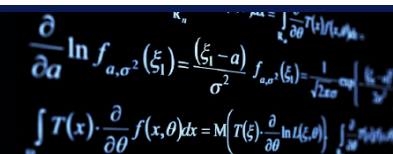
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Cosmological Perturbations from Non-Equilibrium Quantum Fields

Arttu Rajantie

13 July 2012

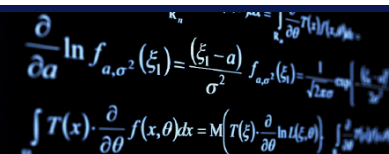


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Particle Physics $\leftrightarrow$ Cosmology

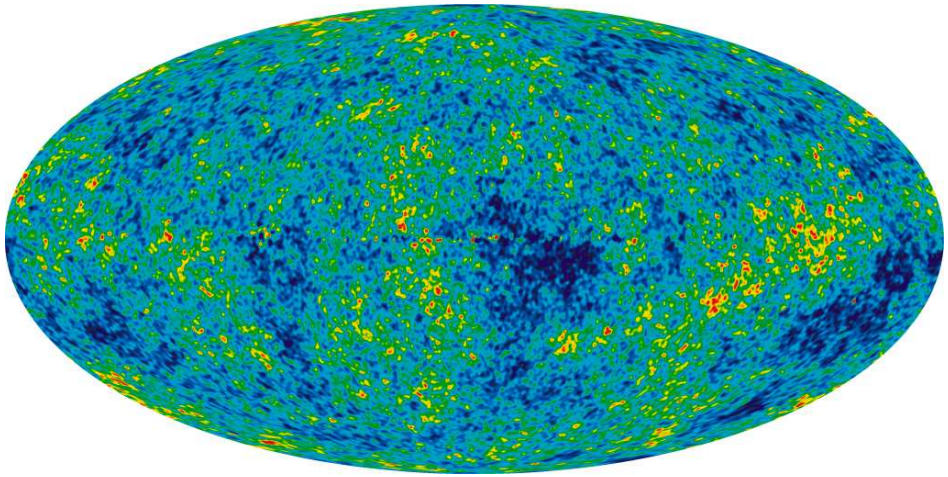
- Explain cosmological observations:
  - Baryon asymmetry (Huber,Tranberg,Saffin)
  - Dark matter (Cirelli)
  - Dark energy
  - Inflation

- Test particle physics theories:
  - Topological defects (monopoles, strings)
  - Gravity waves
  - Perturbations (CMB, LSS)



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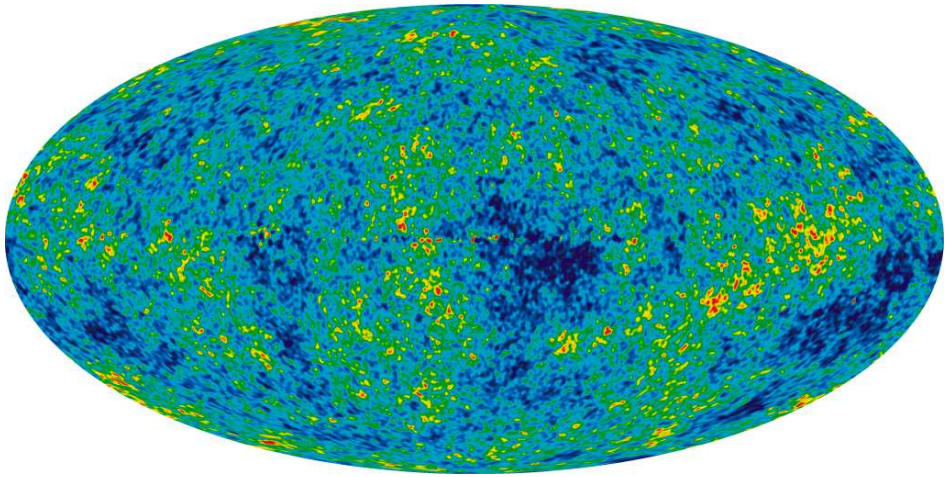
Cosmic Microwave Background



- Penzias&Wilson 1964
- Thermal radiation from recombination $t \approx 300000\mathrm{yrs}$
- Redshifted from $T \approx 4000\mathrm{K}$ to $T \approx 2.7\mathrm{K}$

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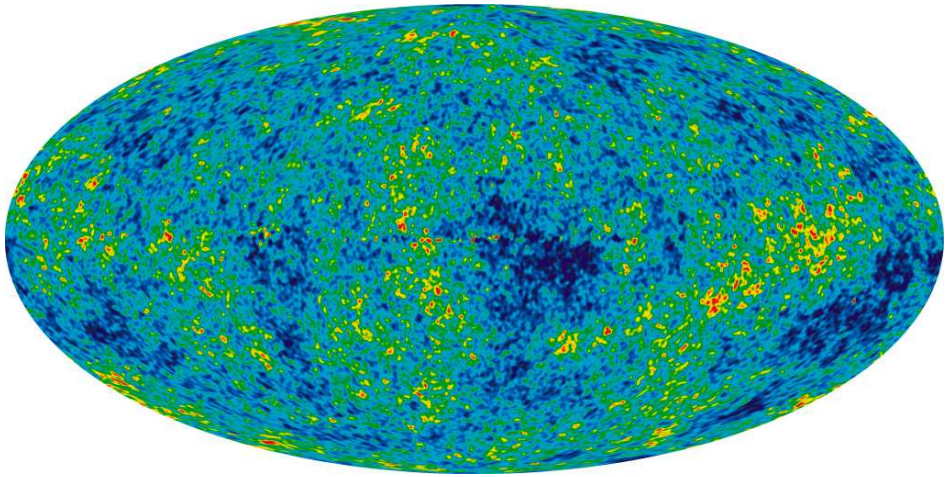
Cosmic Microwave Background



- Temperature anisotropies $\delta T \sim 10^{-5}\mathrm{K}$
- COBE 1992, WMAP 2003–2010
- The best source of information about the early universe

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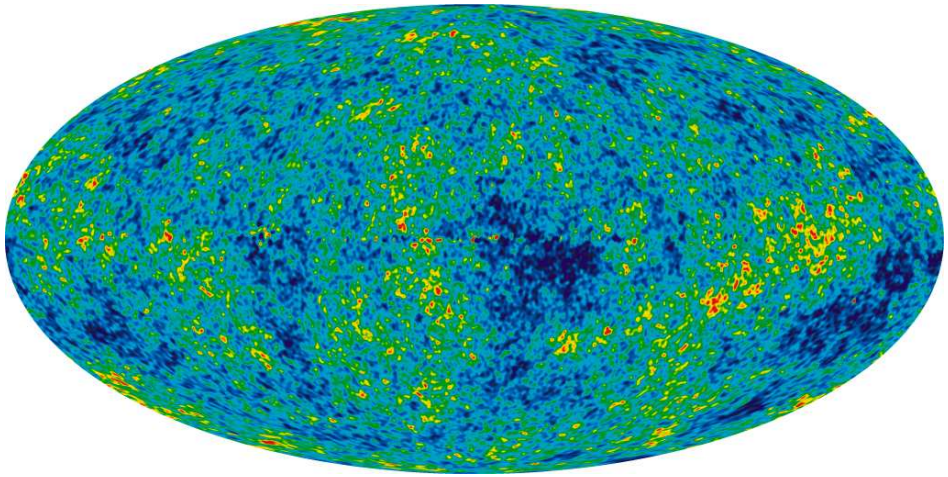
Cosmic Microwave Background



