Anisotropic Hydrodynamics

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Strong and Electroweak Matter 2012
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References:
arXiv:1204.1473 (M. Martinez, R. Ryblewski, MS)
arXiv:1204.2624 (W. Florkowski and R. Ryblewski)
arXiv:1011.3056 (M. Martinez and MS)
arXiv:1007.0889 (M. Martinez and MS)







Motivation

- Hydro for heavy ion collisions (HIC) at RHIC and LHC
- Cannot apply viscous hydrodynamics too early after initial nuclear impact
- Large corrections to ideal energy-momentum tensor due to rapid longitudinal expansion → momentum-space anisotropy
- Corrections grow rapidly as η/S increases
- Also breaks down near transverse and longitudinal edges (forward rapidity) where the system is dilute ~ longitudinal free streaming
- How can we improve things to make dynamical models more phenomenologically reliable for HIC?

Anisotropic Plasma

$$f(\tau,\mathbf{x},\mathbf{p}) = f_{\rm RS}(\mathbf{p},\xi(\tau,\mathbf{x}),p_{\rm hard}(\tau,\mathbf{x})) = f_{\rm iso}([\mathbf{p}^2+\xi(\tau,\mathbf{x})p_z^2]/p_{\rm hard}^2(\tau,\mathbf{x}))$$

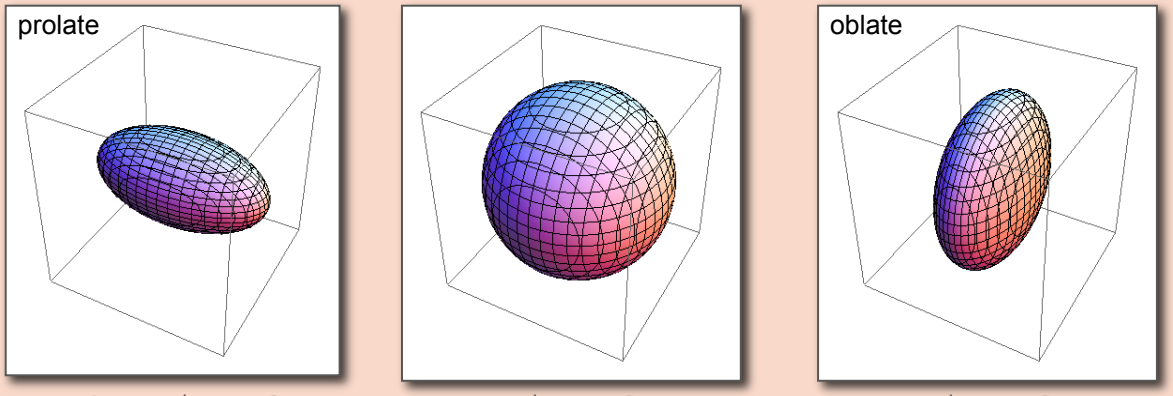
$$\xi = \frac{\langle p_T^2\rangle}{2\langle p_L^2\rangle} - 1$$

Small Anisotropy Limit (Thermal f_{iso})

$$f \approx f_{\rm iso}(p)\left[1-\xi\frac{p_z^2}{2\,p_{\rm hard}\,p}(1\pm f_{\rm iso}(p))\right]$$

Anisotropy parameter, ξ, is related to pressure anisotropy of the system.

$P_{\rm L} > P_{\rm T}$ | $P_{\rm L} = P_{\rm T}$ | $P_{\rm L} < P_{\rm T}$

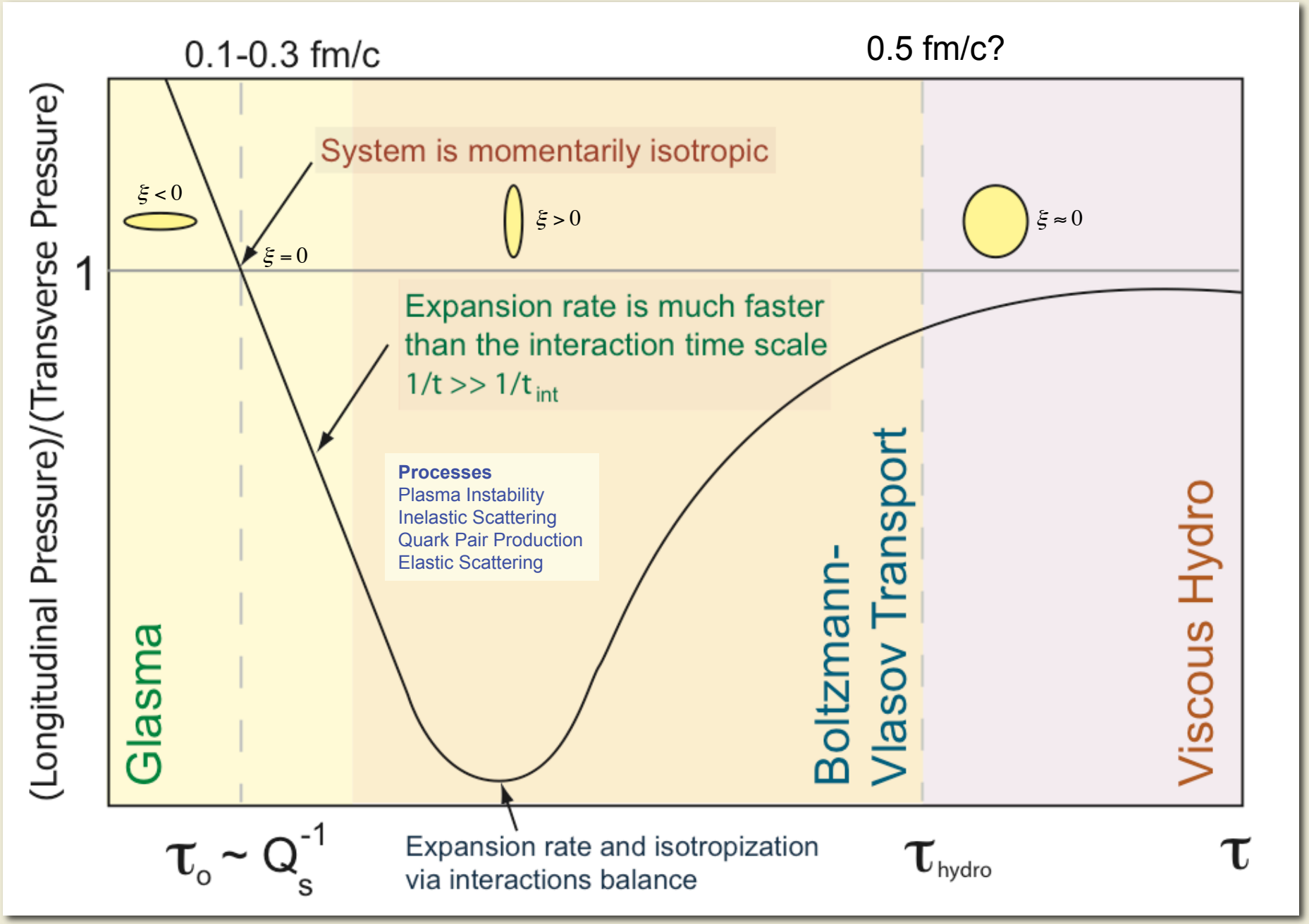


$-1 < \xi < 0$ | $\xi = 0$ | $\xi > 0$

Navier-Stokes Limit

$$\xi_{\rm NS} = \frac{10}{T\tau}\frac{\eta}{\mathcal{S}} \simeq \frac{10}{T_0\tau^{2/3}}\frac{\eta}{\mathcal{S}}$$

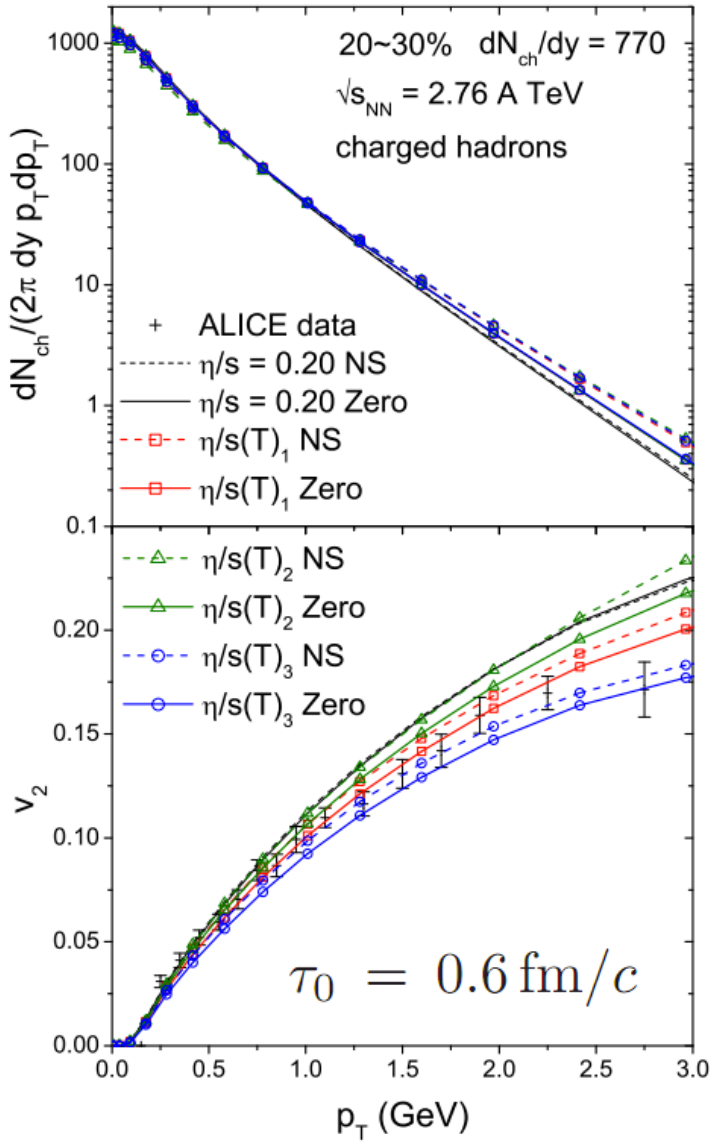
QGP momentum anisotropy



Radial and elliptic flow in Pb+Pb collisions at the Large Hadron Collider from viscous hydrodynamics

