





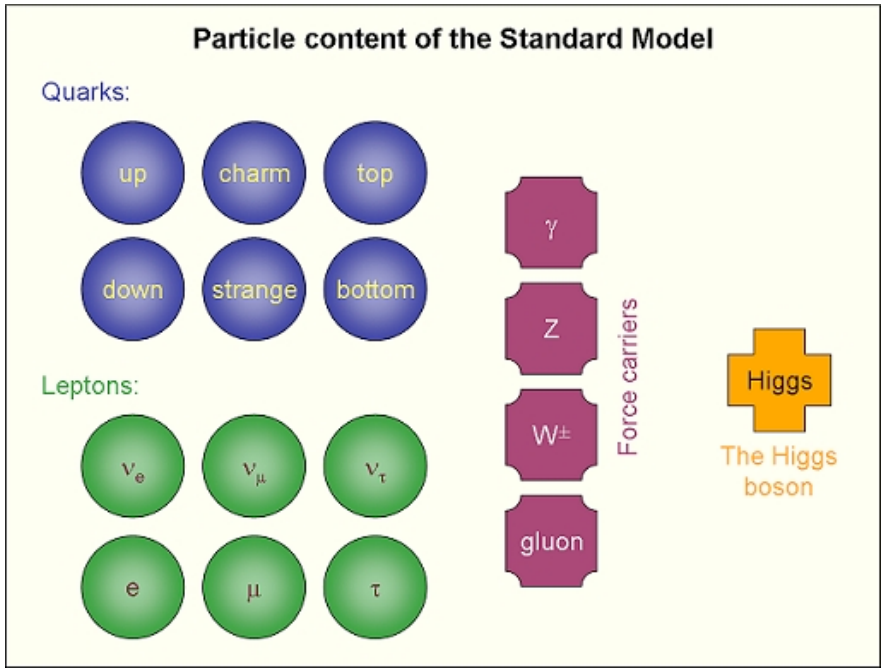
Simulating the standard model

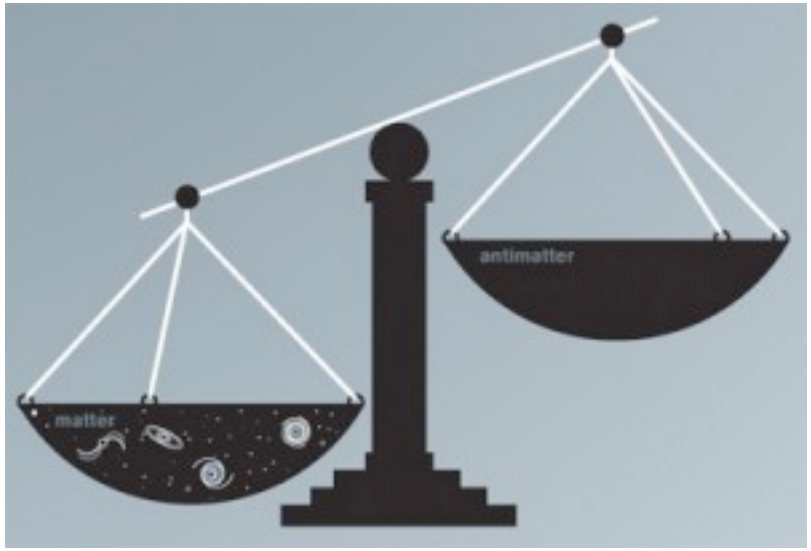
Paul Saffin

University of Nottingham

In collaboration with Anders Tranberg, Zong-Gang Mou

Main aim

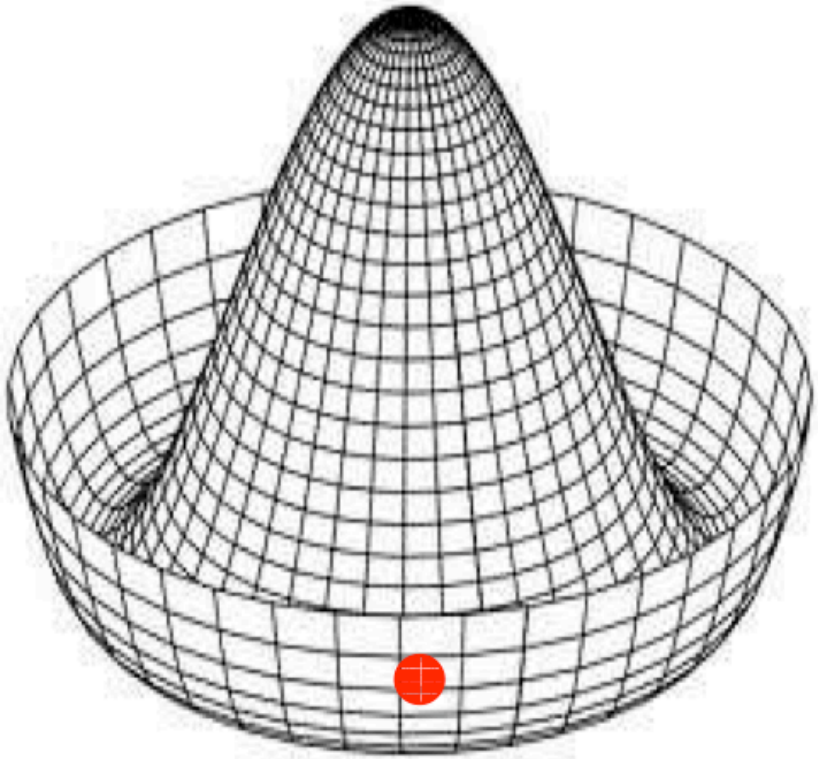

Particle content of the Standard Model
Quarks:
up
charm
top
down
strange
bottom
Leptons:
ν_e
ν_μ
ν_τ
e
μ
τ
γ
Z
W±
gluon
Force carriers
Higgs
The Higgs boson


antimatter
matter

outline of talk

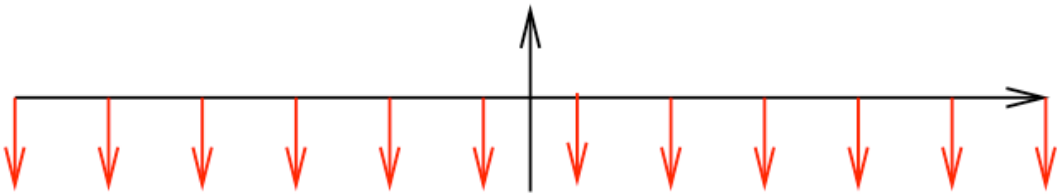
- introduction to main problem
- dynamical fermions on a lattice
    - mode functions
    - ensemble averages
- 1+1 anomaly analogue
- 3+1 lattice EW anomaly
- conclusions