- Primordial density perturbation: $\delta T/T = \delta\rho/\rho$
- Origin of large-scale structure:
  Galaxies, galaxy clusters

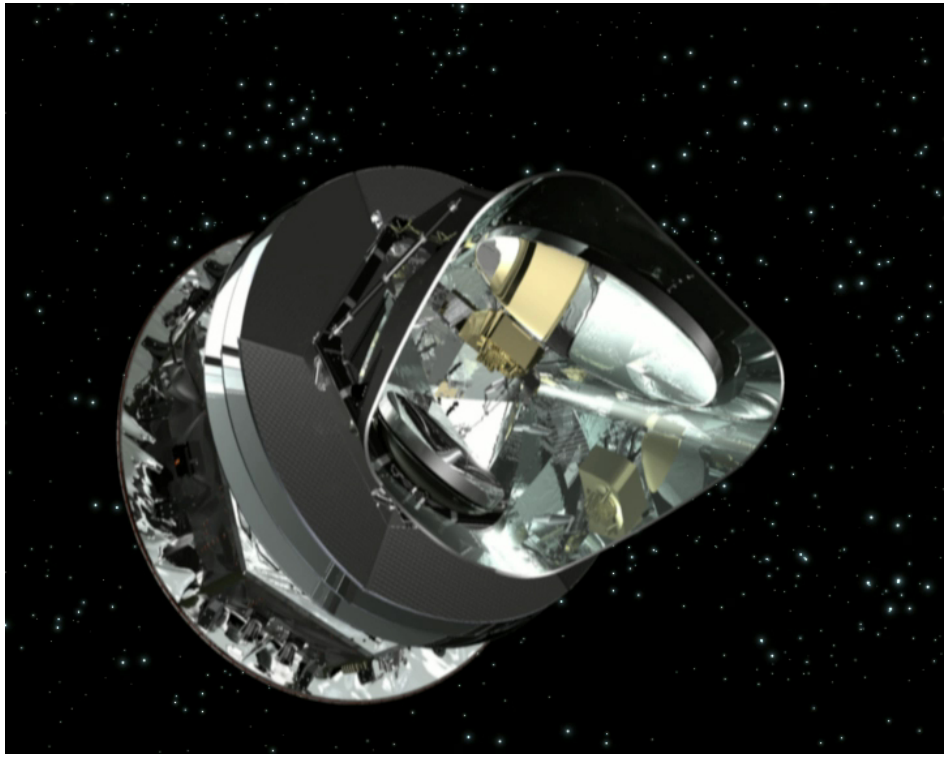
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Cosmic Microwave Background



- Highly Gaussian $-10 < f_{\rm NL} < 74$ (i.e. $\zeta_{\rm NG} \lesssim 10^{-8}$)
- Nearly scale-invariant spectrum $n_{\rm s} = 0.963 \pm 0.012$
- Just as inflation predicts!

Planck Satellite

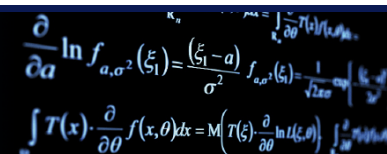


- High angular resolution, polarisation
- Very sensitive to non-Gaussianity ($\zeta_{\rm NG} \sim 10^{-9}$)
- Data released in early 2013

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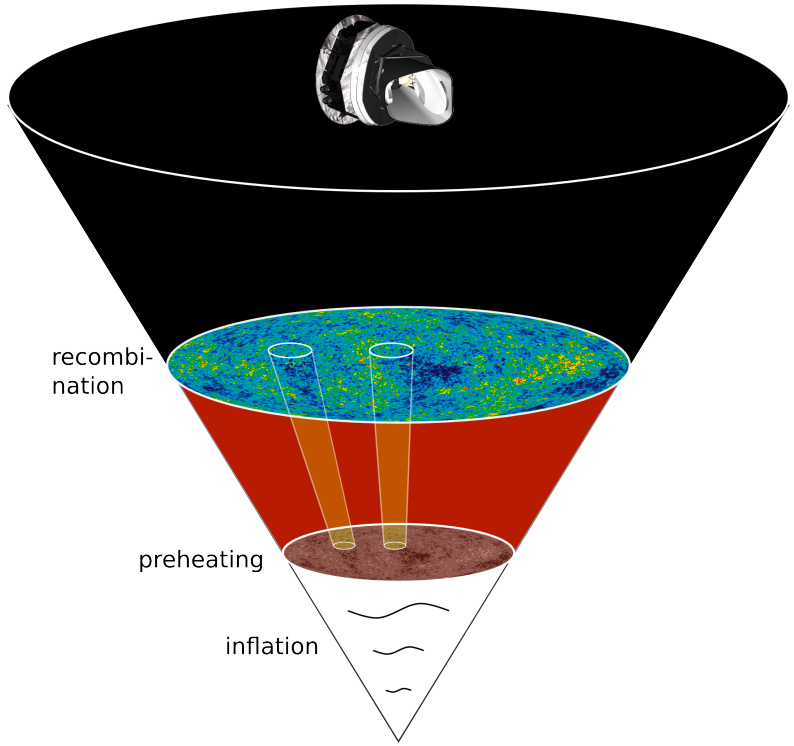
Particle Physics in the CMB?

- Adiabatic evolution $\Rightarrow$ Perturbation conserved
  - $\Rightarrow$ We need non-equilibrium dynamics
- Horizon then $\ll$ Horizon now
  - $\Rightarrow$ We need correlations over many, many horizon lengths
- Causality: Correlations cannot appear after inflation
  - $\Rightarrow$ We need an isocurvature perturbation
    Simplest possibility: Light ($m < H$) scalar field
    - Since last week we know such fields actually exist!