Chun Shen,[1,*] Ulrich Heinz,[1,†] Pasi Huovinen,[2,‡] and Huichao Song[3,§]

arXiv:1105.3226v2 [nucl-th] 9 Sep 2011



η/s model	$\pi_0^{\mu\nu}$	s_0 (fm^{-3})	T_0 (MeV)
$\eta/s = 0.2$	0	191.6	427.9
	NS	172.4	413.9
$(\eta/s)_1(T)$	0	179.6	419.2
	NS	119.3	368.7
$(\eta/s)_2(T)$	0	179.6	419.2
	NS	115.6	365.1
$(\eta/s)_3(T)$	0	175.2	416.0
	NS	116.6	366.1

NS = Navier Stokes

$$\xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1 \qquad \xi_{\rm NS} = \frac{10}{T\tau}\frac{\eta}{\mathcal{S}}$$

Using values from the paper I obtain

$$\xi_{0,\rm NS} \simeq 1.3$$

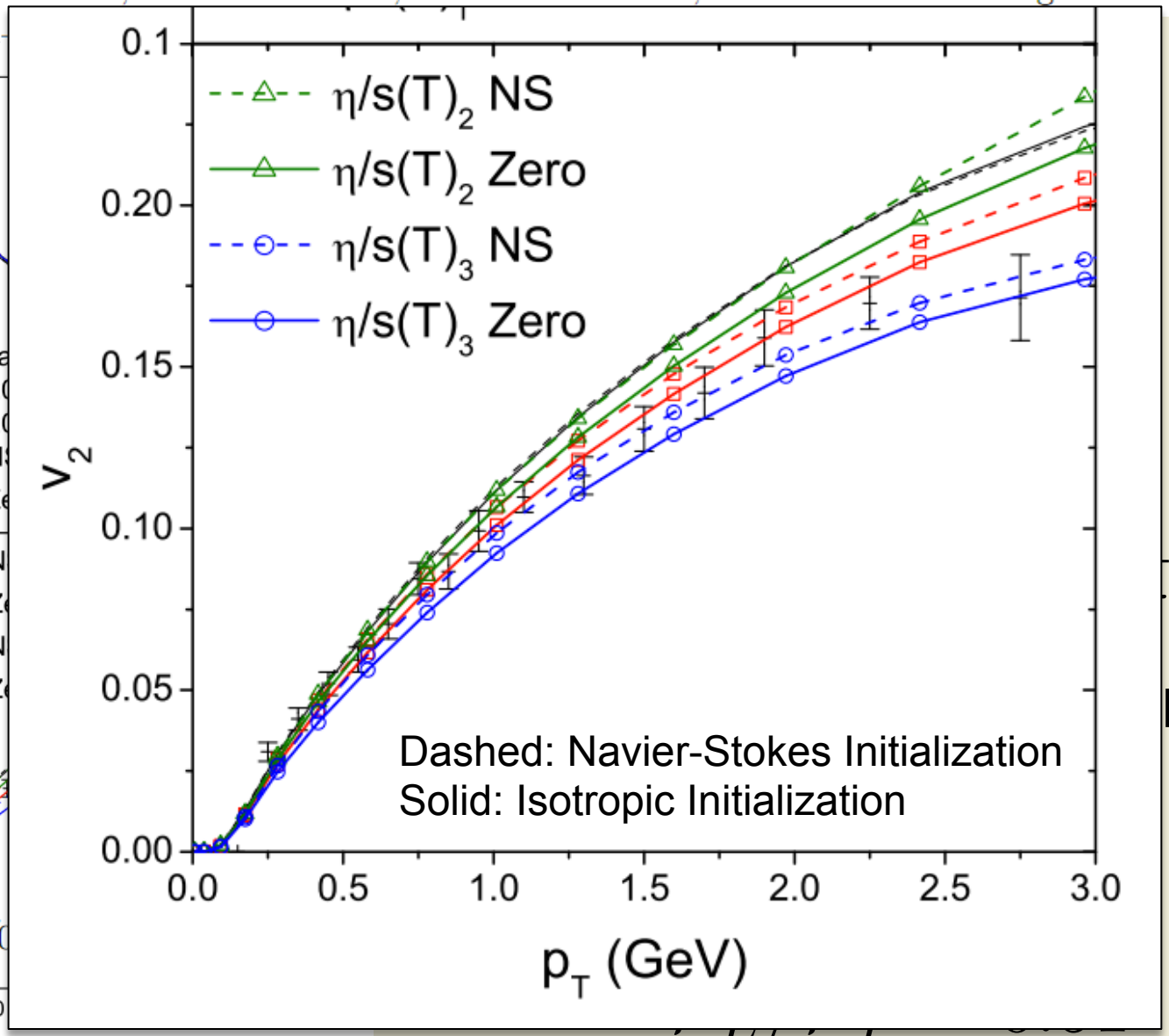
Which corresponds to

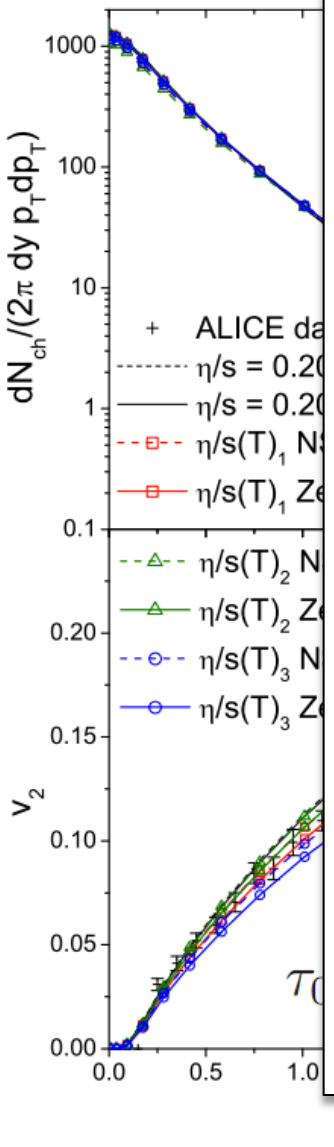
$$\mathcal{P}_L/\mathcal{P}_T \simeq 0.51$$

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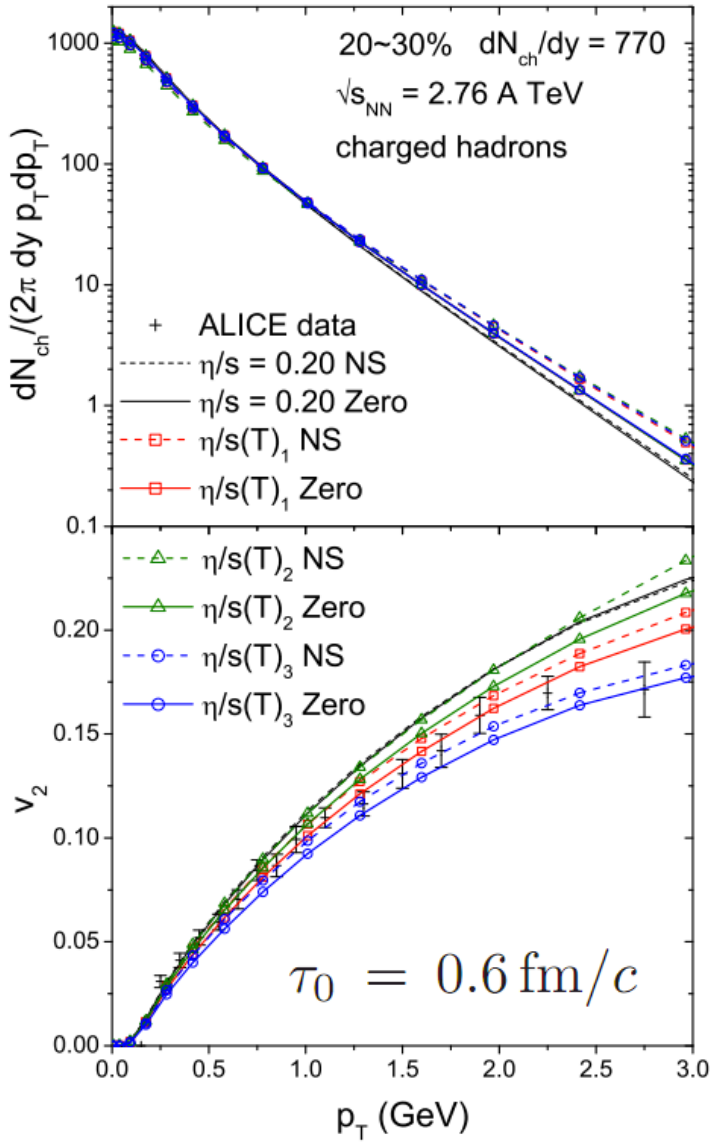
$\frac{\eta}{\mathcal{S}}$