Creating matter in 1+1



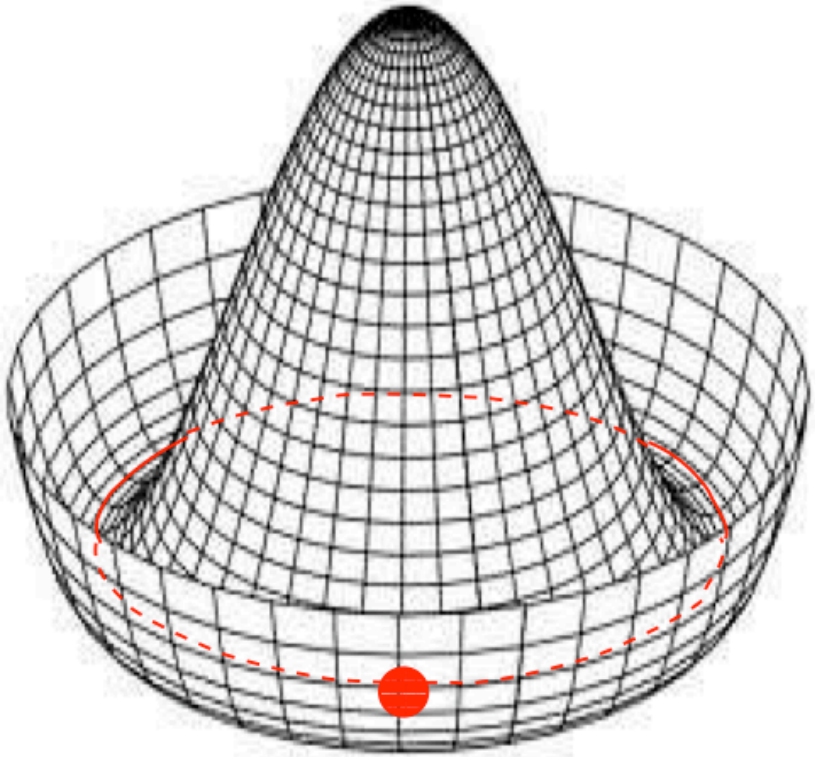
$$(\phi, \psi, A_\mu)$$

N=0 vacuum



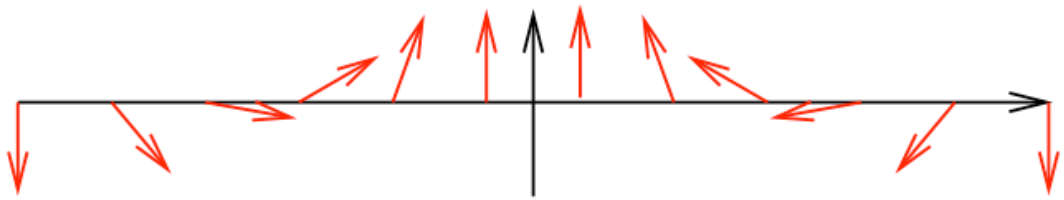
$$\mathcal{L} = -D_\mu \phi^\dagger D^\mu \phi - \bar{\psi}\gamma^\mu(\partial_\mu + iA_\mu\gamma^5)\psi - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + \ldots$$

