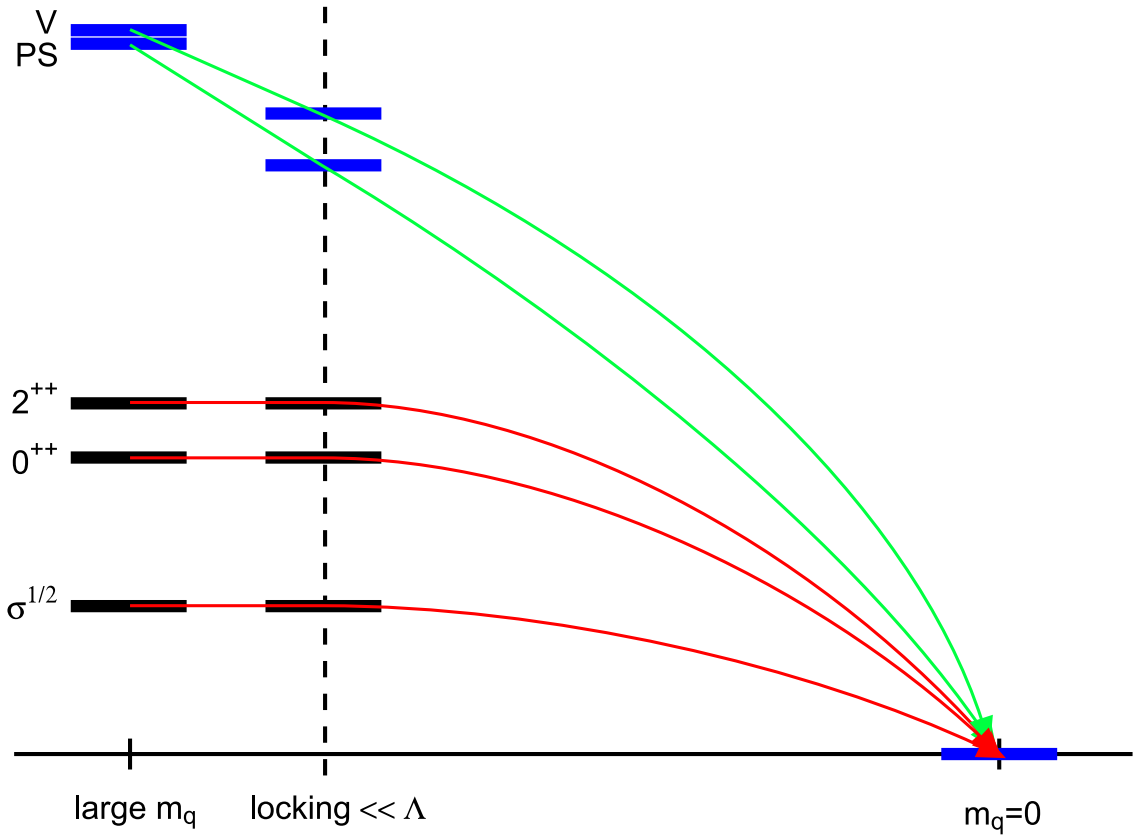
V. A. Miransky. Dynamics in the Conformal Window in QCD like theories. hep-ph/9812350.

$$\Lambda_{YM} = M_q \; e^{-\frac{1}{2b_0^{YM} g_*^2}} \ll M_q \ll \Lambda$$

- At energies much lower than M_q, the original theory is effectively described by a pure Yang-Mills theory with scale Λ_{YM}.
- Glueballs are lighter than mesons.
- A deconfinement transition occurs at a temperature $T_c \simeq \Lambda_{YM}$.
- Mesons are effectively quenched. The mesons are bound states of the quark-antiquark pair interacting via the YM static potential, the bound energy is small with respect to the mass of the constituents, and the correction to the potential due to quark-antiquark pair creation are negligible.
- As the mass M_q is reduced, the IR physics is always the same, provided that all the masses are rescaled with M_q.

Some cartoons again

IR conformality

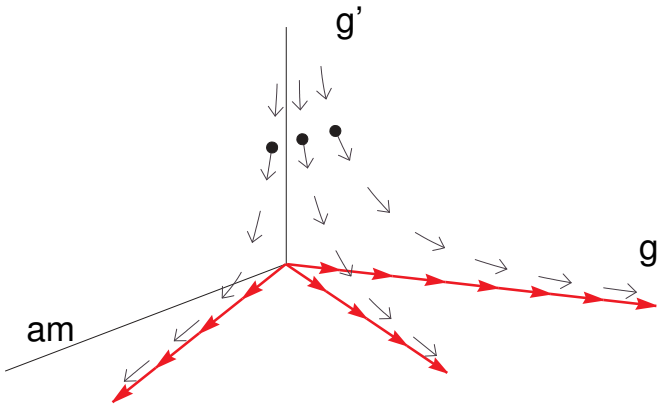


$$M_V/M_{PS} \simeq 1+\epsilon$$

$$M_{PS} \gg \sigma^{1/2}$$

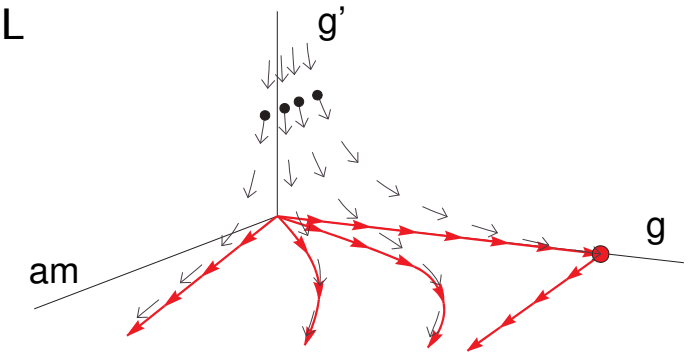
$$M_G/\sigma^{1/2} \simeq \left[M_G/\sigma^{1/2}\right]^{(YM)}$$

And on the lattice?