btain

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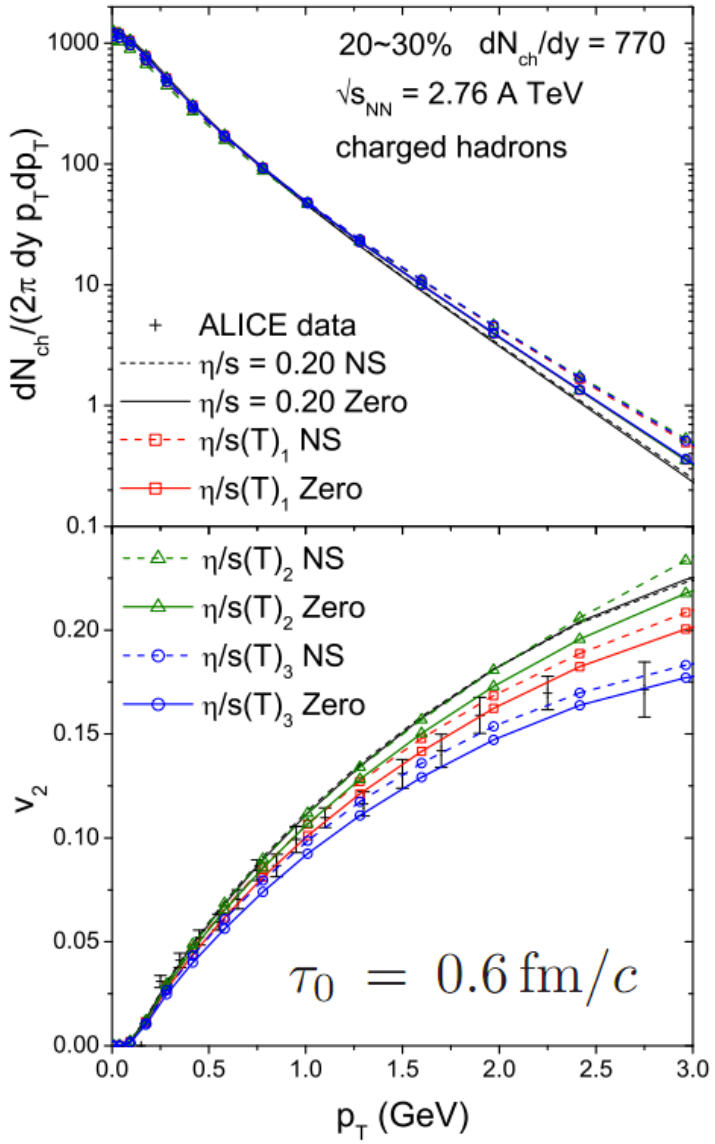
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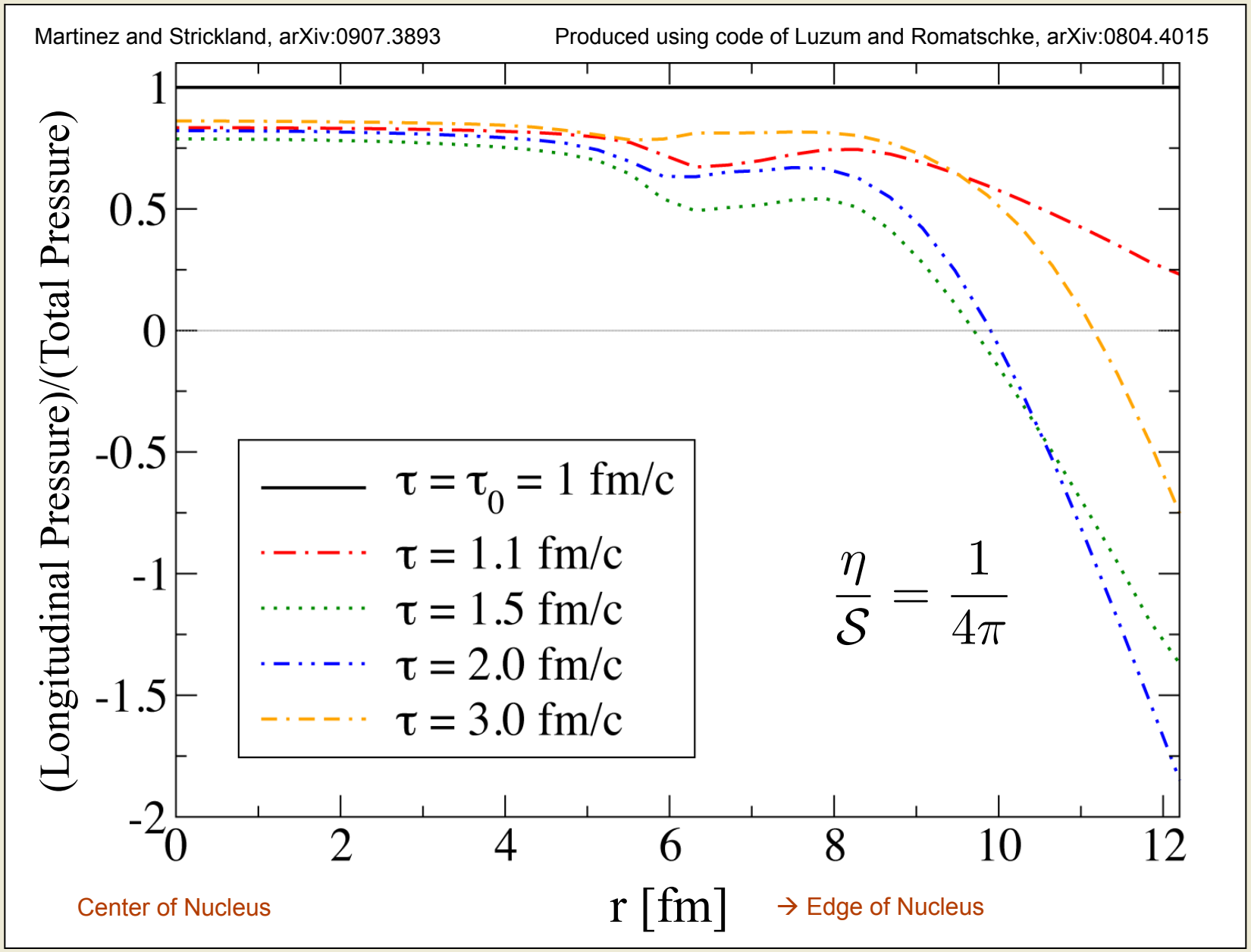
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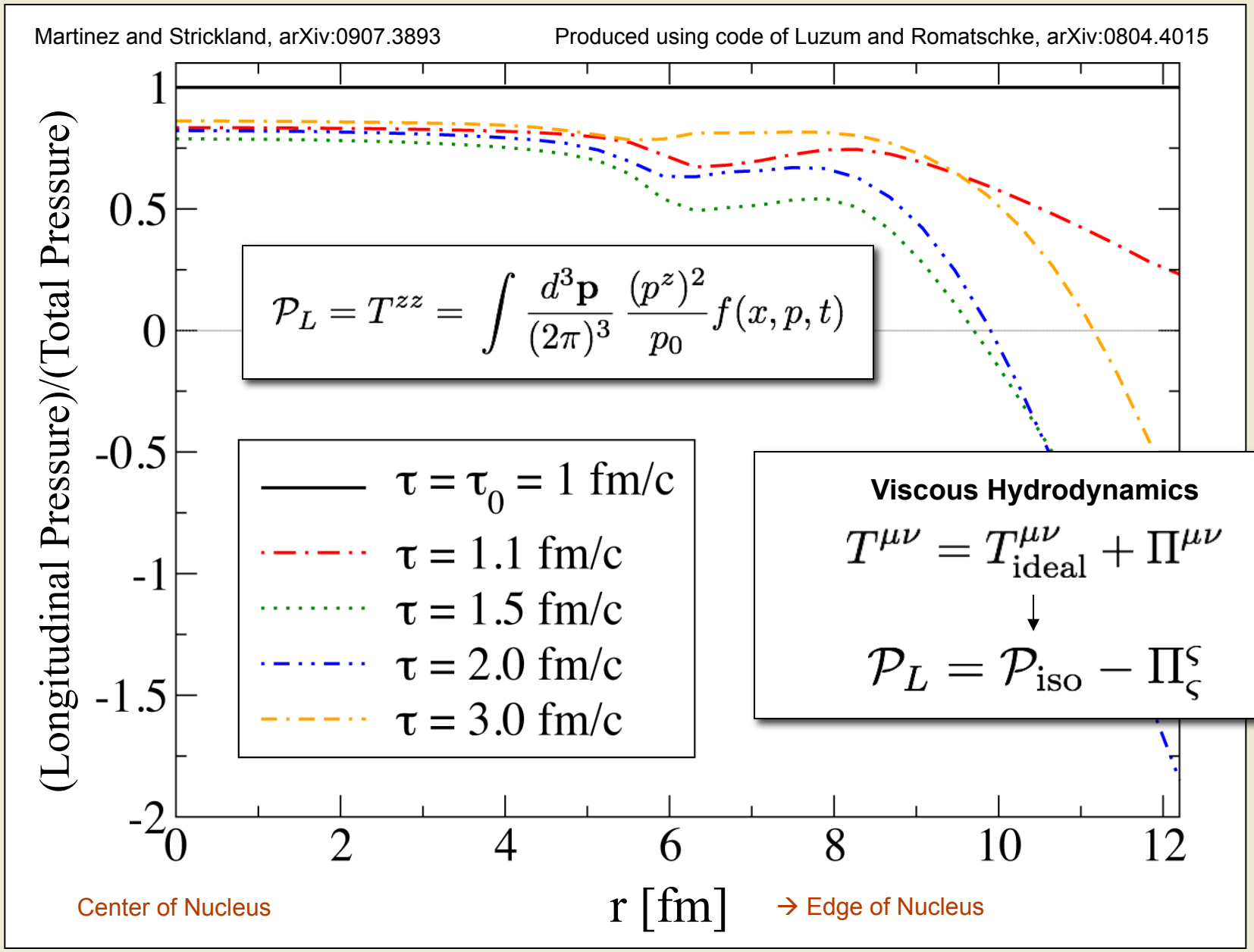
Viscous Hydro situation is even worse than this would make it seem. For Navier-Stokes at the initial time shown, the longitudinal pressure is negative.