Creating matter in 1+1



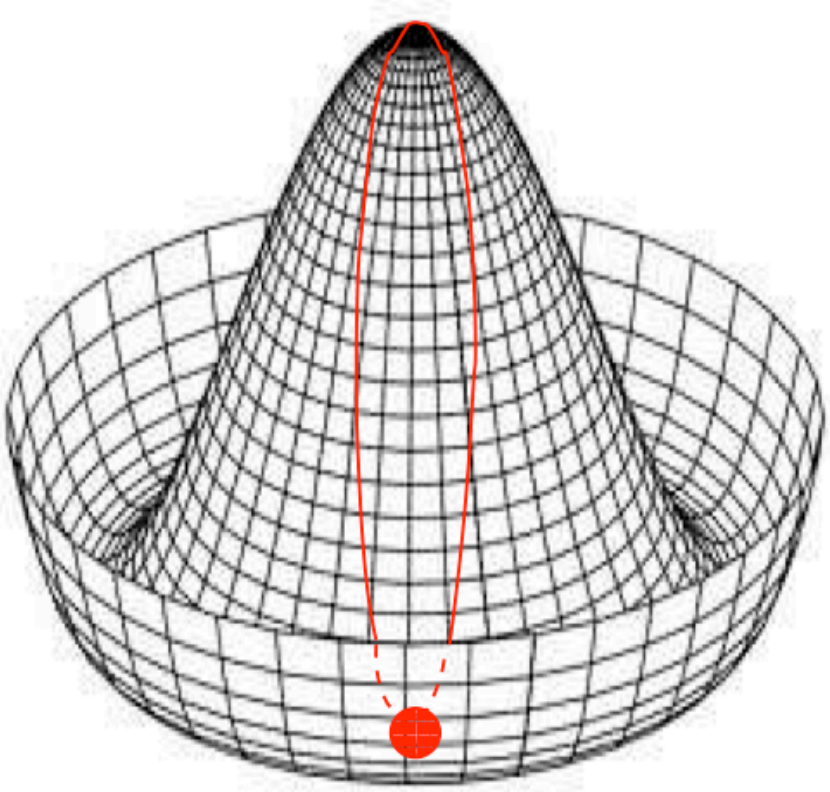
$$(\phi, \psi, A_\mu)$$

N=1 vacuum



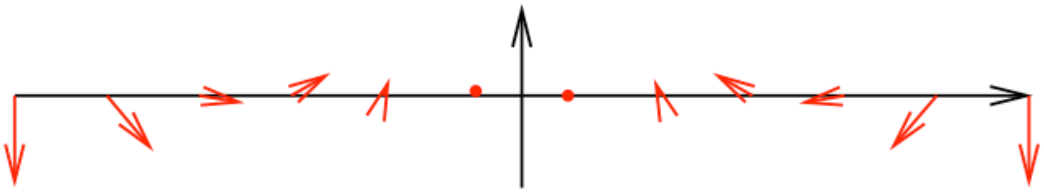
$$\mathcal{L} = -D_\mu \phi^\dagger D^\mu \phi - \bar{\psi}\gamma^\mu(\partial_\mu + iA_\mu\gamma^5)\psi - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + \ldots$$

Creating matter in 1+1



$$(\phi, \psi, A_\mu)$$

N=1/2 sphaleron



$$\mathcal{L} = -D_\mu \phi^\dagger D^\mu \phi - \bar{\psi}\gamma^\mu(\partial_\mu + iA_\mu\gamma^5)\psi - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + \ldots$$

Creating matter

$$\partial_\mu j^\mu = \frac{1}{4\pi}\epsilon^{\mu\nu}F_{\mu\nu}$$ 1+1 dimensions

$$\partial_\mu j^\mu = \frac{1}{32\pi^2}\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}W^a_{\mu\nu}W^{a\rho\sigma}$$ 3+1 dimensions



Creating anti-matter

$$\partial_\mu j^\mu = \frac{1}{4\pi}\epsilon^{\mu\nu}F_{\mu\nu}$$ 1+1 dimensions

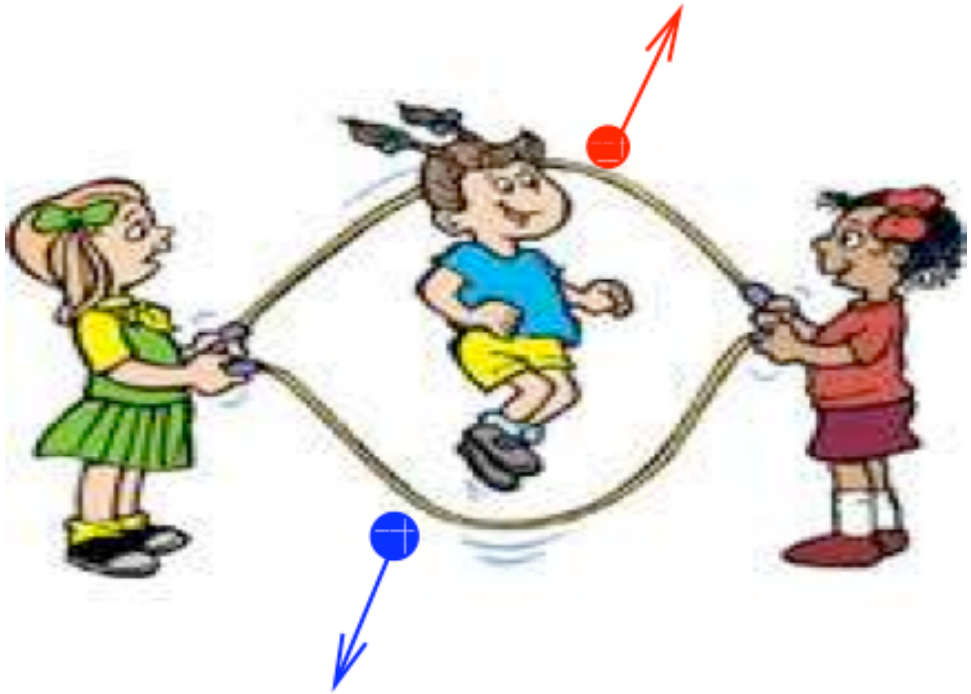
$$\partial_\mu j^\mu = \frac{1}{32\pi^2}\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}W^a_{\mu\nu}W^{a\rho\sigma}$$ 3+1 dimensions



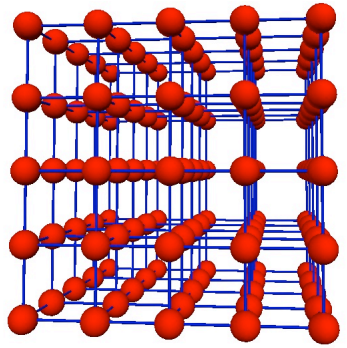
Creating matter

$$\partial_\mu j^\mu = \frac{1}{4\pi}\epsilon^{\mu\nu}F_{\mu\nu}$$ 1+1 dimensions

$$\partial_\mu j^\mu = \frac{1}{32\pi^2}\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}W^a_{\mu\nu}W^{a\rho\sigma}$$ 3+1 dimensions



Fermions on a lattice



Inherently quantum mechanical:

$$\Psi = \sum_{\alpha=1}^{\alpha=N_{mode}} \hat{b}_\alpha U_\alpha + \hat{d}_\alpha^\dagger V_\alpha$$

Must evolve lots of mode functions:

$$\alpha \to \underline{p} \Rightarrow N_{mode} = (n_x)^d$$

e.g. $32^3 = 32,768$ $64^3 = 262,144$

Equations of motion:

$$e^{i\Gamma[\bar{\phi},\phi]} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{iS[\bar{\phi},\phi,\bar{\psi}\psi]} \qquad \frac{\delta\Gamma}{\delta\bar{\phi}} = 0$$

$$\gamma^\mu \partial_\mu \psi + m(\phi,\bar{\phi})\psi = 0$$

$$-\Box\phi + V' + gTr(D(x,x)) + g'Tr(D(x,x)\gamma^5) = 0$$

$$D^<(x,y)_{\alpha\beta} = \langle \bar{\psi}_\beta(y)\psi_\alpha(x)\rangle$$
$$D^>(x,y)_{\alpha\beta} = \langle \psi_\alpha(x)\bar{\psi}_\beta(y)\rangle$$

$$D(x,y)_{\alpha\beta} = \frac{1}{2}\left(D^>(x,y)_{\alpha\beta} - D^<(x,y)_{\alpha\beta}\right)$$