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Separate Universes



- Each horizon volume $\sim$ separate FRW universe (Salopek&Bond 1990)

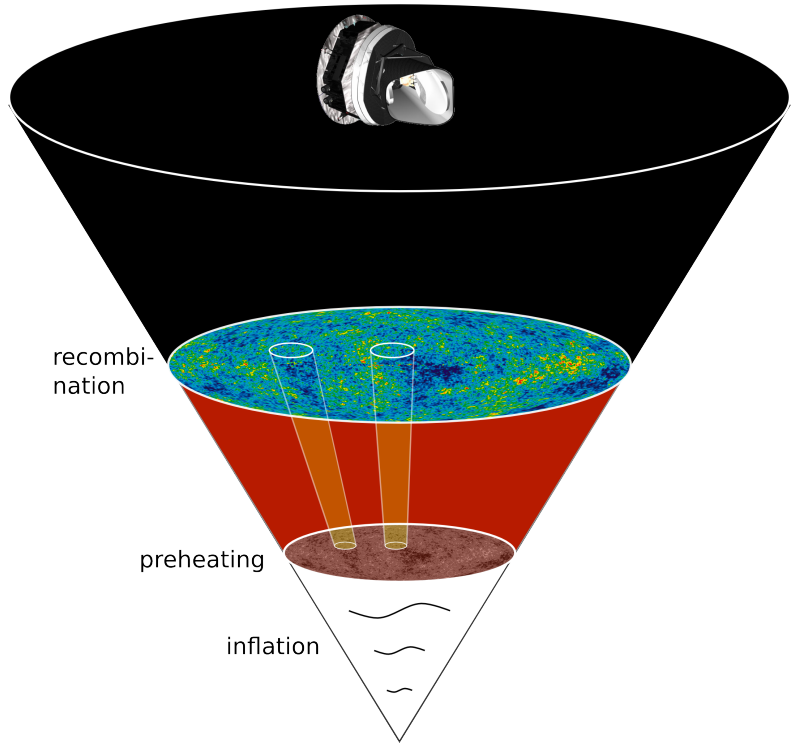
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Separate Universes

- Each horizon volume $\sim$ separate FRW universe (Salopek&Bond 1990)
- Described by a separate Friedmann equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3M_{Pl}^2}$$

- Curvature perturbation $\zeta = \delta T/T = \delta \ln a|_{\rho=\rho_*}$
- Valid at distances $d \gg 1/H$

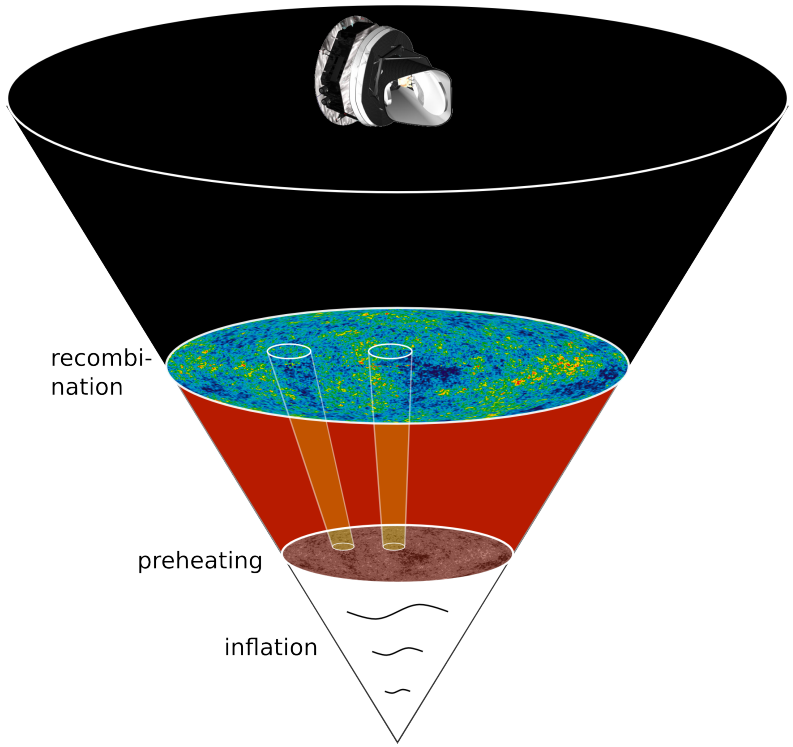


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Conservation of ζ

- Curvature perturbation
  $\zeta = \delta T/T = \delta \ln a|_{\rho=\rho_*}$
- If the equation of state $P = P(\rho)$ is the same everywhere,

$$\frac{d\ln a}{d\rho} = -\frac{1}{3(\rho + P(\rho))} \Rightarrow \frac{d\zeta}{d\rho_*} = 0$$