$$T^{\mu\nu} = T^{\mu\nu}_{\rm ideal} + \Pi^{\mu\nu}$$
$$\downarrow$$
$$\mathcal{P}_L = \mathcal{P}_{\rm iso} - \Pi^{\varsigma}_{\varsigma}$$

Hydro Results - Strong Coupling

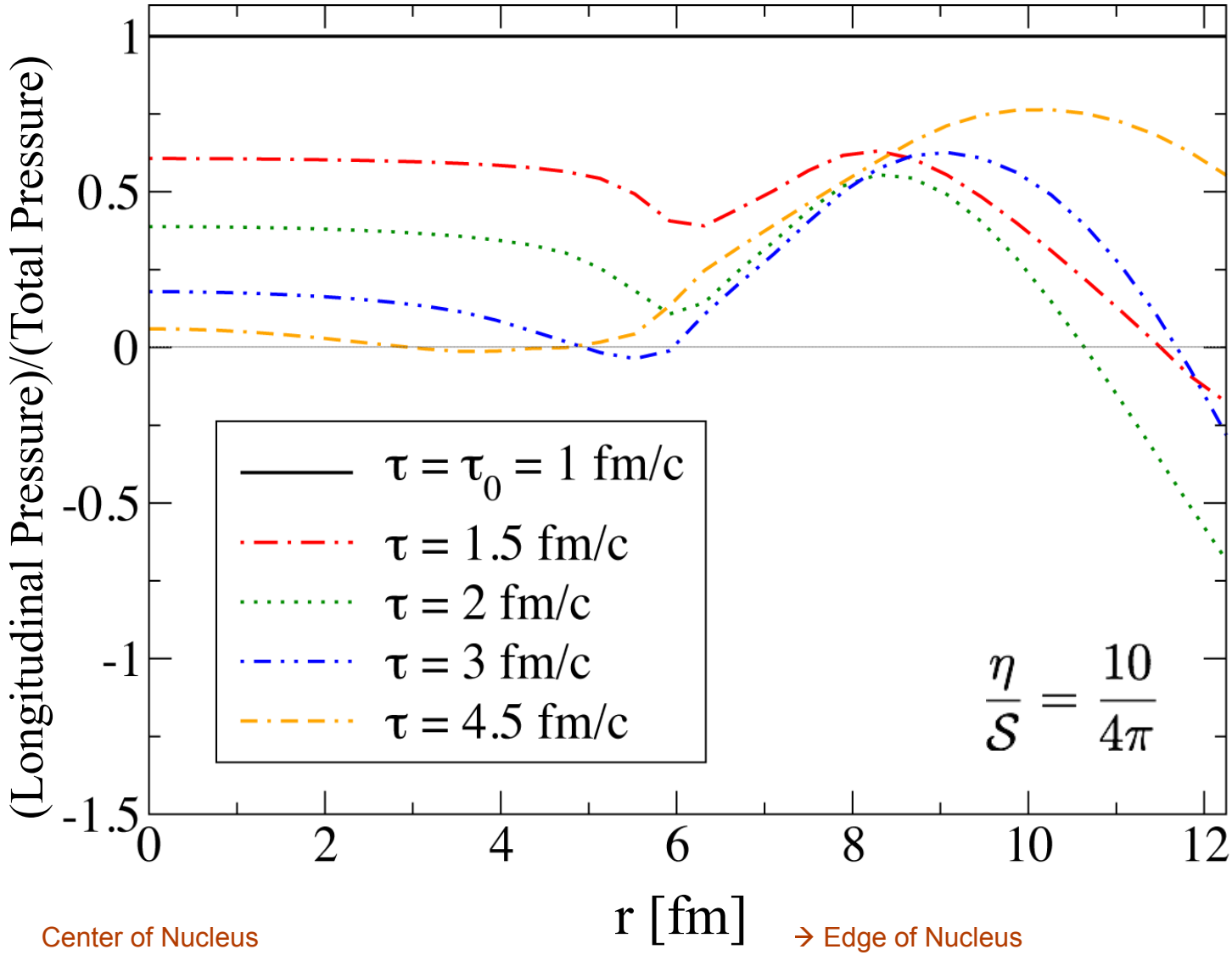


Hydro Results - Strong Coupling



Hydro Results - Weak Coupling

Martinez and Strickland, arXiv:0907.3893 Produced using code of Luzum and Romatschke, arXiv:0804.4015



Center of Nucleus → Edge of Nucleus

Start over from scratch

Viscous Hydrodynamics Expansion

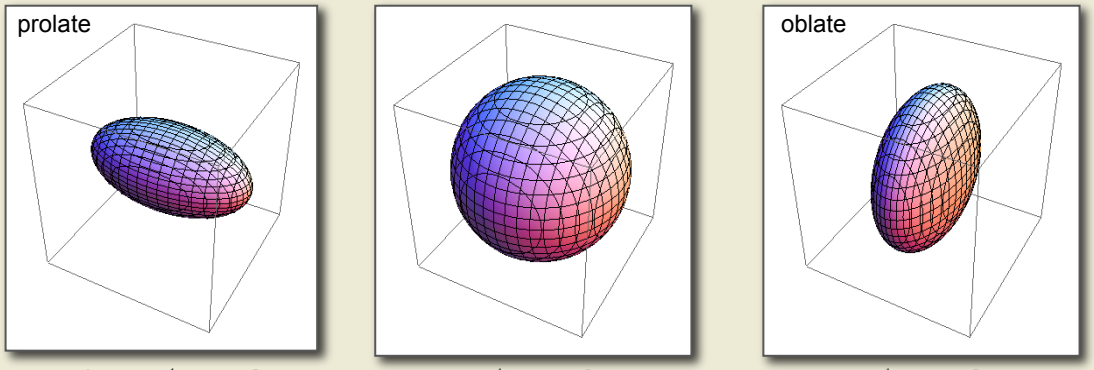
$$f(\tau, \mathbf{x}, \mathbf{p}) = f_{\rm eq}(|\mathbf{p}|, T(\tau, \mathbf{x})) + \delta f_1 + \delta f_2 + \cdots$$

Isotropic in momentum space

Anisotropic Hydrodynamics (aHydro) Expansion

$$f(\tau, \mathbf{x}, \mathbf{p}) = f_{\rm aniso}(\mathbf{p}, p_{\rm hard}(\tau, \mathbf{x}), \xi(\tau, \mathbf{x})) + \delta f_1' + \delta f_2' + \cdots$$

$$\xi = \frac{\langle p_T^2 \rangle}{2\langle p_L^2 \rangle} - 1$$



$-1 < \xi < 0$ $\quad \xi = 0$ $\quad \xi > 0$

Using relaxation-time approximation scattering kernel gives

0th Moment of Boltzmann EQ

$$\partial_\alpha N^\alpha \neq 0$$

$$\frac{1}{1+\xi}\partial_\tau \xi - \frac{2}{\tau} - \frac{6}{p_{\rm hard}}\partial_\tau p_{\rm hard} = 2\Gamma\left[1 - \mathcal{R}^{3/4}(\xi)\sqrt{1+\xi}\right]$$

1st Moment of Boltzmann EQ

$$\partial_\alpha T^{\alpha\beta} = 0$$

$$\frac{\mathcal{R}'(\xi)}{\mathcal{R}(\xi)}\partial_\tau \xi + \frac{4}{p_{\rm hard}}\partial_\tau p_{\rm hard} = \frac{1}{\tau}\left[\frac{1}{\xi(1+\xi)\mathcal{R}(\xi)} - \frac{1}{\xi} - 1\right]$$

where

$$\Gamma = \frac{2T(\tau)}{5\bar{\eta}} = \frac{2\mathcal{R}^{1/4}(\xi)p_{\rm hard}}{5\bar{\eta}}$$

$$\mathcal{R}(\xi) \equiv \frac{1}{2}\left(\frac{1}{1+\xi} + \frac{\arctan\sqrt{\xi}}{\sqrt{\xi}}\right)$$

$$\mathcal{E}(p_{\rm hard}, \xi) = \mathcal{R}(\xi)\mathcal{E}_{\rm iso}(p_{\rm hard})$$