Equations of motion:

$$e^{i\Gamma[\bar{\phi},\phi]} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{iS[\bar{\phi},\phi,\bar{\psi}\psi]} \qquad \frac{\delta\Gamma}{\delta\bar{\phi}} = 0$$

$$\gamma^{\mu}\partial_{x\mu}D(x,y) + m(\phi(x),\bar{\phi}(x))D(x,y) = 0$$

$$-\Box\phi + V' + gTr(D(x,x)) + g'Tr(D(x,x)\gamma^5) = 0$$

Fermion initial conditions:

In the vacuum use

$$\Psi = \sum_p \hat{b}_p U_p e^{ip.x} + \hat{d}_p^\dagger V_p e^{-ip.x}$$

$$b|0\rangle = 0, \; d|0\rangle = 0$$

$$D(x,y) = \int_p \mathcal{M}\gamma^0 e^{i(x-y).p} \qquad \mathcal{M} = \frac{1}{\omega}(p.\gamma + m)\gamma^0$$

Fermion initial conditions:

$$\text{“}\mathcal{M} = \langle|\psi(p)\psi^\dagger(q)|\rangle\text{”}$$

eigenVal($\mathcal{M}$) = +1,+1,-1,-1

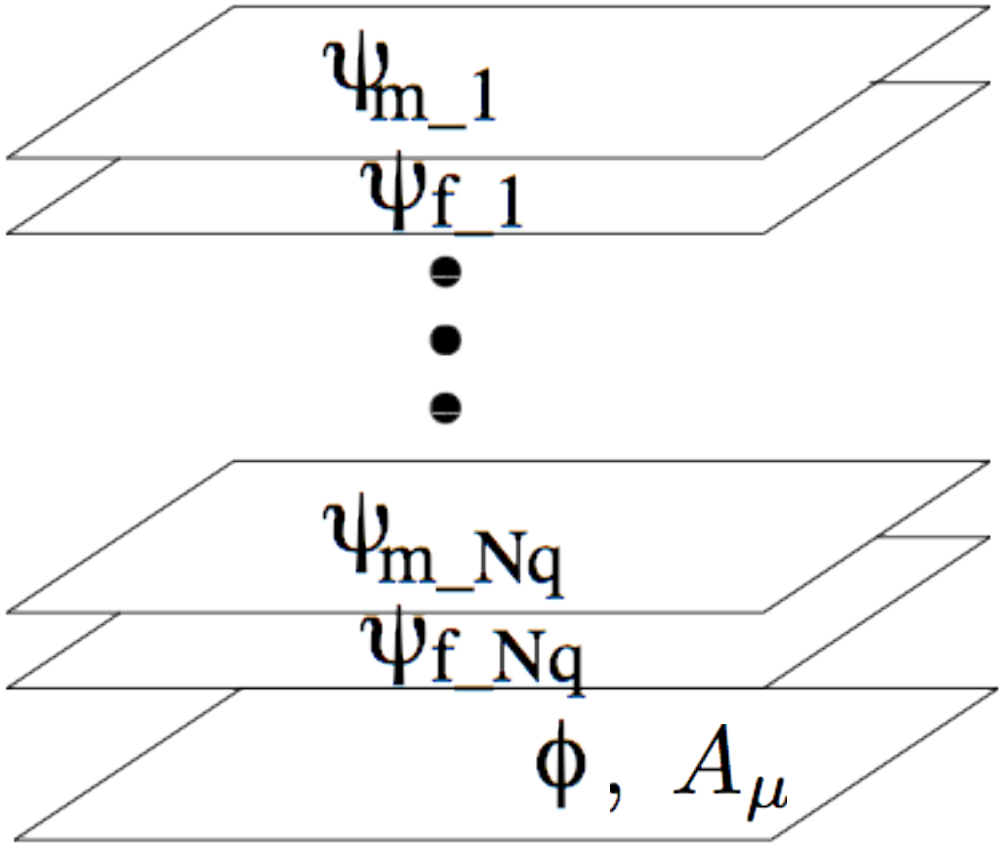
$$\mathcal{M} = \langle\psi_m(p)\psi_f^\dagger(q)\rangle_e$$

$$\psi_{m,f} = \sum_p \xi_p U_p e^{ip.x} \pm \eta_p^\star V_p e^{-ip.x}$$

$$\langle\xi(p)\xi^\star(p')\rangle_e = \delta(p-p')$$

$$\langle\eta(p)\eta^\star(p')\rangle_e = \delta(p-p')$$

Borsanyi, Hindmarsh, Phys. Rev. D79, (2009), 045022



- Nq pairs of c-number fermions
- each gender satisfies Dirac equation
- $D(x,y) = \langle \psi_m(x)\bar{\psi}_f(y)\rangle_e$
- if Nq < (n_x)^3 we have won



doublers

- Wilson in space

$$\mathcal{L}_W = \frac{1}{2} r_1 a \bar{\psi} D_x D_x \psi$$

- staggered in time

Aarts and Smit, Nucl. Phys. B555 (1999) 355

1+1 anomaly analogue

$$(\phi,\ \psi,\ A_\mu)$$

$$D_\mu\phi = (\partial_\mu - iA_\mu)\phi \qquad D_\mu\psi = (\partial_\mu + iA_\mu\gamma^5)\psi$$