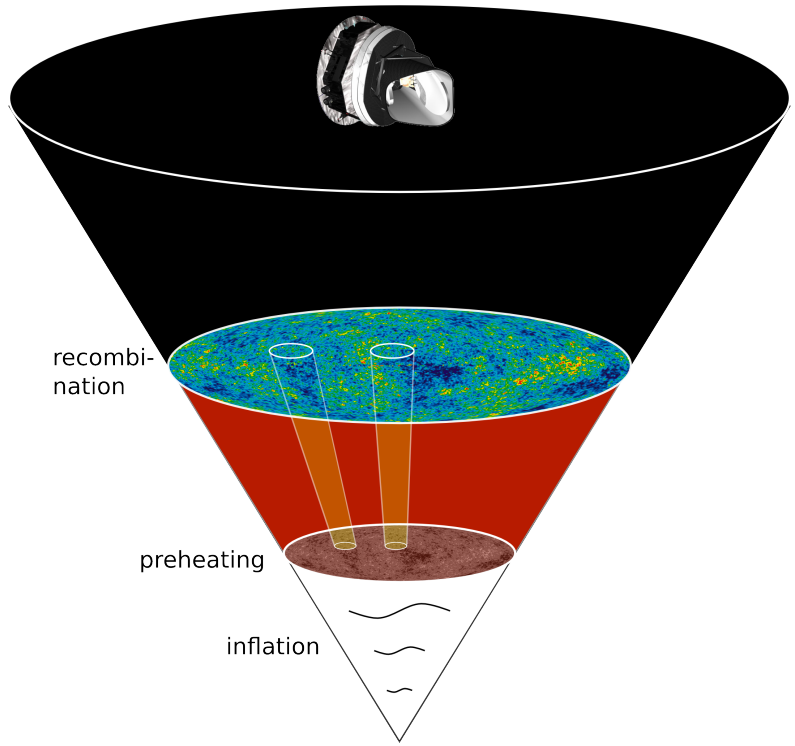


Another Light Scalar χ

- Light scalar $m_\chi < H$
  $\Rightarrow$ Scale-invariant perturbations

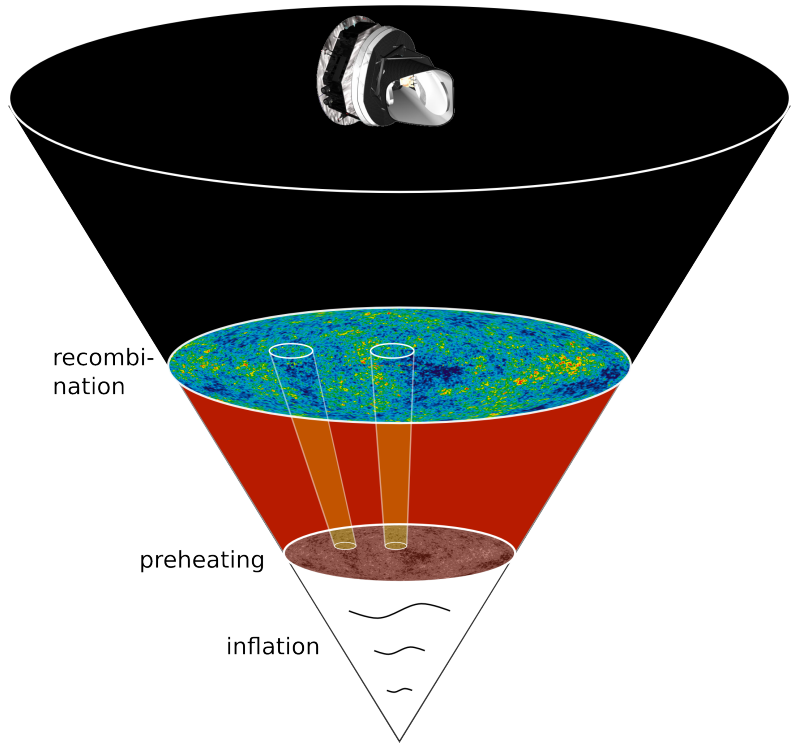
$$\mathcal{P}_\chi(k) \approx \frac{H^2}{4\pi^2}$$

- Each separate "universe" has different initial value χ_0
- Affects expansion
  $\Rightarrow$ Scale factor depends on χ_0



Another Light Scalar χ

- Each separate ”universe” has different initial value χ_0
- Affects expansion
  $\Rightarrow$ Scale factor depends on χ_0
- Thermalisation erases memory of $\chi_0 \Rightarrow P = P(\rho)$
- Curvature perturbation determined at thermalisation:
  $\zeta_{\rm rec} = \zeta_{\rm therm} = \delta \ln a|_{\rho=\rho_{\rm therm}}$



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Calculating the Perturbation

- Solve Friedmann+field eqs for each separate universe

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho(\phi,\chi)}{3M_{Pl}^2}$$

  - Gives $a(t)$, $\rho(t)$
  - Non-linear, includes gravity, valid at $d \gg H^{-1}$
- Pick $\rho_* < \rho_{\rm therm}$, and calculate $\zeta = \delta \ln a|_{\rho=\rho_*}$

  $\Rightarrow$ Gives curvature perturbation for given ϕ_0, χ_0

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Calculating the Perturbation

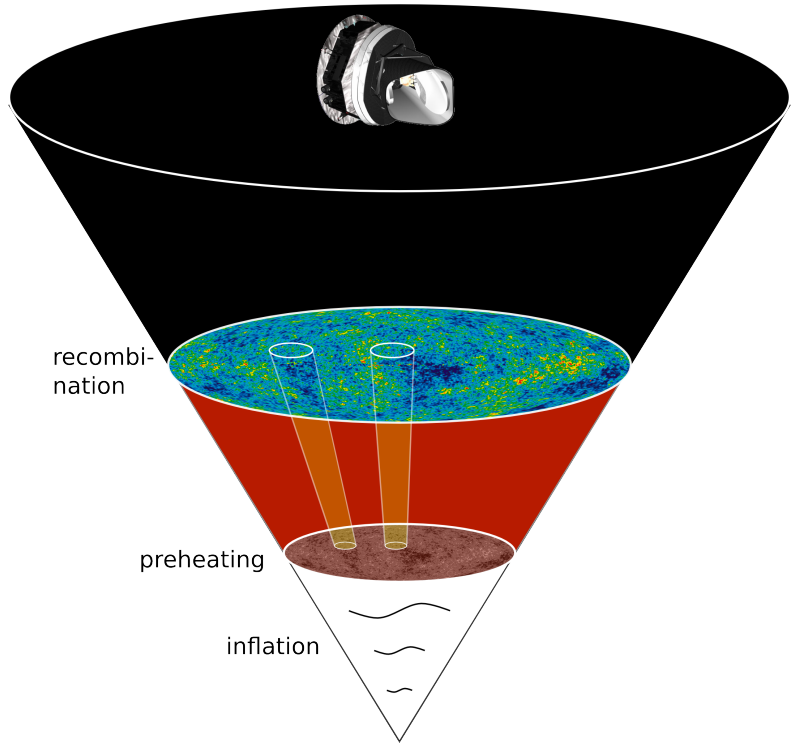
- Solve Friedmann+field eqs for each separate universe
- Pick $\rho_* < \rho_{\rm therm}$, and calculate $\zeta = \delta \ln a|_{\rho=\rho_*}$
  - Two fields: $\zeta = \zeta(\phi_0, \chi_0)$
  - $\delta\phi_0 \Rightarrow$ Usual inflationary perturbations
  - $\delta\chi_0 \Rightarrow$ New contribution
- Need to calculate $N(\chi_0) = \ln a(\rho_*, \chi_0)$
  - Perturbative approach: δN formalism
  - Fully non-linear:
    Solve field and Friedmann eqs numerically
    for many different initial values χ_0

  $\Rightarrow$ Whole non-linear function $N(\chi_0)$ (Bassett&Tanaka 2003, Suyama&Yokoyama 2006)

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Lattice Calculation

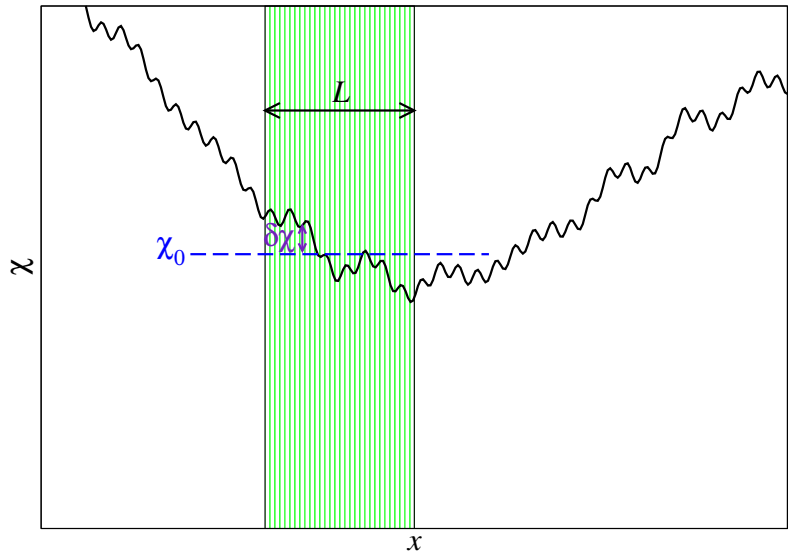
- Describe each “universe” as a lattice (Chambers&AR 2007)
- Inhomogeneous fields, FRW metric
- Lattice size $L \sim 1/H$



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Lattice Calculation

- Solve field + Friedmann eqs numerically on lattice
- Initial conditions $\chi(x) = \chi_0 + \delta\chi(x)$:
  - $\delta\chi(x)$: subhorizon quantum fluctuations
  - χ_0: mean value over horizon
- Find curvature perturbation as $\zeta(\chi_0) = \delta \ln a(\chi_0)|_{\rho=\rho_*}$
  - Function of χ_0, not x