Linearized Equations

If we expand the energy-momentum tensor, keep only the lowest order in the anisotropy parameter, and match to 2nd-order viscous hydro we find

$$\frac{\Pi}{\mathcal{E}_{\rm eq}} = \frac{8}{45}\xi + \mathcal{O}(\xi^2)$$

and similarly the coupled nonlinear differential equations reduce to

$$\partial_\tau \mathcal{E} = -\frac{\mathcal{E}+\mathcal{P}}{\tau} + \frac{\Pi}{\tau}$$

$$\partial_\tau \Pi = -\frac{\Pi}{\tau_\pi} + \frac{4}{3}\frac{\eta}{\tau_\pi \tau} - \frac{4}{3}\frac{\Pi}{\tau}$$

$$\Gamma = \frac{2}{\tau_\pi}$$

$$\tau_\pi = \frac{5}{4}\frac{\eta}{\mathcal{P}}$$

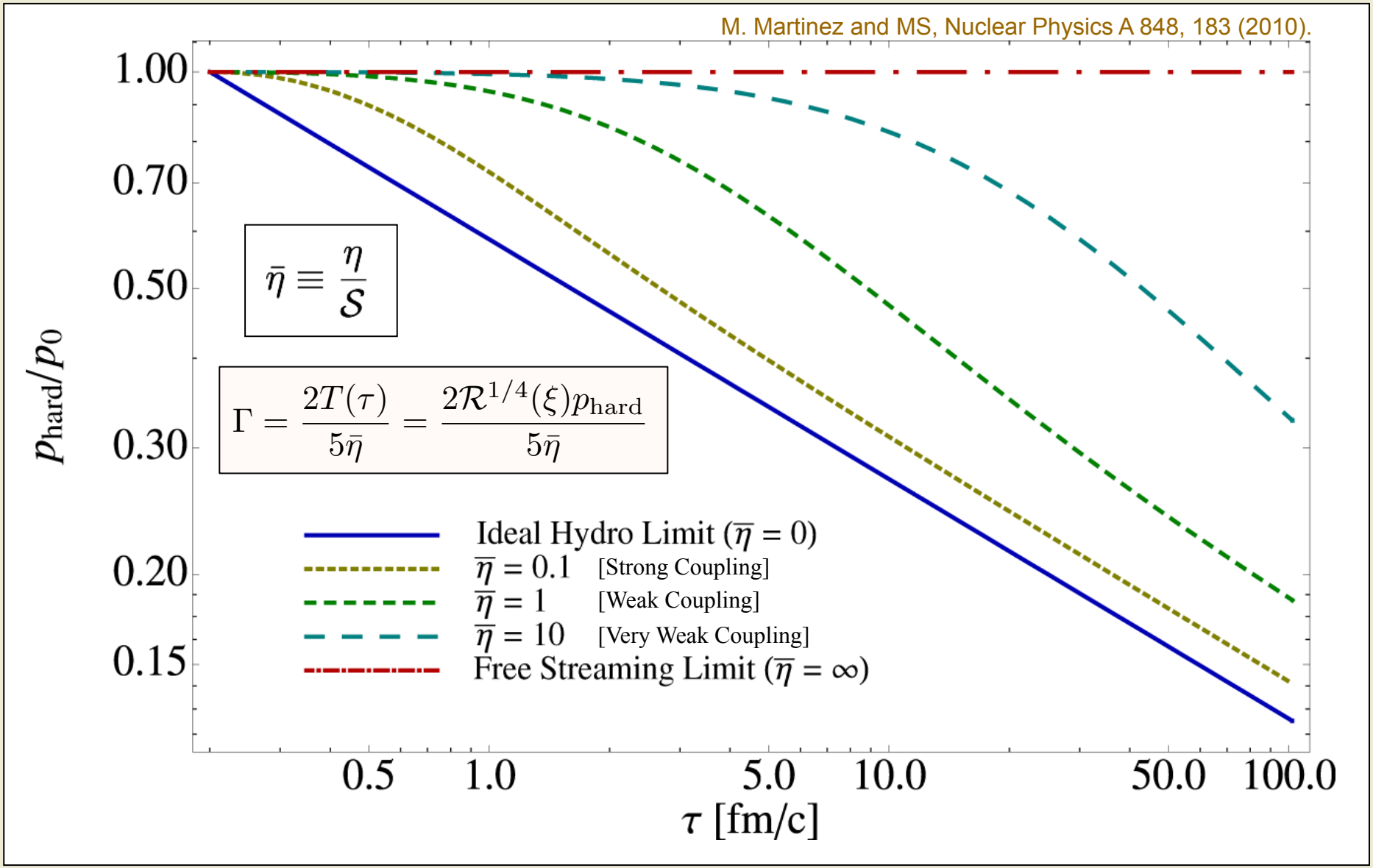
Reproduces 2nd order viscous hydro in small anisotropy limit!

Hard Momentum vs Time

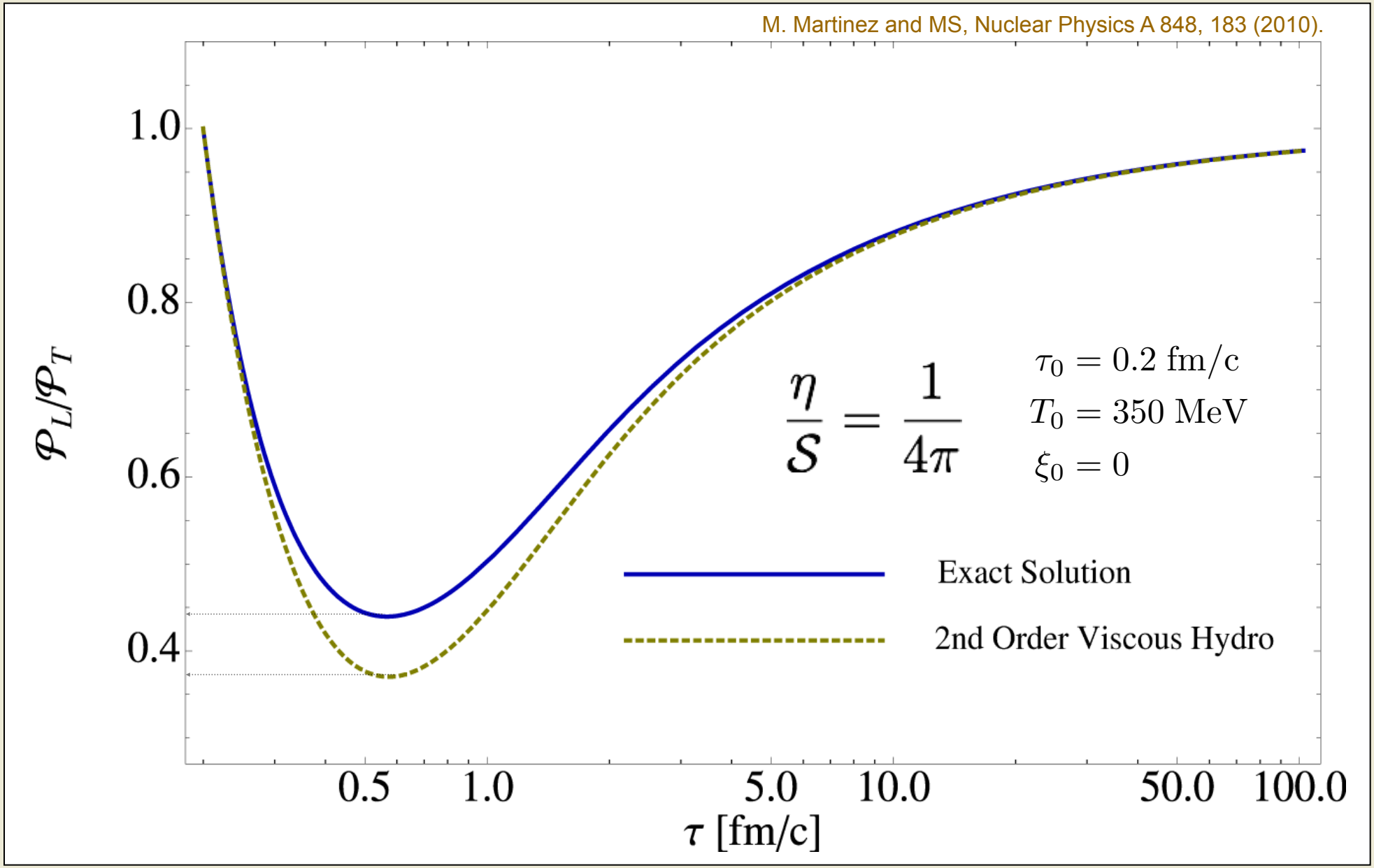
$$\frac{1}{1+\xi}\partial_\tau \xi - \frac{2}{\tau} - \frac{6}{p_{\rm hard}}\partial_\tau p_{\rm hard} = 2\Gamma\left[1 - \mathcal{R}^{3/4}(\xi)\sqrt{1+\xi}\right]$$

$$\frac{\mathcal{R}'(\xi)}{\mathcal{R}(\xi)}\partial_\tau \xi + \frac{4}{p_{\rm hard}}\partial_\tau p_{\rm hard} = \frac{1}{\tau}\left[\frac{1}{\xi(1+\xi)\mathcal{R}(\xi)} - \frac{1}{\xi} - 1\right]$$