- the theory has two UV-relevant parameters g, m
- renormalized trajectories lie in the (g, m) plane
- simulations are performed away from this plane
- am small in order to have small discretization effects

- scale invariance is broken by m AND 1/L
- large physical volume + light masses!
- deviations from QCD spectrum



General strategy: go chiral! (if you can)

- In order to distinguish between confinement and IR-conformality, the study of the chiral limit is essential.
- The IR-conformality is characterised by the presence of a scaling region (all the masses go to zero with the same power law).
- In principle one could investigate the existence of a power-law behaviour, but unfortunately the quality of the fits is poor. A stabler numerical strategy is to look for plateaux in ratios of masses.
- In the case of a perturbative-like IR fixed point, the quasi-degeneracy of PS and V mesons is a common feature to the high-mass and chiral regimes. Instead the glueballs and the string tension have drammatically different behaviours in the two opposite regimes.

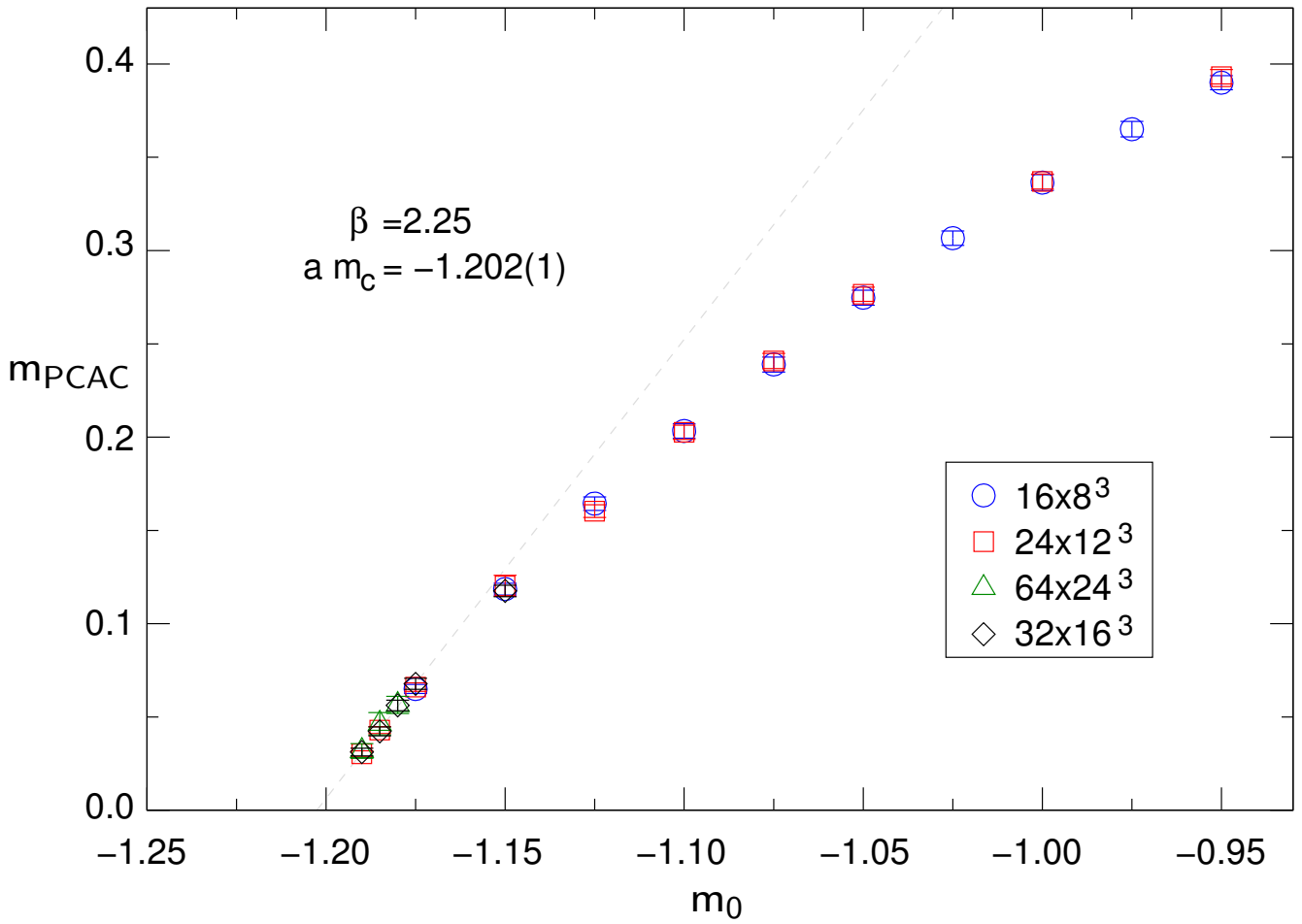
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Many of these behaviors can be masked in numerical simulations
by finite-volume effects and discretization artifacts.