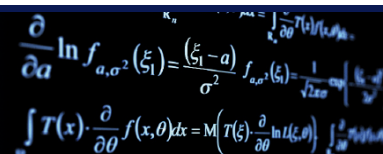
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Applications

- Light scalar χ + nonequilibrium dynamics
  $\Rightarrow$ Perturbations

- Examples:
  - Massless preheating
  - Resonant curvaton
  - Electrically charged curvaton

- Future: Concrete particle physics models

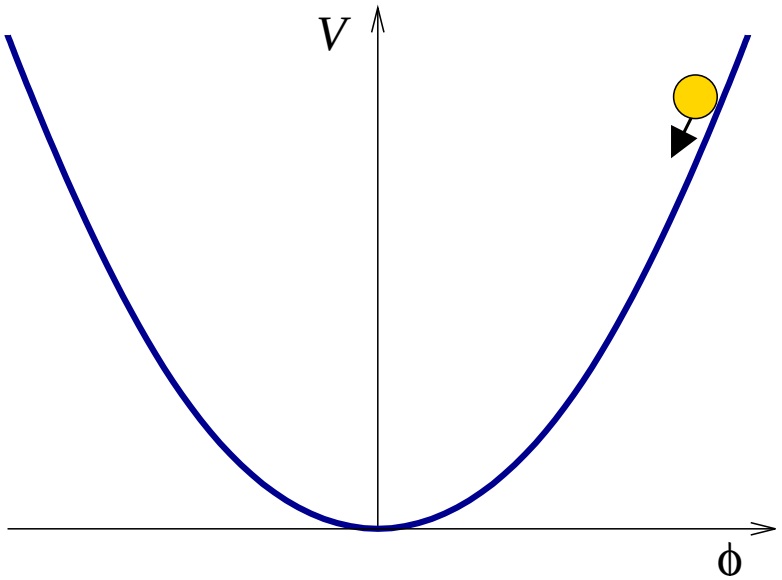


Massless Preheating

- Chaotic inflation + massless scalar χ (Prokopec&Roos 1997)

$$V(\phi,\chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2, \quad \lambda \sim 7\times 10^{-14}$$

- Inflation: $\phi > 2\sqrt{3}M_{\rm Pl}$

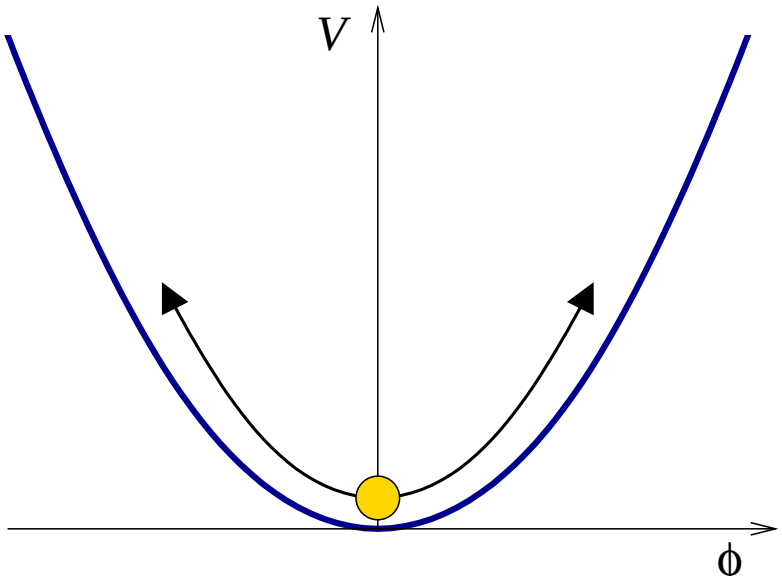


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- After inflation: Radiation domination $a \propto t^{1/2}$



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Massless Preheating

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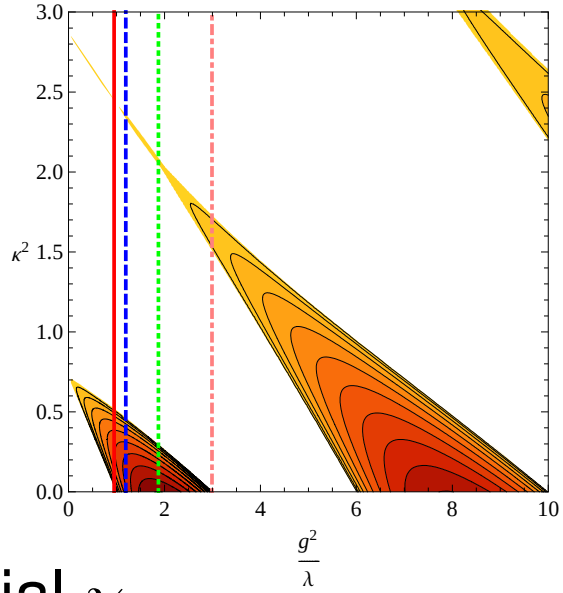
- After inflation: Radiation domination $a \propto t^{1/2}$
- Inhomogeneous χ modes $\chi_k = a^{-1}\tilde{\chi}_k$

$$\tilde{\chi}_k'' + \left[\kappa^2 + \frac{g^2}{\lambda}\mathrm{cn}^2(\tau;1/\sqrt{2})\right]\tilde{\chi}_k = 0, \quad \kappa^2 = \frac{k^2}{\lambda\tilde{\phi}_{\mathrm{ini}}^2}$$

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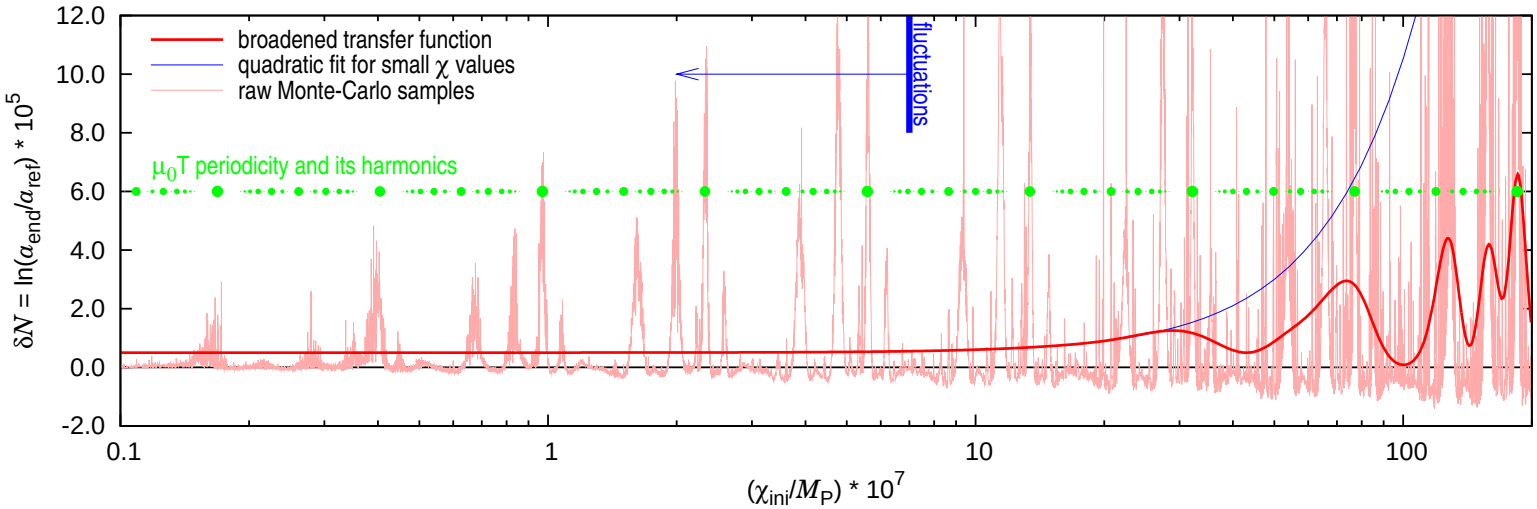
Parametric Resonance