M. Martinez and MS, Nuclear Physics A 848, 183 (2010).

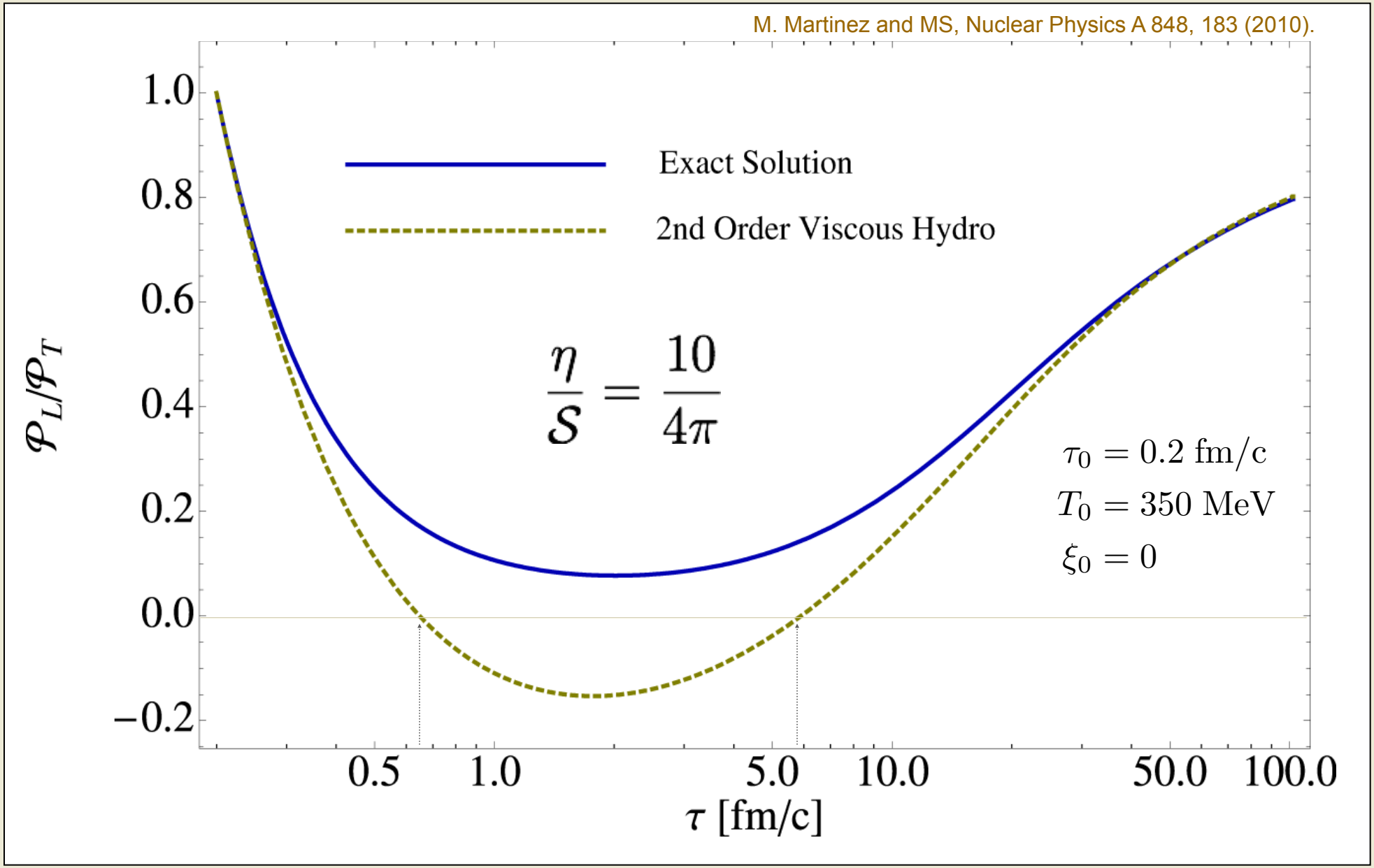


Hydro vs AD : Strong Coupling

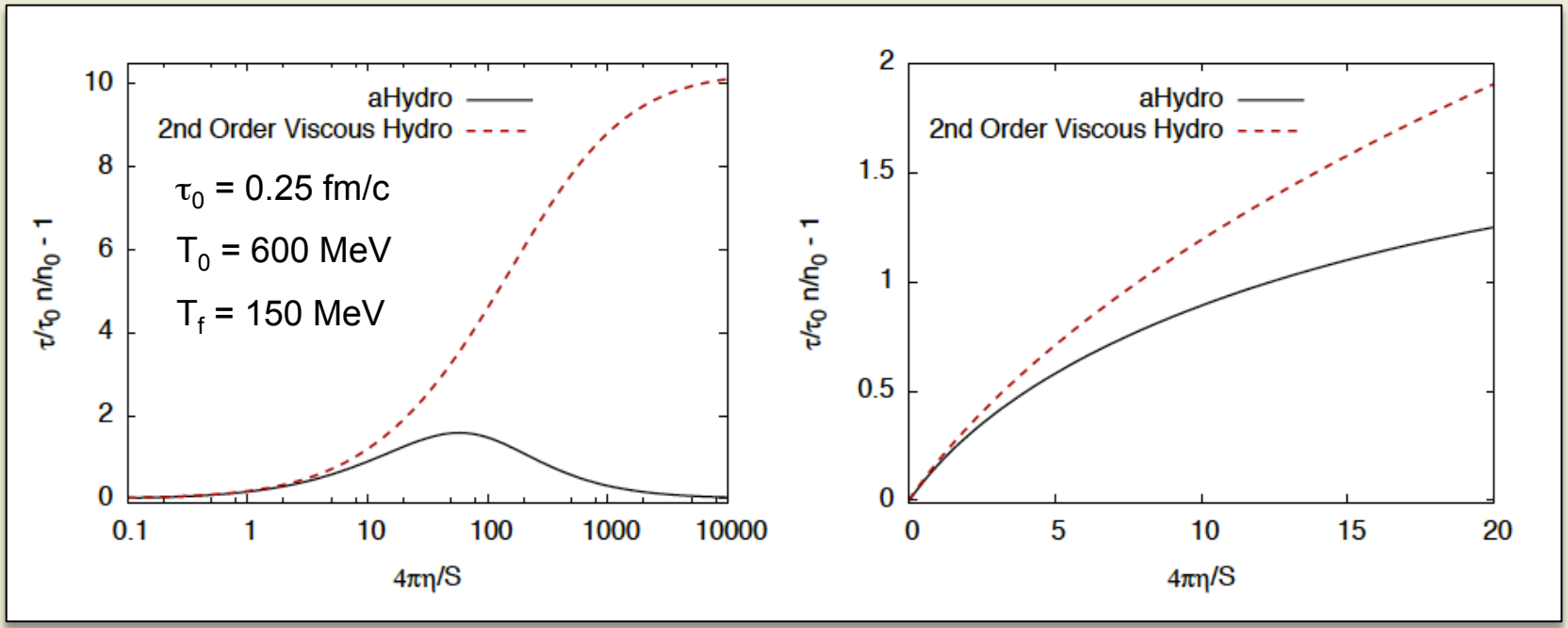


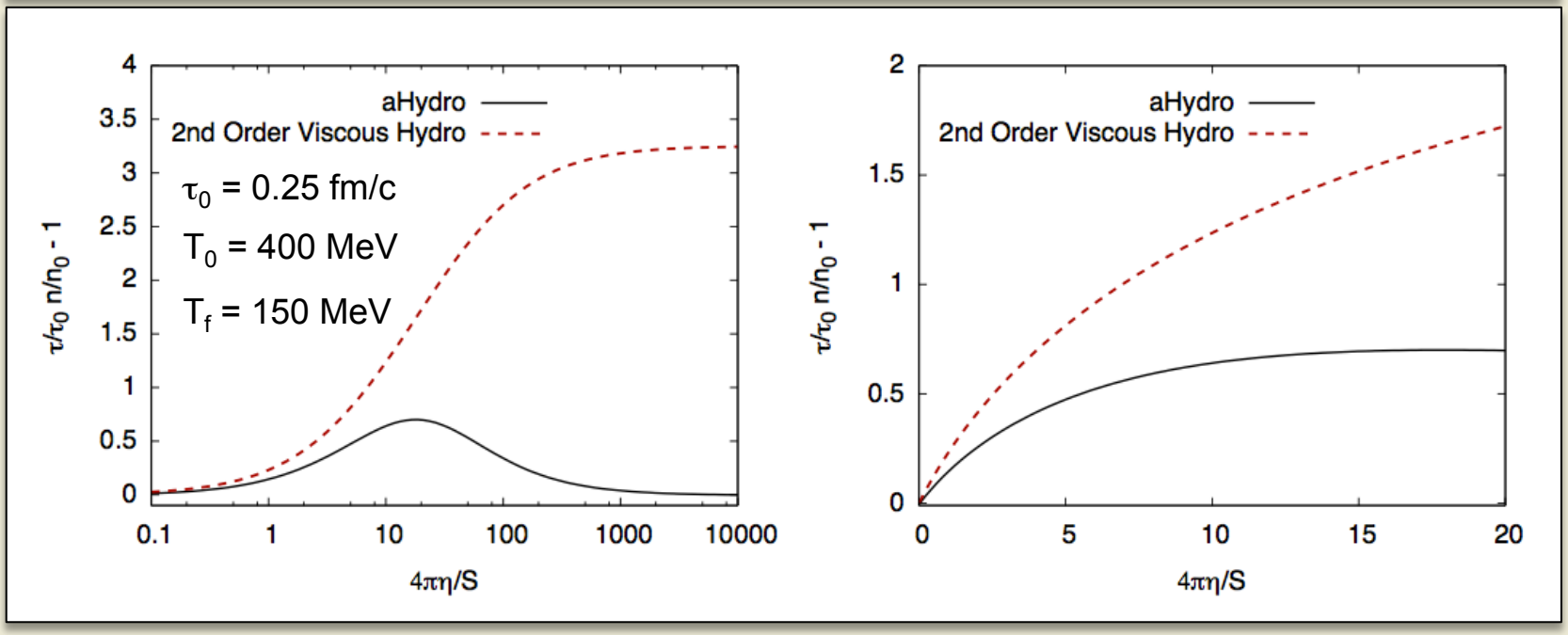
Hydro vs AD : Weak Coupling

M. Martinez and MS, Nuclear Physics A 848, 183 (2010).



Entropy Production





N=4 SUSY using AdS/CFT

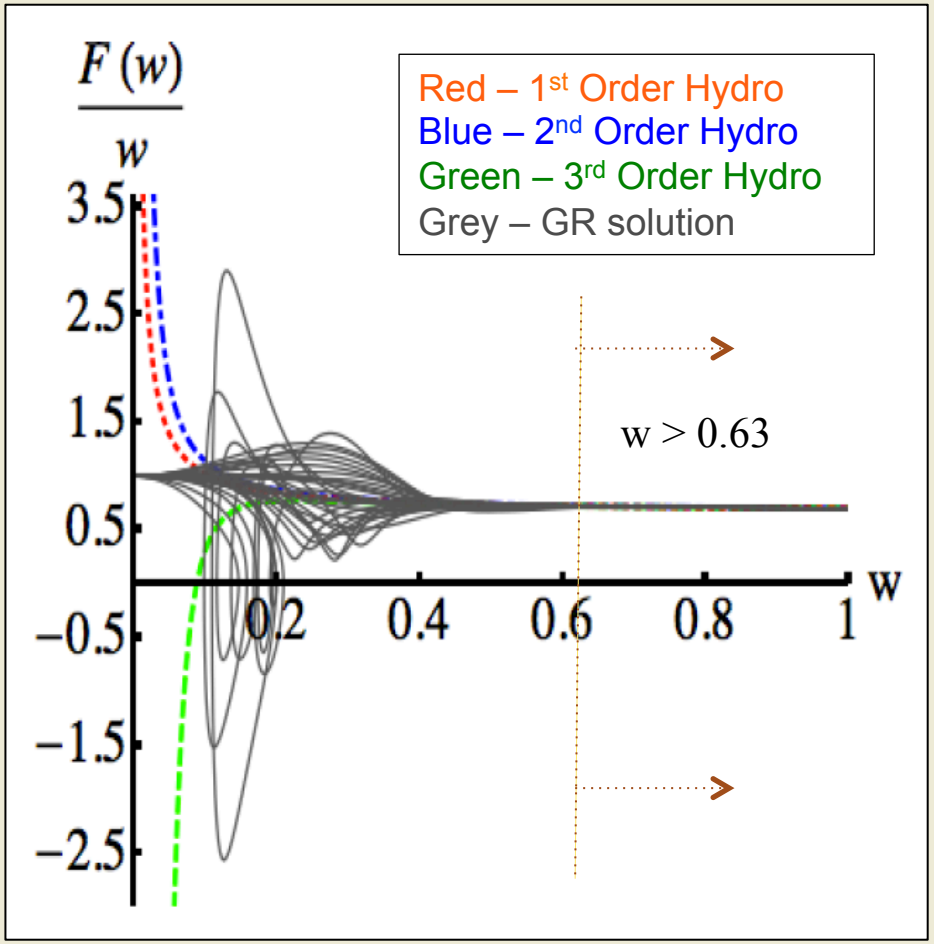
- In 0+1 case there are now numerical solutions of Einstein's equations to compare with.
  [Heller, Janik, and Witaszczyk, arXiv:1103.3452]
- They study a wide variety of initial conditions and find a kind of universal lower bound for the thermalization time

$$\langle T_{\tau\tau}\rangle \equiv \varepsilon(\tau) \equiv N_c^2 \cdot \frac{3}{8}\pi^2 \cdot T_{eff}^4 \cdot$$

$$w = T_{eff} \cdot \tau$$

$$\frac{\tau}{w}\frac{d}{d\tau}w = \frac{F_{hydro}(w)}{w},$$

F_{hydro} known up to 3rd order hydro analytically

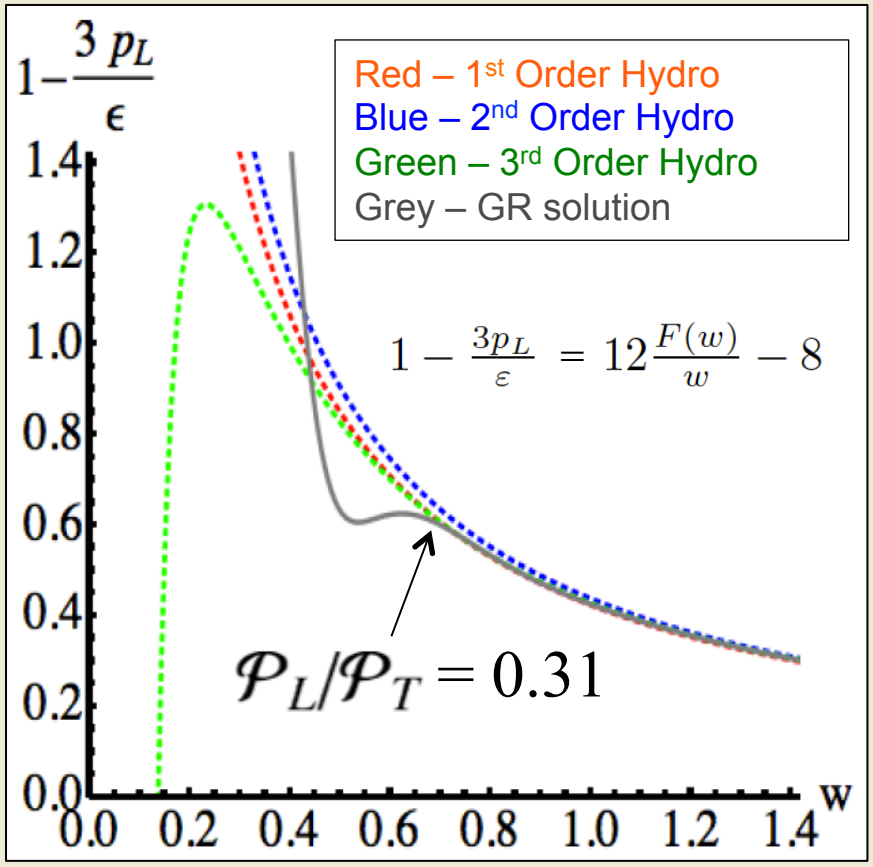


RHIC 200 GeV/nucleon:
T_0 = 350 MeV, τ_0 > 0.35 fm/c

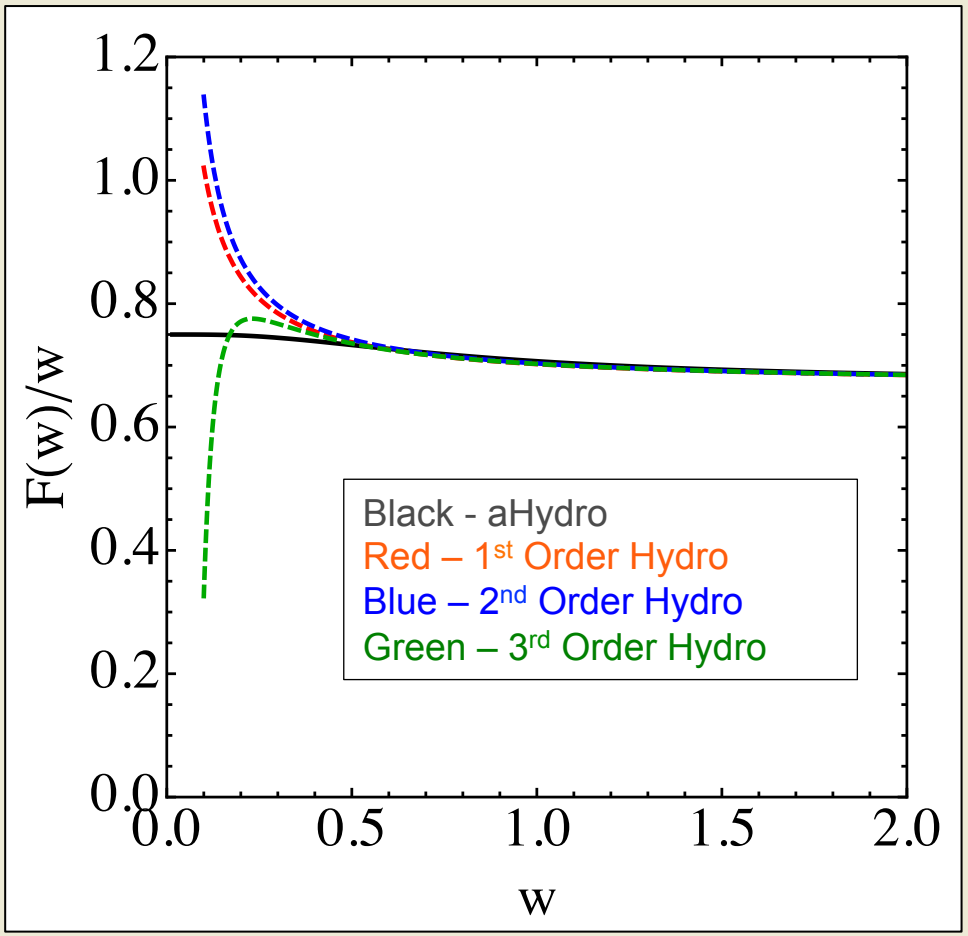
LHC 2.76 TeV/nucleon:
T_0 = 600 MeV, τ_0 > 0.2 fm/c

N=4 SUSY using AdS/CFT

However, at that time the system is not isotropic and remains anisotropic for the entirety of the evolution



How well does the `aHydro` framework compare?



Including Transverse Dynamics

M. Martinez, R. Ryblewski, and MS, arXiv:1204.1473.