Chiral Limit

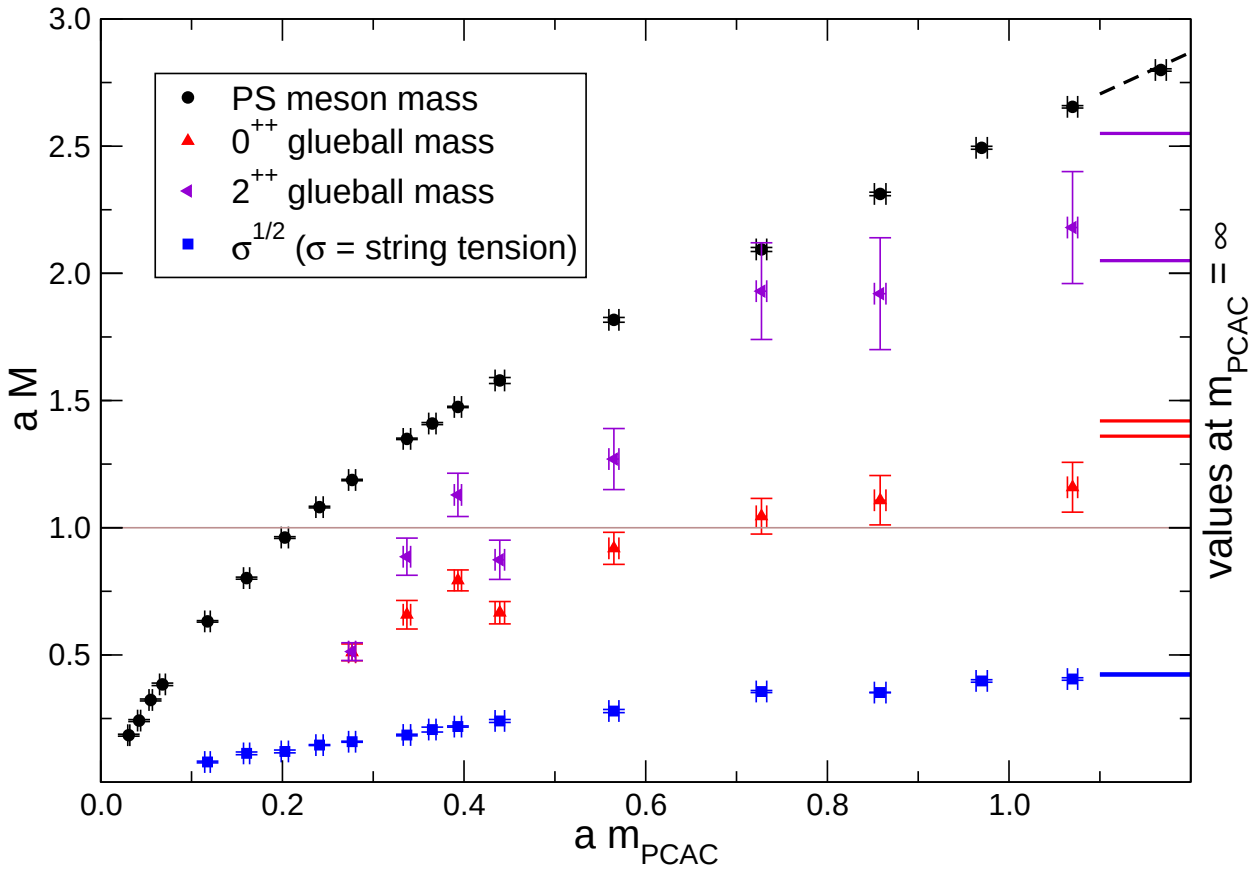
The quark mass from the axial Ward identity (PCAC mass) is used.