- Floquet theorem:
  - $\tilde{\chi}_k(\tau) = e^{\mu\tau} f(\tau)$ with periodic $f(\tau)$
- Real μ: Exponential growth = resonance
  - Resonance transfers energy to χ
- When χ grows enough, evolution becomes non-linear
  $\Rightarrow$ Resonance ends – Thermalisation
- Amount of growth needed depends on initial χ
  $\Rightarrow$ Curvature perturbation (Chambers&AR,2007)
- Effect strongest when $\kappa = 0$ dominates the resonance



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Massless Preheating



(Bond et al 2009)

- Chaotic behaviour: Highly non-Gaussian (Chambers&AR 2007)
- Not well approximated by a quadratic $f_{\rm NL}$
- Amplitude $\sim 10^{-4}$ – Observable!?

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Curvaton

- Scalar field σ (Mollerach 1990; Linde&Mukhanov 1997; Enqvist&Sloth 2002; Lyth&Wands 2002)

$$V(\phi,\sigma) = V(\phi) + \frac{1}{2}m^2\sigma^2 + h\sigma\bar{\psi}\psi$$

- During inflation $H > m$: Scale-invariant fluctuations
- After inflation $m > H$: Behaves like matter

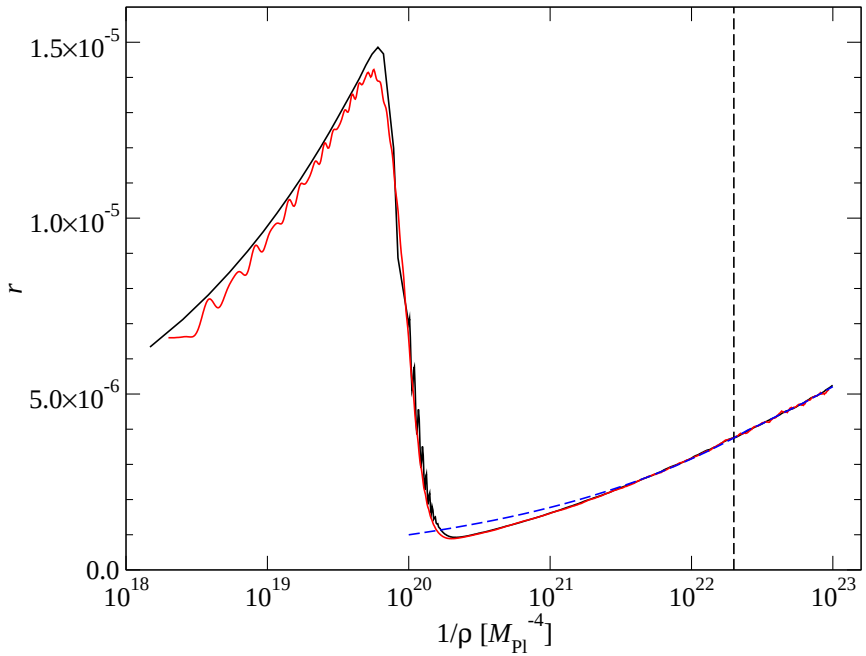
$$P = \frac{1-r}{3}\rho_{\rm tot}, \quad r = \frac{\rho_\chi}{\rho_{\rm tot}}$$

- Decays perturbatively to ψ particles $\Rightarrow \zeta$ conserved

$$\zeta = N(\overline{\chi} + \delta\chi) = \frac{r_{\rm decay}}{2}\frac{\delta\chi}{\overline{\chi}} + \frac{r_{\rm decay}}{4}\left(\frac{\delta\chi}{\overline{\chi}}\right)^2 + \ldots$$

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Curvaton Resonance

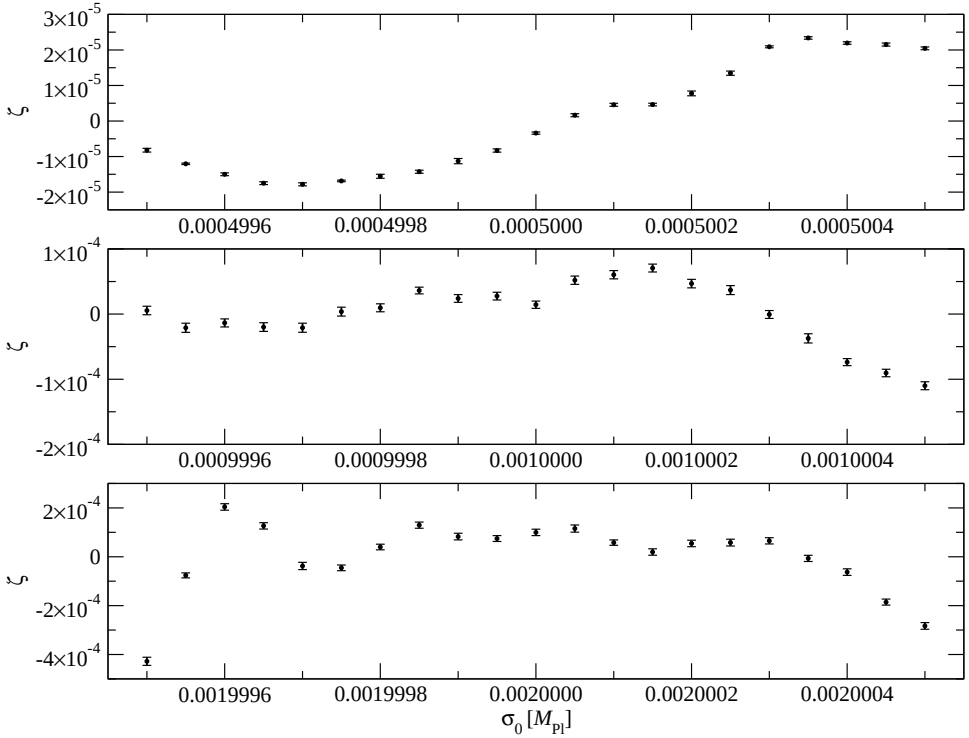


(Chambers,Nurmi&AR, 2009)