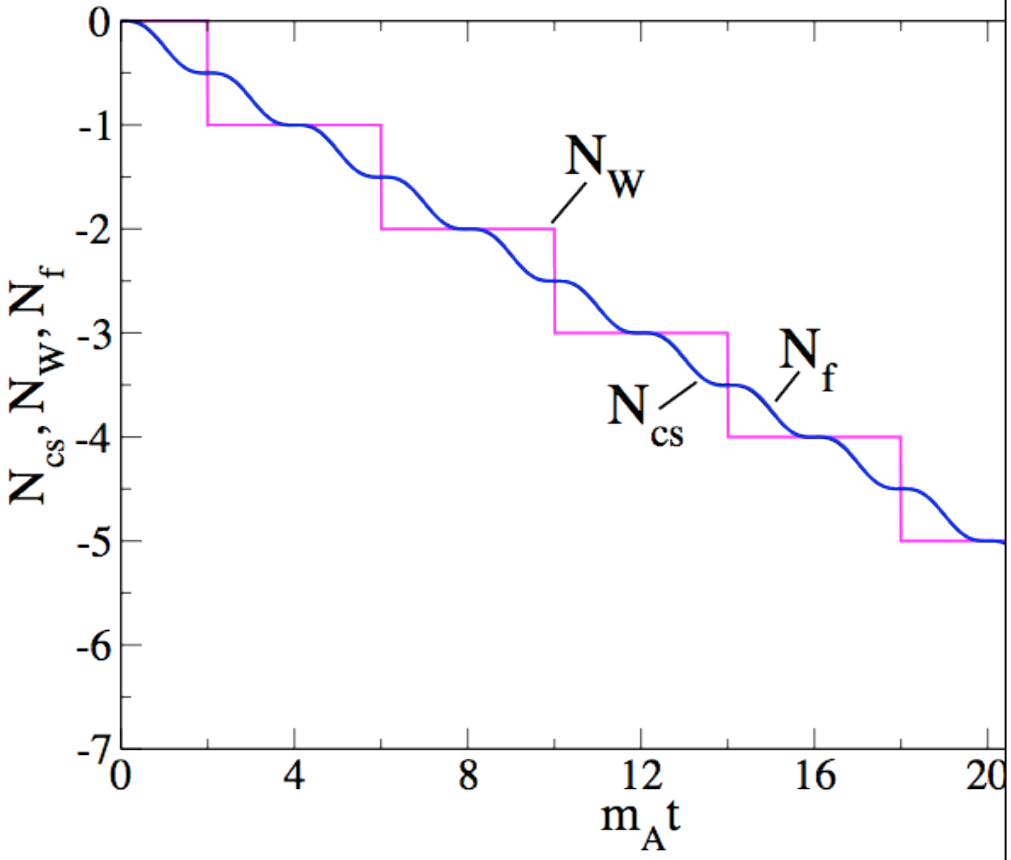
$$\partial_\mu j_5^\mu = \frac{1}{4\pi}\epsilon^{\mu\nu}F_{\mu\nu} = \partial_\mu C^\mu$$