L. Del Debbio, B. Lucini, A. Patella, C. Pica and A.R, Phys. Rev. D **82** (2010) 014509

Old results: the spectrum

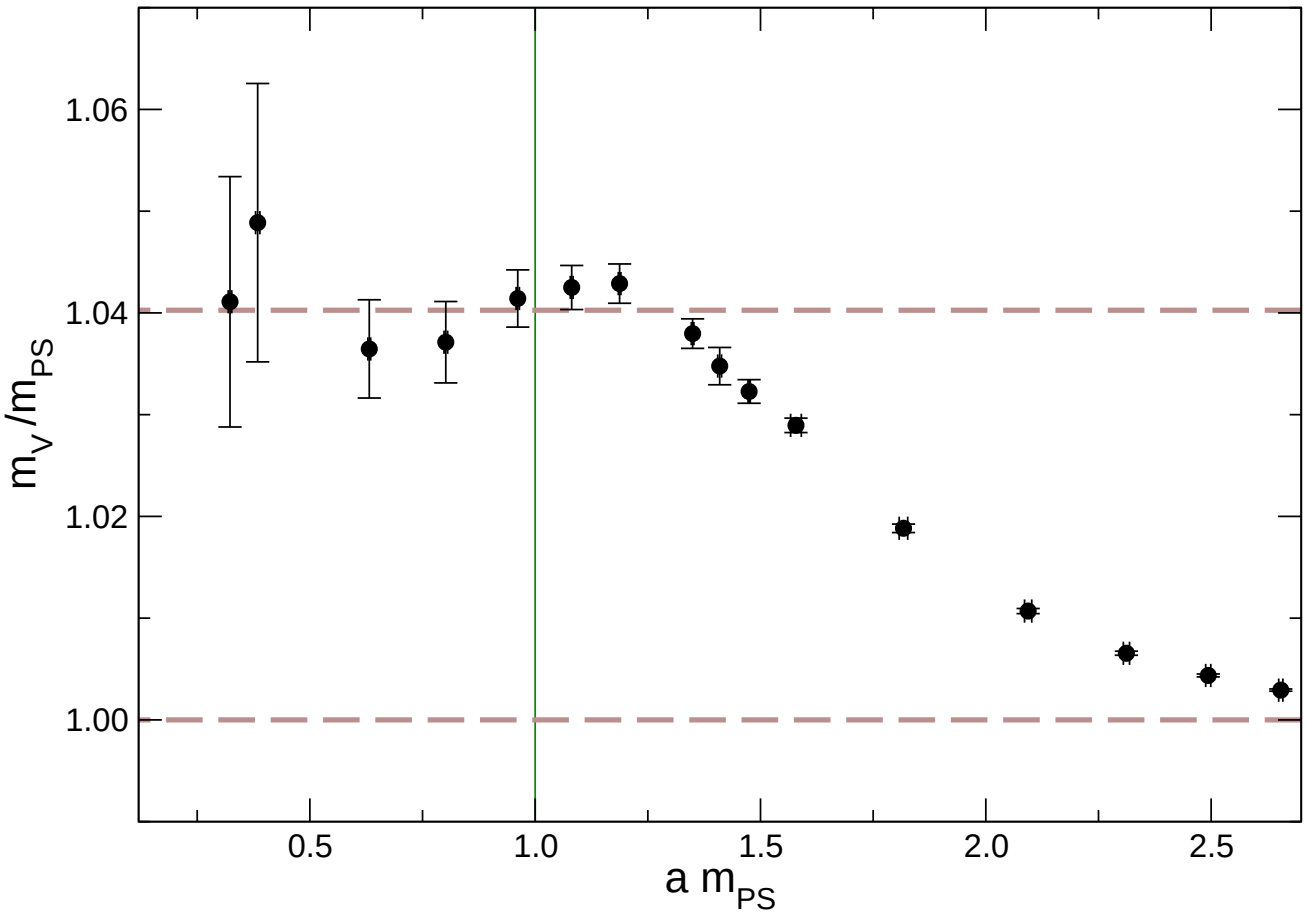
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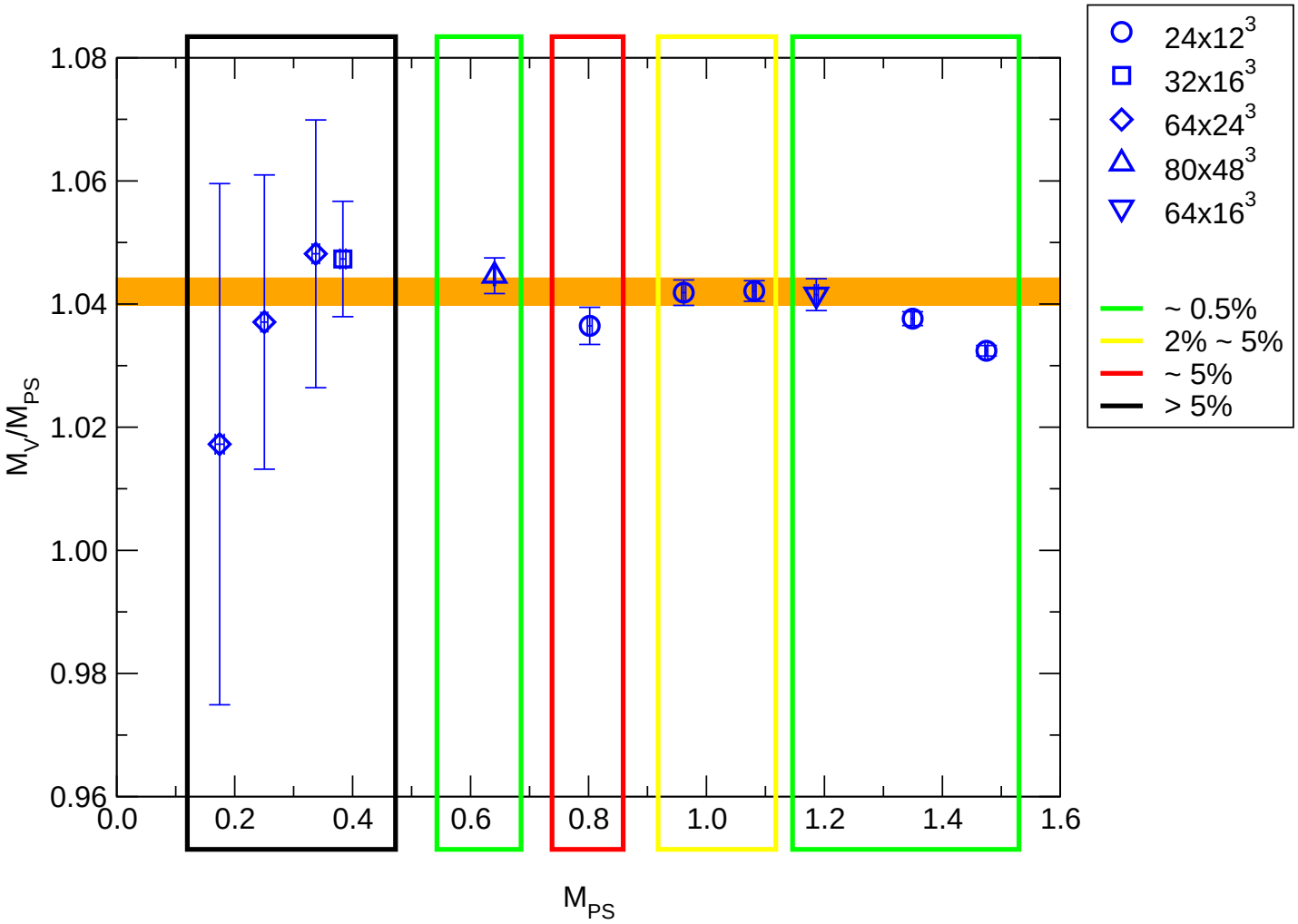
L. Del Debbio, B. Lucini, A. Patella, C. Pica and A.R., Phys. Rev. D **80** (2009) 074507

Old results: ratios

All the masses must scale with the same exponent, ratios have to be constant.