- Curvaton σ coupled to another scalar (Enqvist et al 2009)
- Decays through parametric resonance

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Curvaton Resonance



- Nonlinear $N(\sigma_0) \Rightarrow$ Highly non-Gaussian (Chambers,Nurmi&AR, 2009)

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Electrically Charged Curvaton

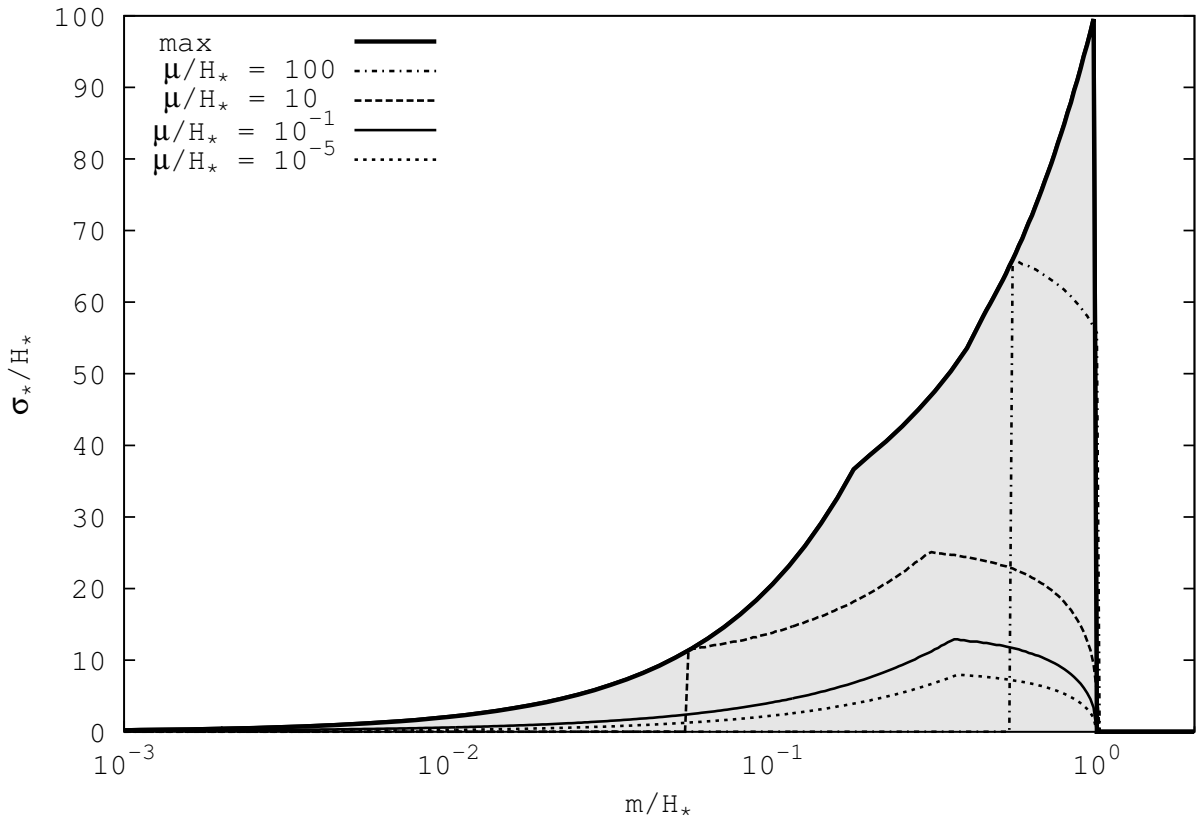
- Charged under electroweak $\mathrm{U}(1)$ (D'Onofrio,Lerner&AR 2012)
  - Coleman-Weinberg potential

$$V_{\mathrm{eff}}(\sigma) = m^2|\sigma|^2 + \frac{3g'^4}{64\pi^2}|\sigma|^4 \ln\left(\frac{|\sigma|^2}{\mu^2}\right)$$

  - Known coupling $g' \approx 0.357$ (or integer multiple)
- During inflation:
  - Perturbations mainly neutral ("Higgs mode")
  - Charge density screened by other charged particles
- After inflation:
  - Perturbative decay (as usual)
  - Interaction with thermal bath
  - Parametric resonance with the gauge field

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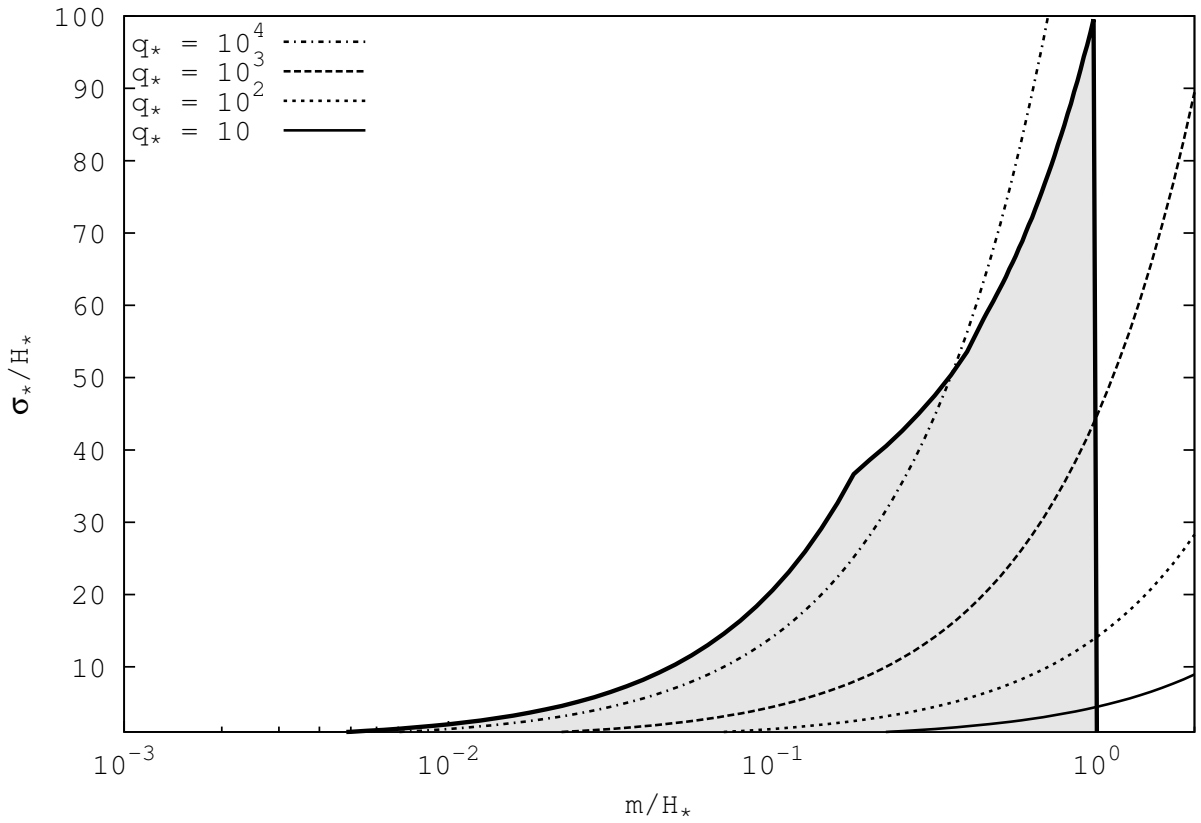
Qualitative Constraints