$$Q = \int dx\ j_5^0$$

$$N_{CS} = \int dx\ C^0 = -\frac{1}{2\pi}\int dx\ A_x$$

$$N_W = \frac{1}{2\pi}\int dx\ \partial_x\theta$$

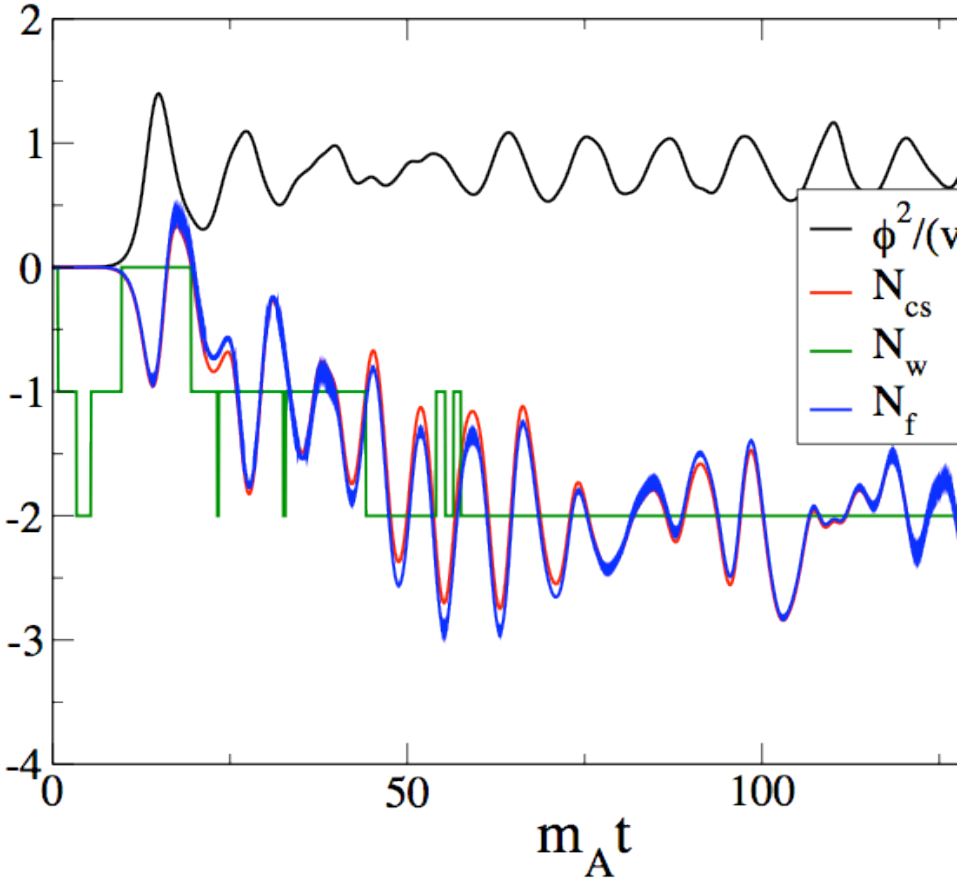
1+1 anomaly analogue



Fermion reaction to
Hand-made sphalerons

nx=128
Nq=90

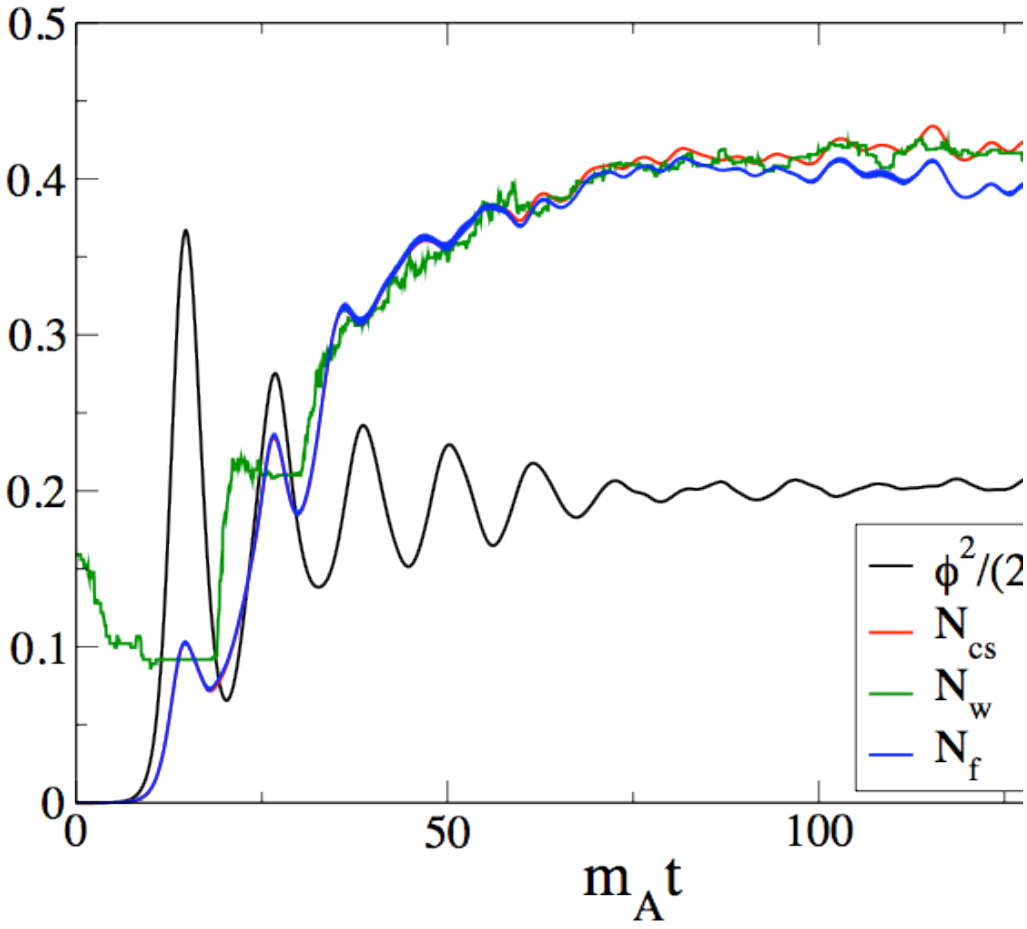
1+1 anomaly analogue



Full dynamics

- nx=256
- G/e=0.1
- Nq=2430

1+1 anomaly analogue



Full dynamics + C(P) violation

$$\mathcal{L}_\kappa = \frac{\kappa}{4\pi}\phi^\star\phi\epsilon_{\mu\nu}F^{\mu\nu}$$

- nx=256
- G/e=0.1
- Nq=2430
- k=0.04
- 64+64* simulations

3+1, EW, one family

$$\left\{W_\mu^a,\ q_L=(u_L,d_L),\ u_R,\ d_R,\ l_L=(\nu_L,e_L),\ e_R,\ \nu_R\right\}$$

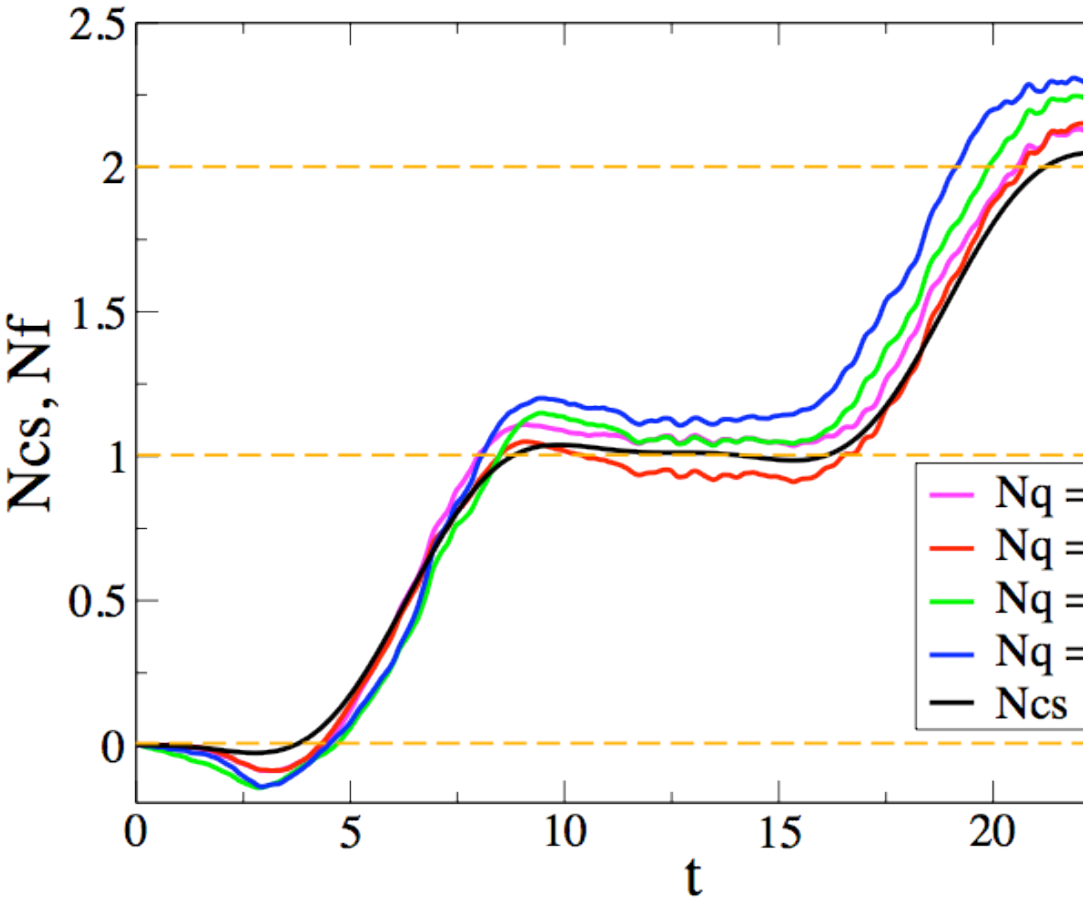
$$\mathcal{L}=-\bar{q}_L\gamma^\mu D_\mu q_L-\bar{u}_R\gamma^\mu D_\mu u_R+\ldots$$