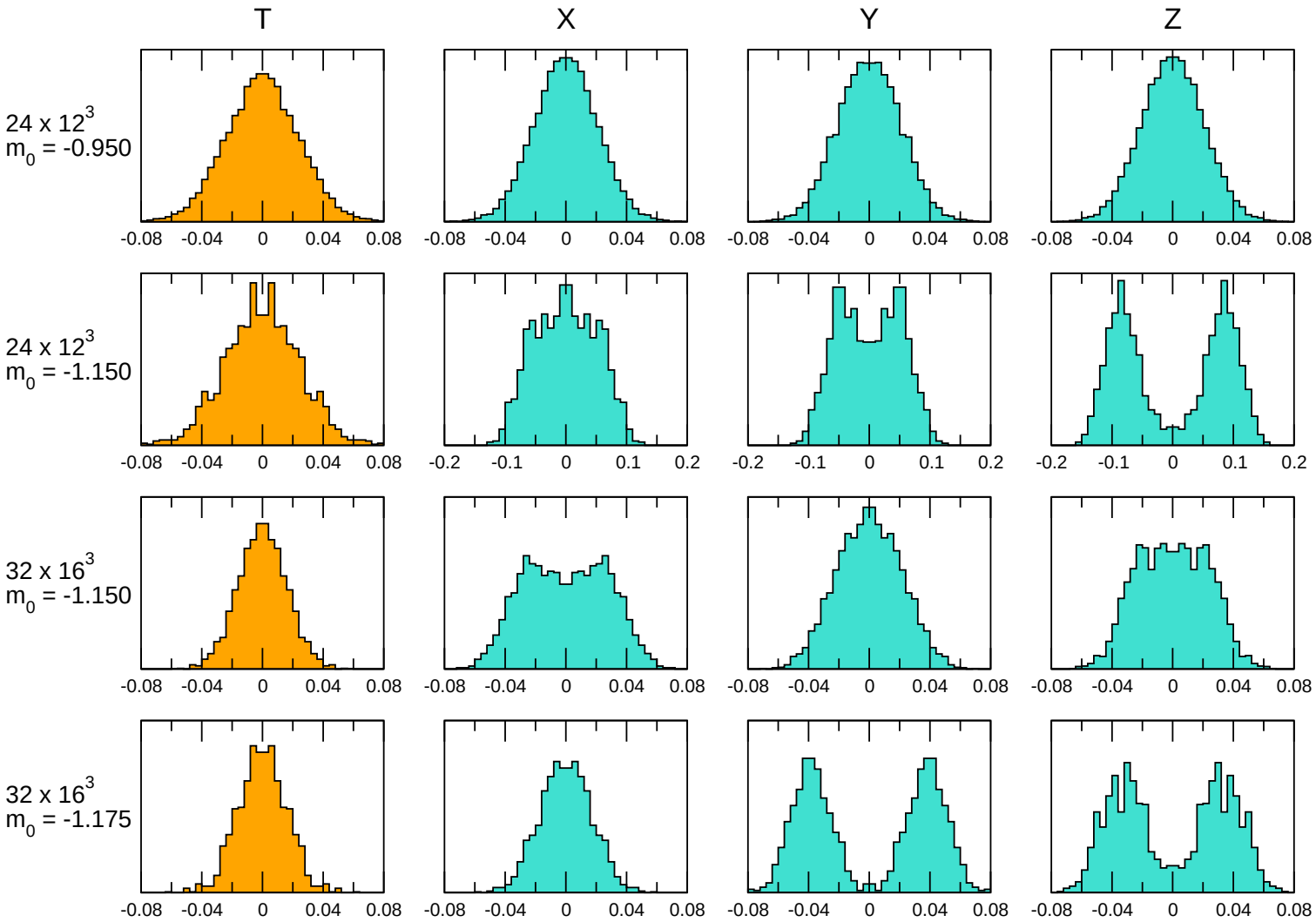


Finite Volume: $M_\pi L \sim 14$

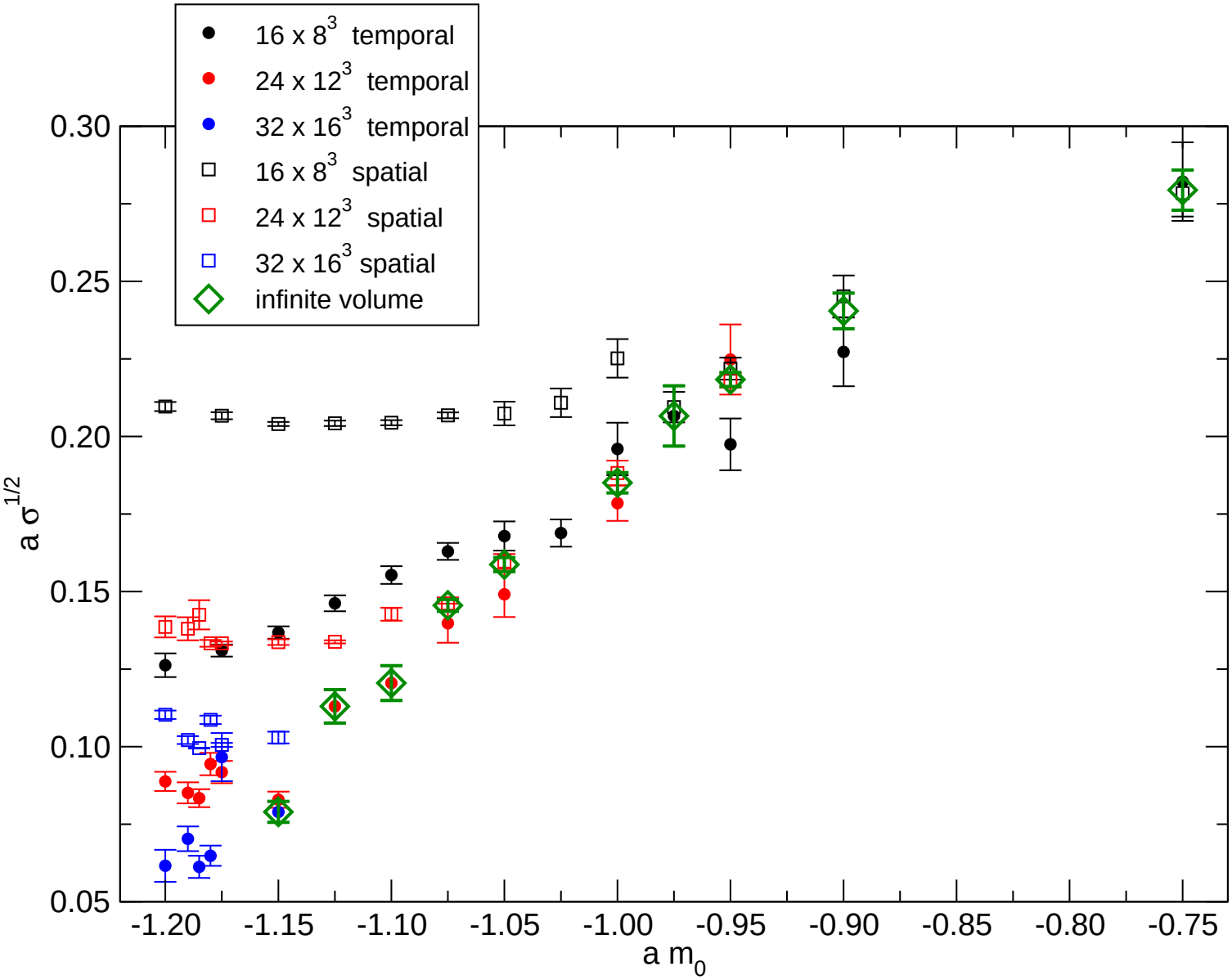


see F.Bursa's Poster

Polyakov distribution

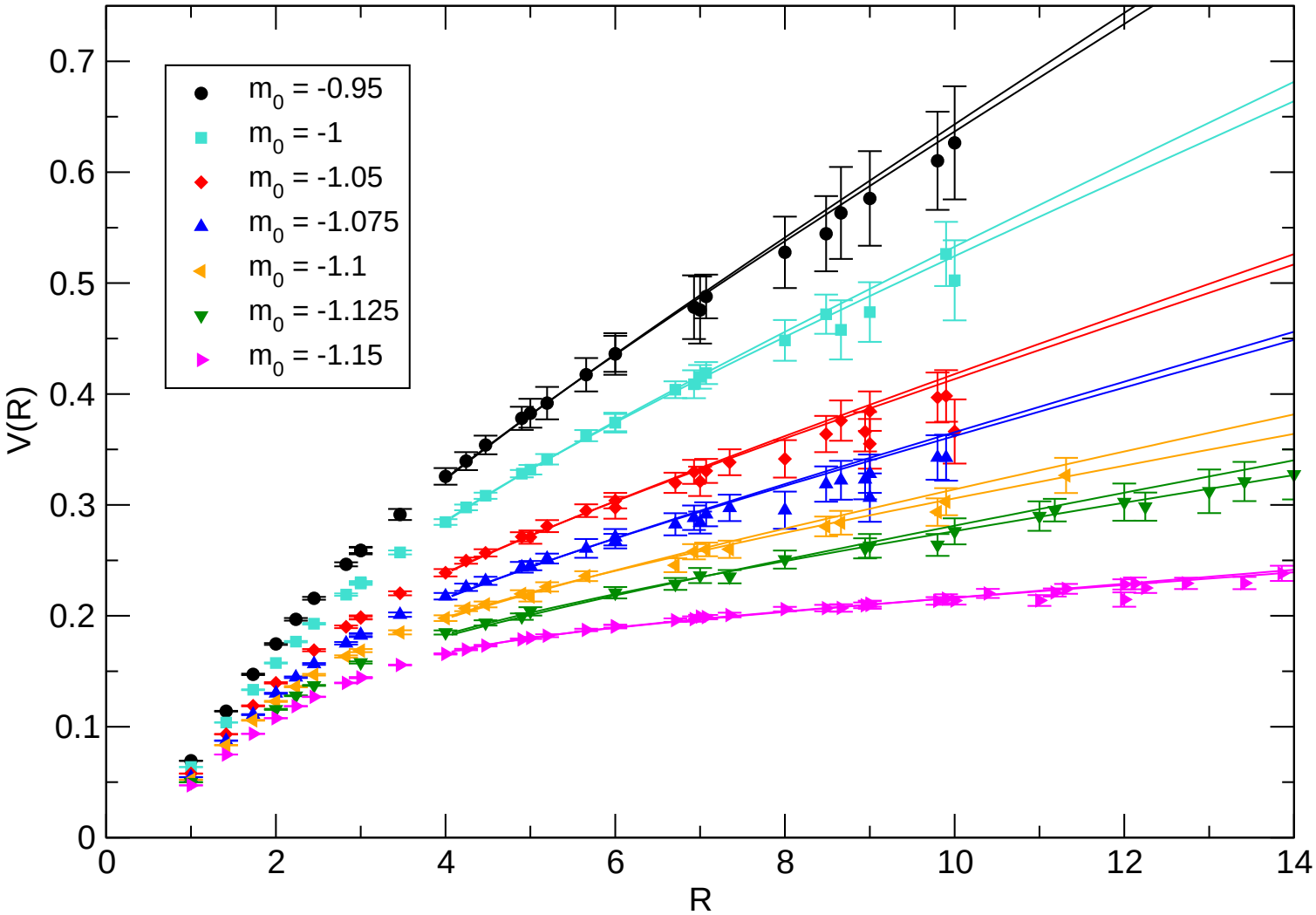