- Light curvaton, stable vacuum, linear evolution
- Shaded=allowed

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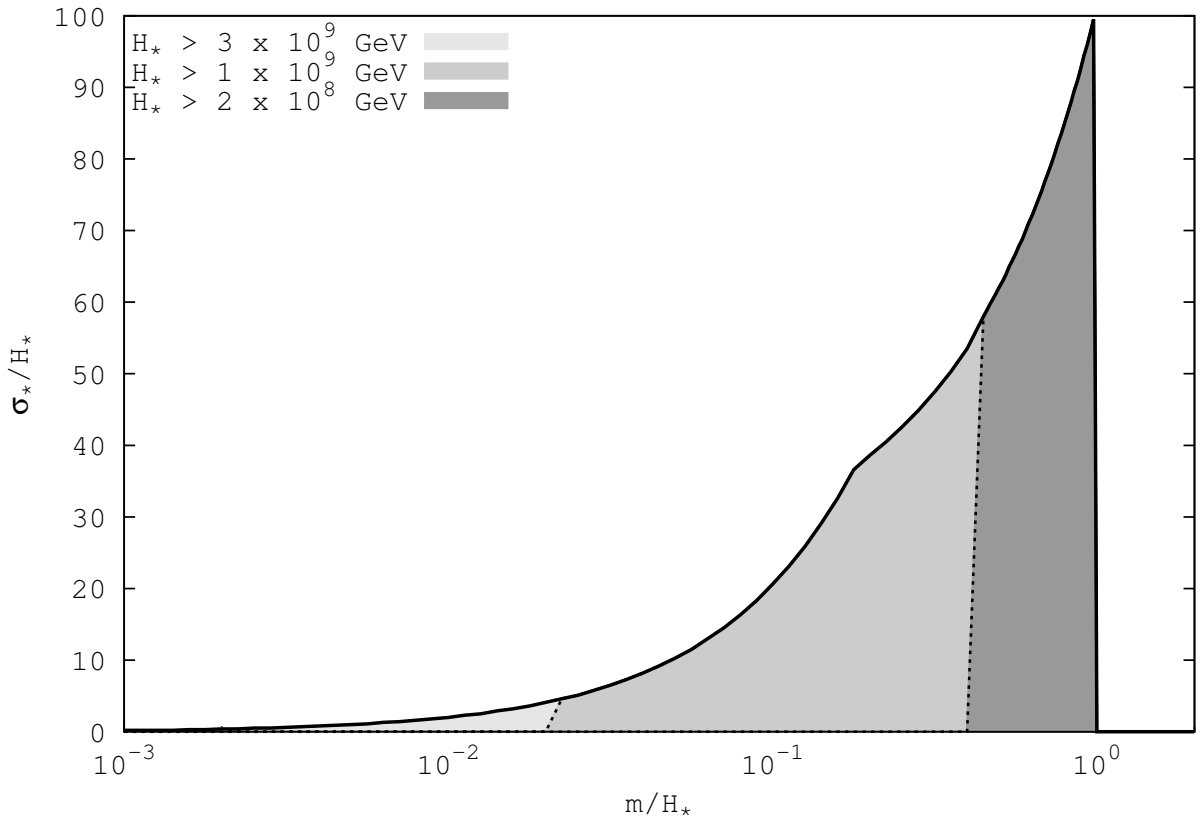
Resonance Parameter



- Strong resonance ($q_* \gg 1$): Lattice simulations needed

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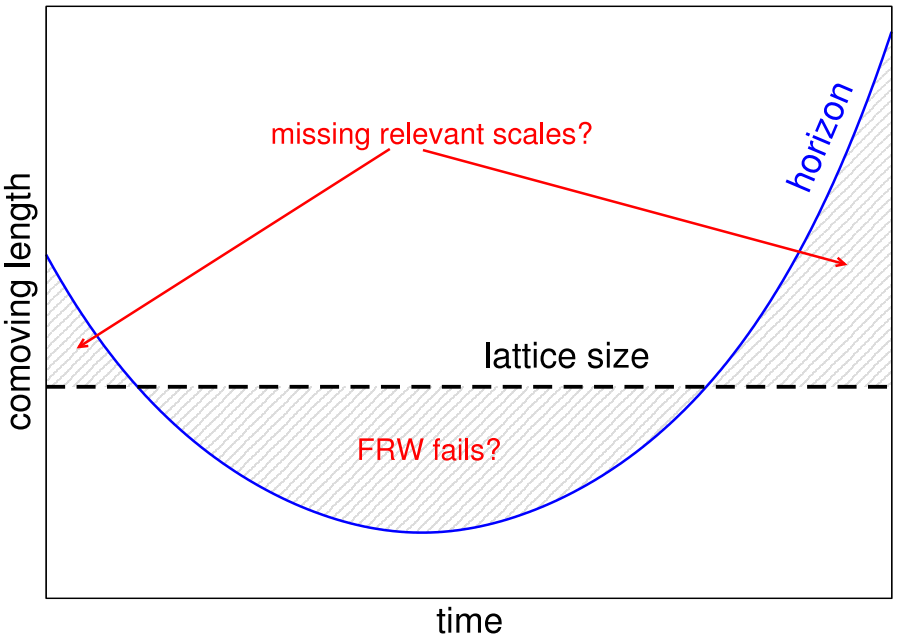
Quantitative Constraints



- Assume the curvaton is the only source of perturbations: Has to have the observed amplitude $\zeta \sim 10^{-5}$

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Caveat with FRW



- Finite-volume error when long wavelengths dominate
- Dynamical metric with Einstein eq (Bastero-Gil,AR&Szmigiel, in progress)
- No limitation on lattice size, in principle exact

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Summary

- Light scalar + non-equilibrium physics $\Rightarrow$ Perturbations
  - Separate universes + lattice field theory
- Massless preheating, Curvaton resonance, Electrically charged curvaton:
  - Possibly observable effects $\zeta \sim 10^{-5}$
  - Highly non-Gaussian
- Realistic particle physics models
  - Higgs, SUSY scalars?
  - Potentially testing physics far beyond the Standard Model