$$\mathcal{L}_Y=G^u\bar{q}_L\phi u_R+\ldots$$

$$D_\mu q_L=\left(\partial_\mu-i\frac{g}{2}\sigma^a W_\mu^a\right)q_L \qquad D_\mu u_R=\partial_\mu u_R$$

$$D_\mu\phi=\left(\partial_\mu-i\frac{g}{2}\sigma^a W_\mu^a\right)\phi$$

3+1, EW, one family

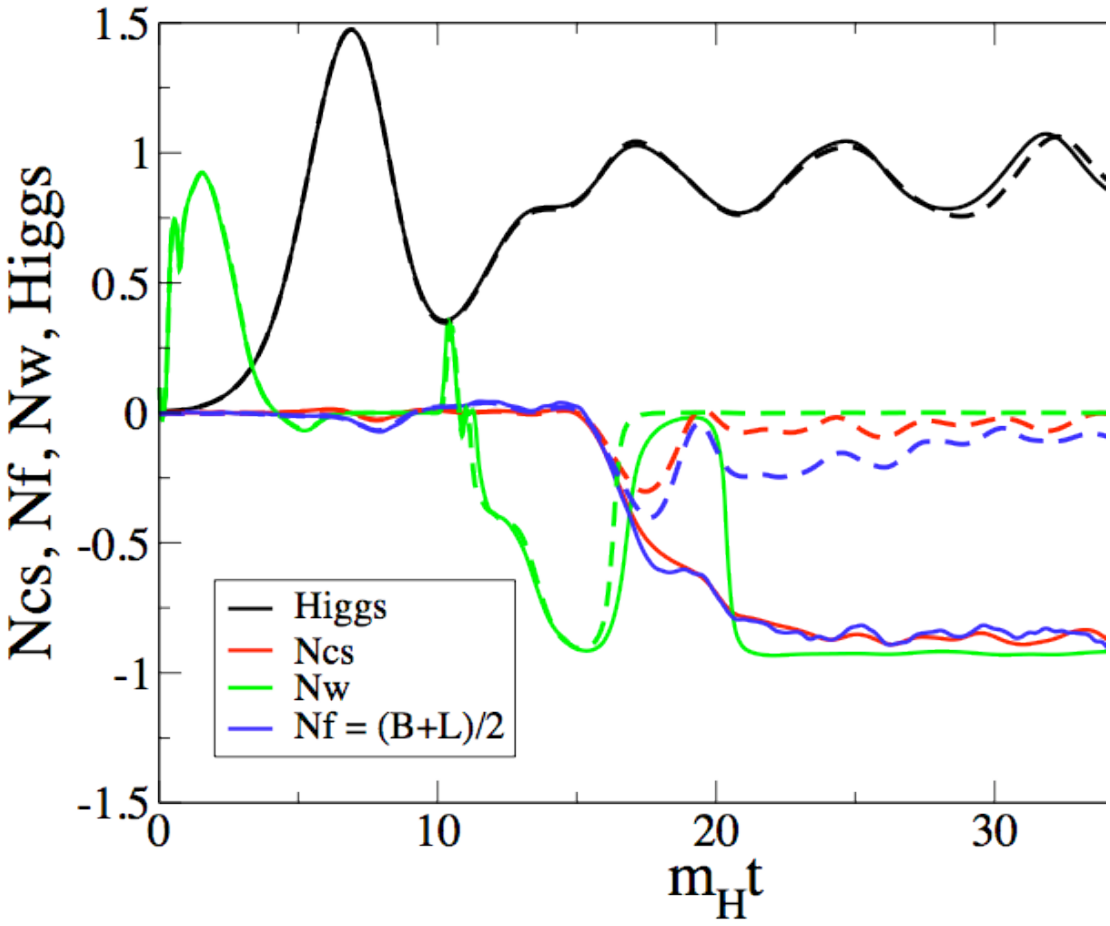


Fermion reaction to hand-made sphalerons

- am=0.42
- Nx^3=32^3=32,768

- Nq=10240
- Nq=2560
- Nq=1280
- Nq=640

3+1, EW, one family



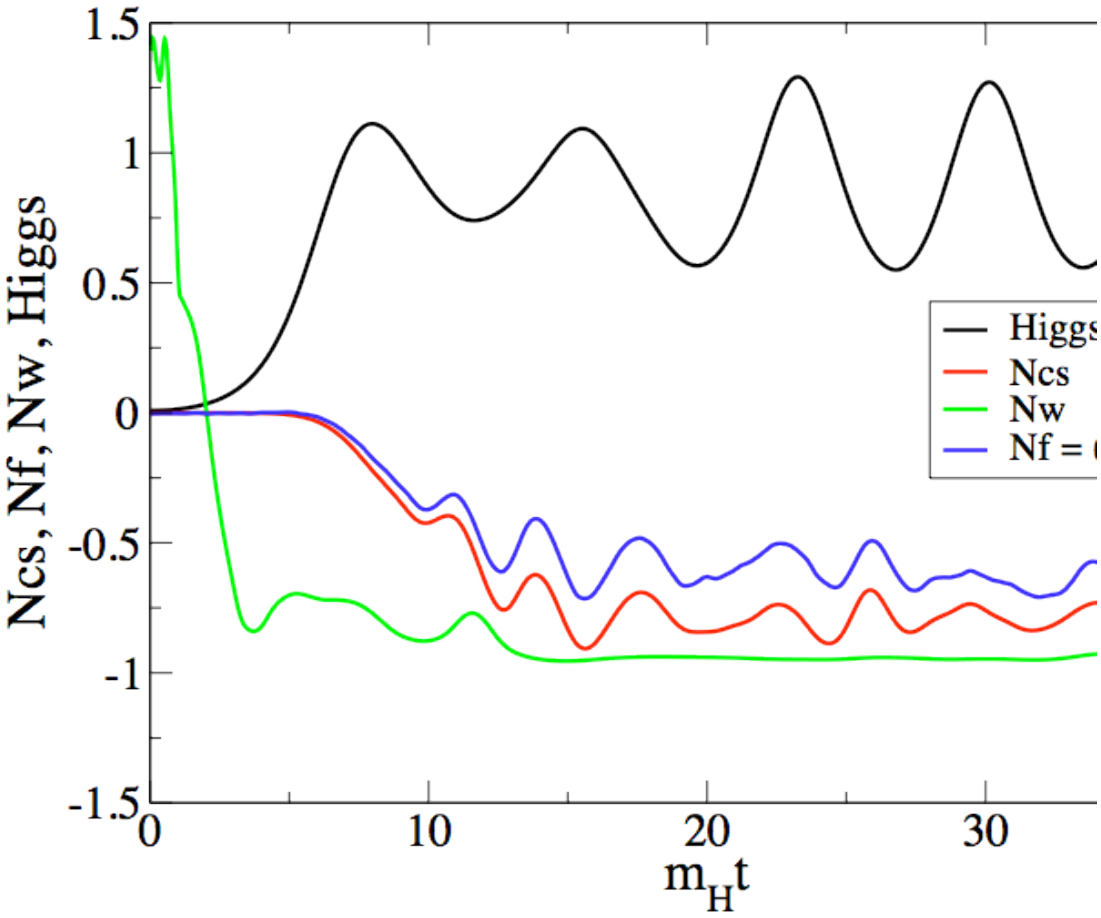
With vs without back-reaction