String tension from Polyakov loops



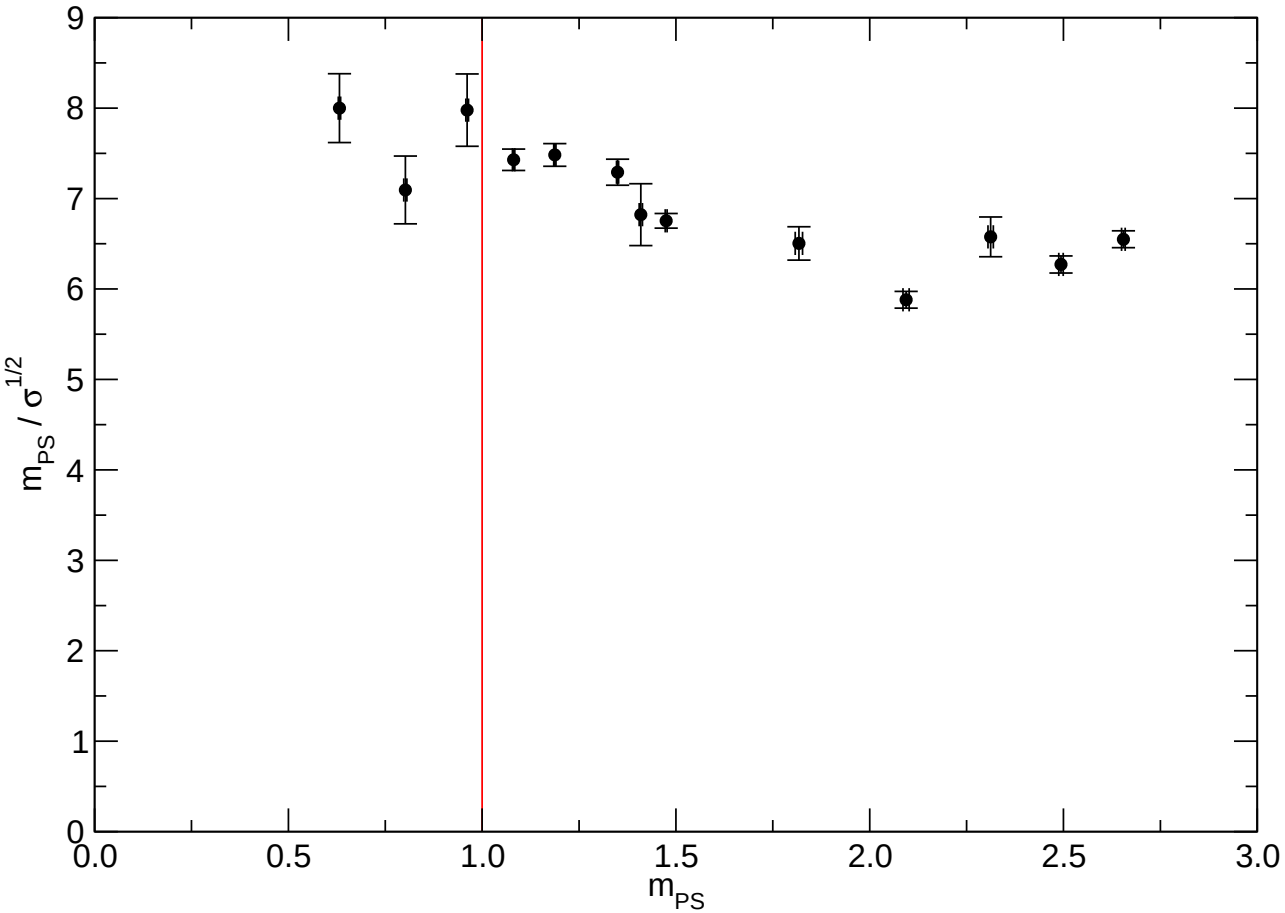
L. Del Debbio, B. Lucini, A. Patella, C. Pica and A.R., Phys. Rev. D **82** (2010) 014510