- Now we consider boost invariant dynamics with transverse flow.
- Four equations for four variables u_x, u_y, ξ, and Λ.

$$T^{\mu\nu} = (\mathcal{E} + \mathcal{P}_\perp) u^\mu u^\nu - \mathcal{P}_\perp g^{\mu\nu} + (\mathcal{P}_L - \mathcal{P}_\perp) z^\mu z^\nu$$

$$D = u^\mu \partial_\mu$$

$$\tilde{\Delta} = \partial_\mu u^\mu$$

$$Dn + n\tilde{\Delta} = J_0$$

$$D\mathcal{E} + (\mathcal{E} + \mathcal{P}_\perp)\tilde{\Delta} + (\mathcal{P}_L - \mathcal{P}_\perp)\frac{u_0}{\tau} = 0$$

$$(\mathcal{E} + \mathcal{P}_\perp) D u_x + \partial_x \mathcal{P}_\perp + u_x D\mathcal{P}_\perp + (\mathcal{P}_\perp - \mathcal{P}_L)\frac{u_0 u_x}{\tau} = 0$$

$$(\mathcal{E} + \mathcal{P}_\perp) D u_y + \partial_y \mathcal{P}_\perp + u_y D\mathcal{P}_\perp + (\mathcal{P}_\perp - \mathcal{P}_L)\frac{u_0 u_y}{\tau} = 0$$

$$n(\xi, \Lambda) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} f_{\rm RS} = \frac{n_{\rm iso}(\Lambda)}{\sqrt{1+\xi}}$$

$$\mathcal{E}(\Lambda, \xi) = T^{\tau\tau} = \mathcal{R}(\xi)\, \mathcal{E}_{\rm iso}(\Lambda)$$

$$\mathcal{P}_\perp(\Lambda, \xi) = \frac{1}{2}\left(T^{xx} + T^{yy}\right) = \mathcal{R}_\perp(\xi)\mathcal{P}_{\rm iso}(\Lambda)$$

$$\mathcal{P}_L(\Lambda, \xi) = -T^{\varsigma}_{\varsigma} = \mathcal{R}_{\rm L}(\xi)\mathcal{P}_{\rm iso}(\Lambda)$$

$$\mathcal{R}(\xi) \equiv \frac{1}{2}\left(\frac{1}{1+\xi} + \frac{\arctan\sqrt{\xi}}{\sqrt{\xi}}\right)$$