- Nq=5,120
- Nx=32
- no Yukawa

Conclusions and to-do list

- Can simulate dynamical fermions cheaply
- Simulations reproduce quantum anomaly
- Other observables
- Generate an asymmetry
    - two-Higgs doublet model
    - three generations, CKM
- Bubble-fermion dynamics
- Include U(1) - magnetic fields

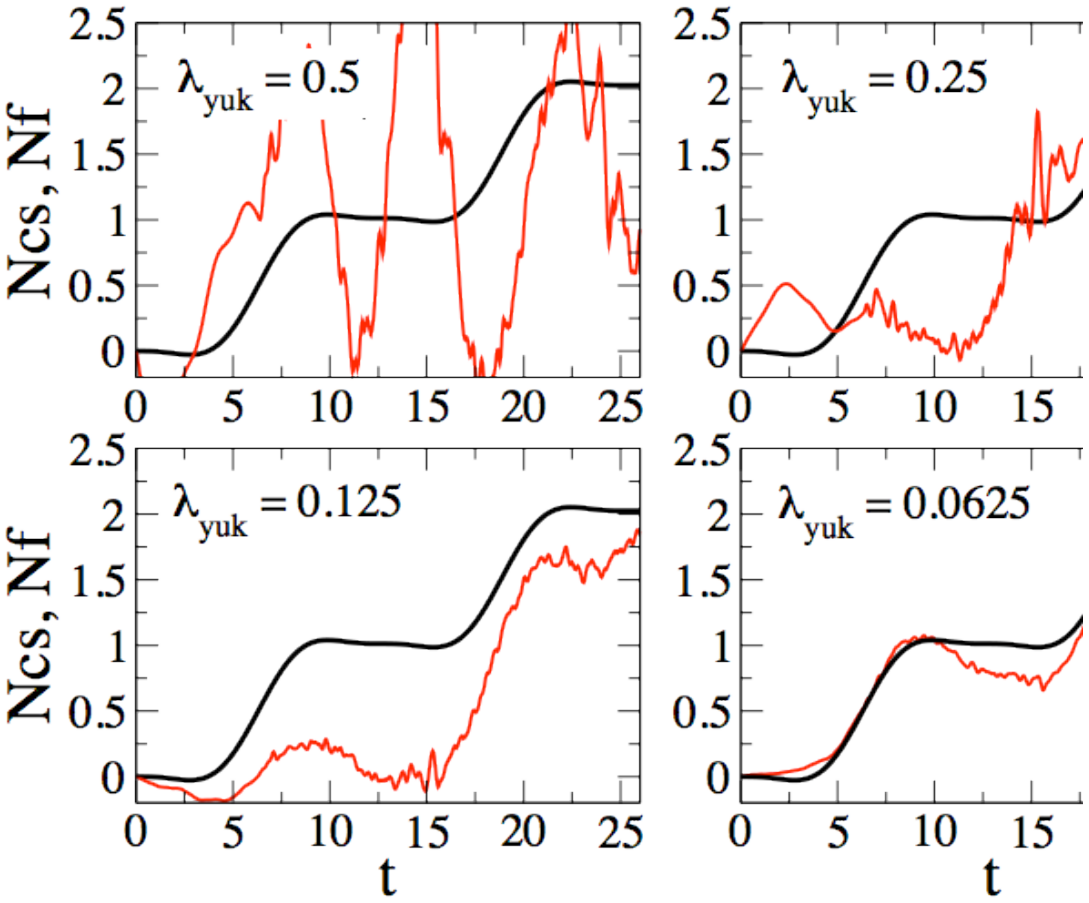
3+1, EW, one family



full simulation

- Nq=10,240
- Nx=32
- mf~1.4GeV

3+1, EW, one family



Yukawa dependence

- Nq=10,240
- Nx=32
- mf=87,44,22,11GeV