Perfect agreement with the prediction of the Wilson loop

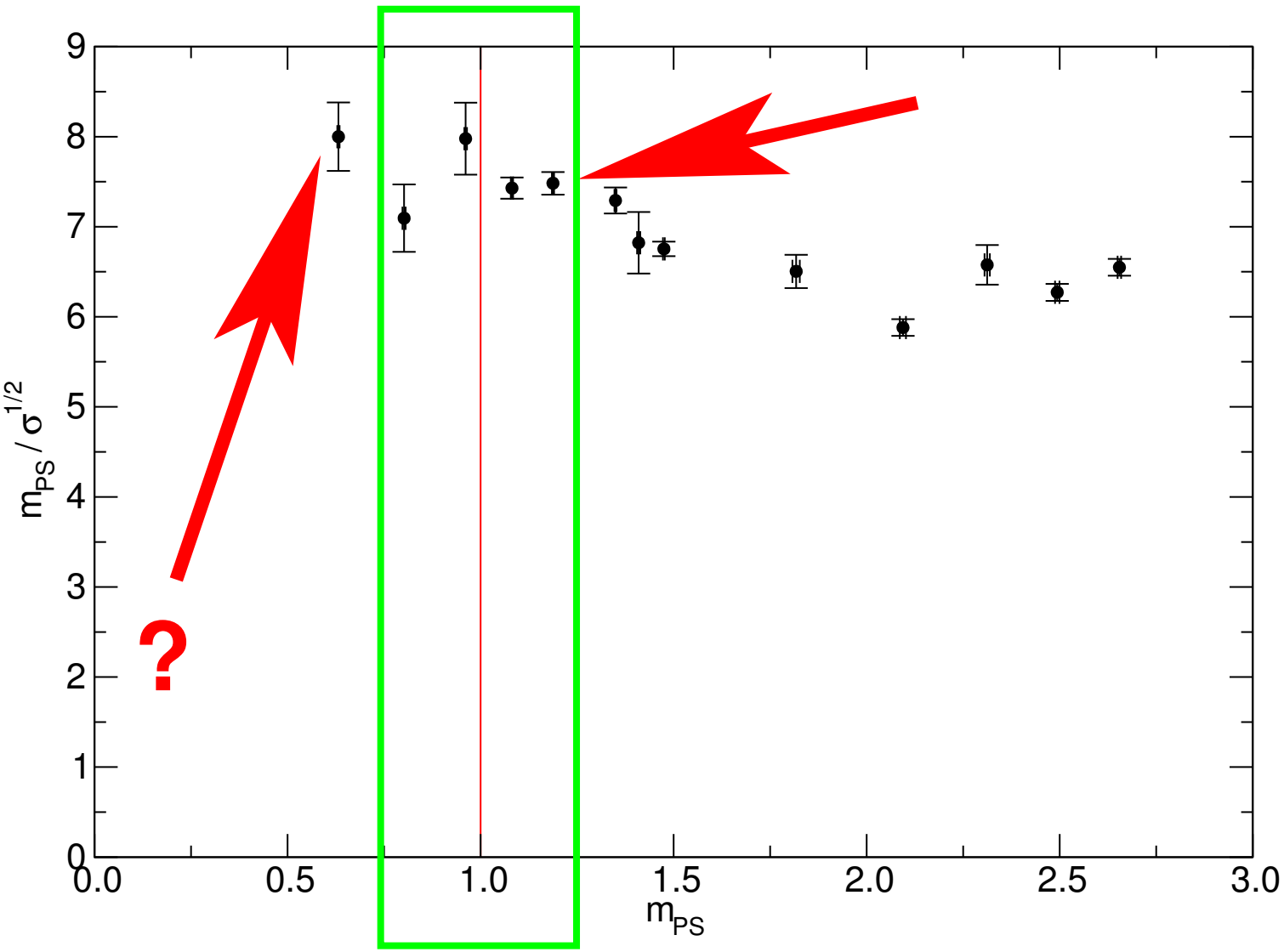


Old results: ratios

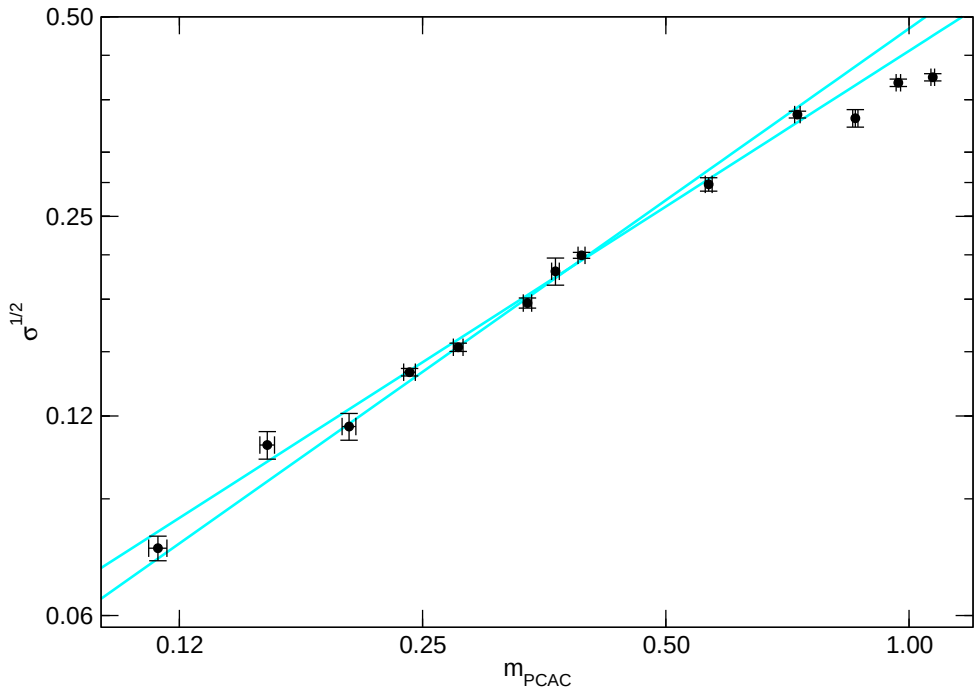
All the masses must scale with the same exponent, ratios have to be constant.



$M_\pi L \sim 16$



Scaling region



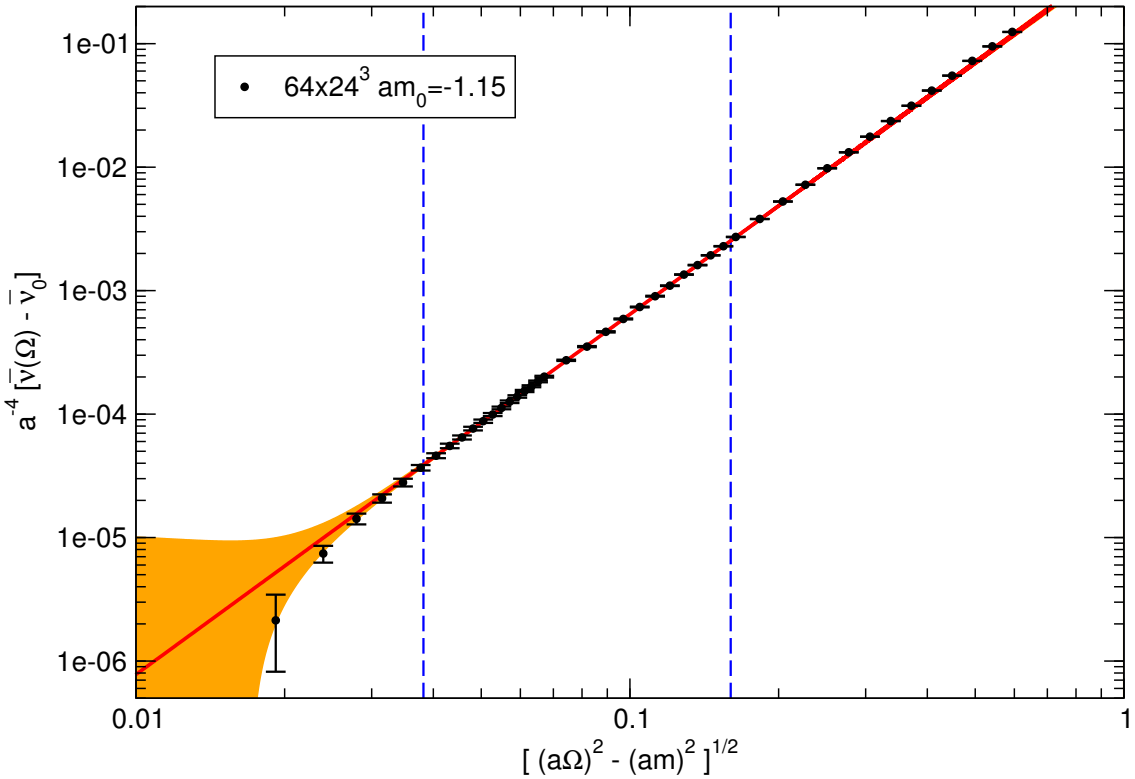
$$a\sigma^{1/2} = A_\sigma (am_q)^{\frac{1}{1+\gamma}} \qquad \gamma \simeq 0.16 \ \ldots \ 0.28$$

Not efficient!

A smart idea!

Study the scaling of the integral of the spectral density of the Dirac operator.

- The mode number $\bar{\nu}(\Omega) = 2\int_0^{\sqrt{\Omega^2-m^2}} \rho(\omega)d\omega$
- It can be shown $\bar{\nu}(\Omega) \simeq \bar{\nu}_0 + A[\Omega^2 - m^2]^{\frac{2}{1+\gamma_*}}$



A. Patella, arXiv:1204.4432 [hep-lat].

Conclusions

- **Control the Volume finite size effects.**
  In every channel interesting for you theory.
- **Use smart measurement.**
  An interesting method to evaluate the anomalous dimensions.

Plans

- **At least another β**
  The ratios in the scaling regime should not depend on the value of β.
- **Scale setting**
  A reliable way to set the scale is still missing.