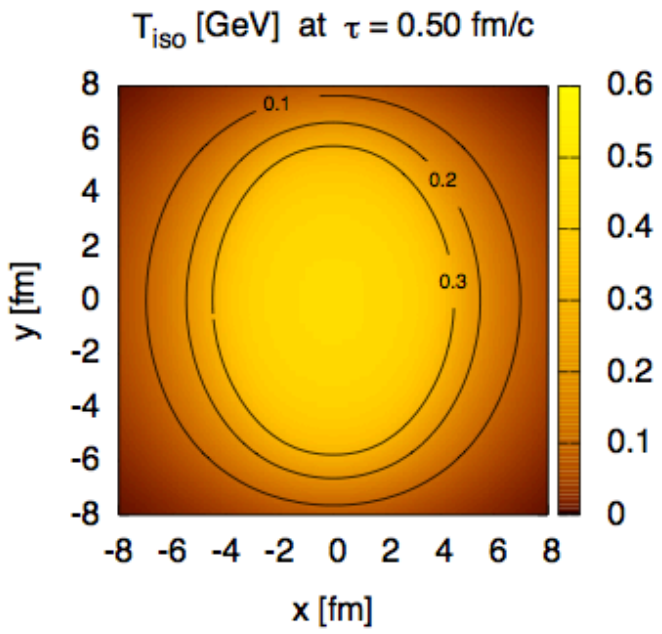
$$\mathcal{R}_\perp(\xi) \equiv \frac{3}{2\xi}\left(\frac{1 + (\xi^2 - 1)\mathcal{R}(\xi)}{\xi + 1}\right)$$

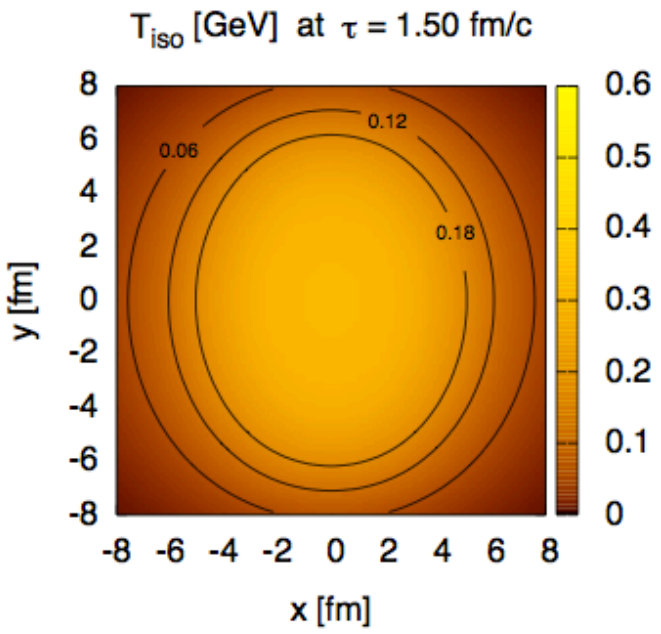
$$\mathcal{R}_L(\xi) \equiv \frac{3}{\xi}\left(\frac{(\xi+1)\mathcal{R}(\xi) - 1}{\xi + 1}\right)$$

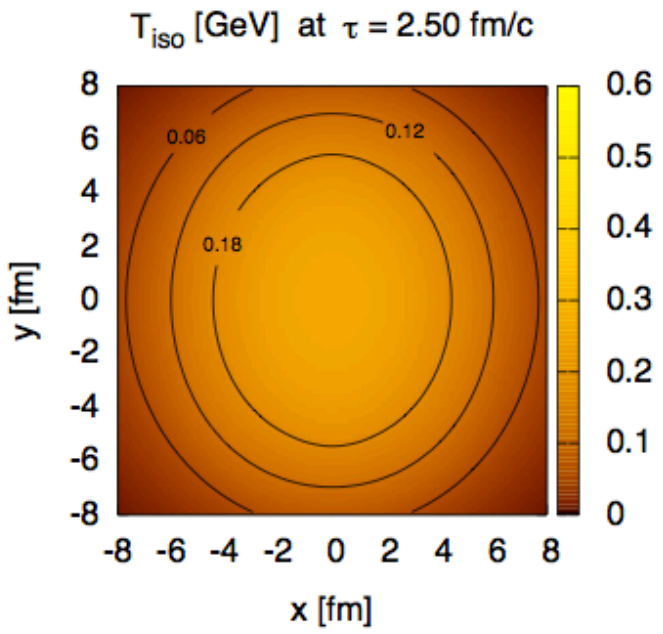
Transverse Dynamics

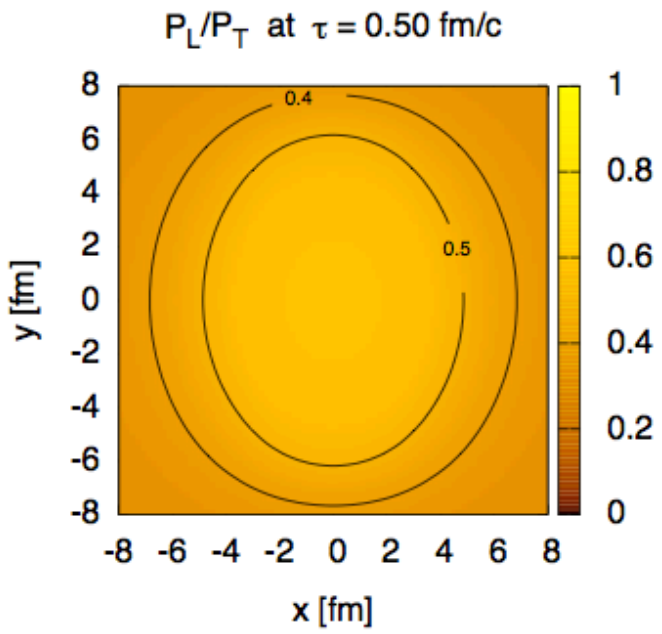
Pb-Pb @ 2.76 TeV
T_0 = 600 MeV
τ_0 = 0.25 fm/c
b = 7 fm

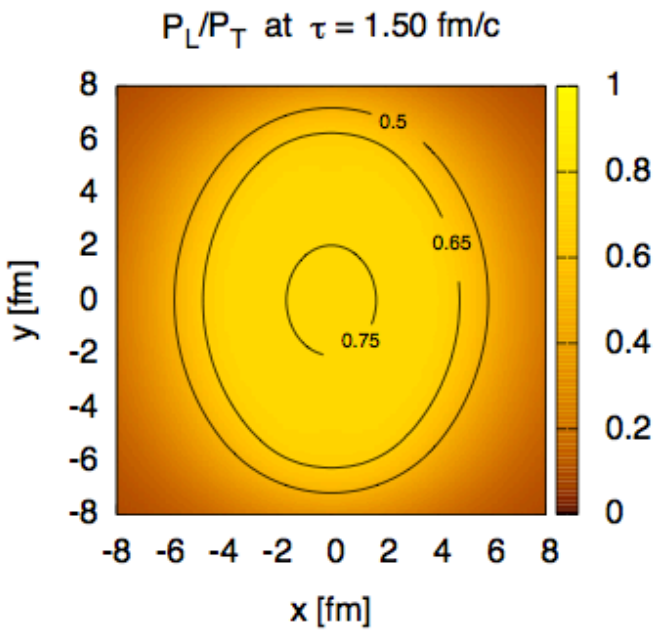
$$\frac{\eta}{s} = \frac{1}{4\pi}$$

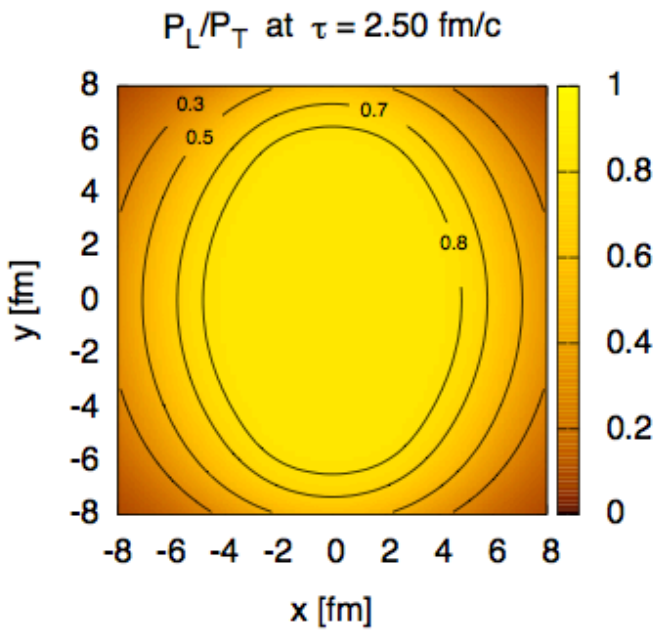










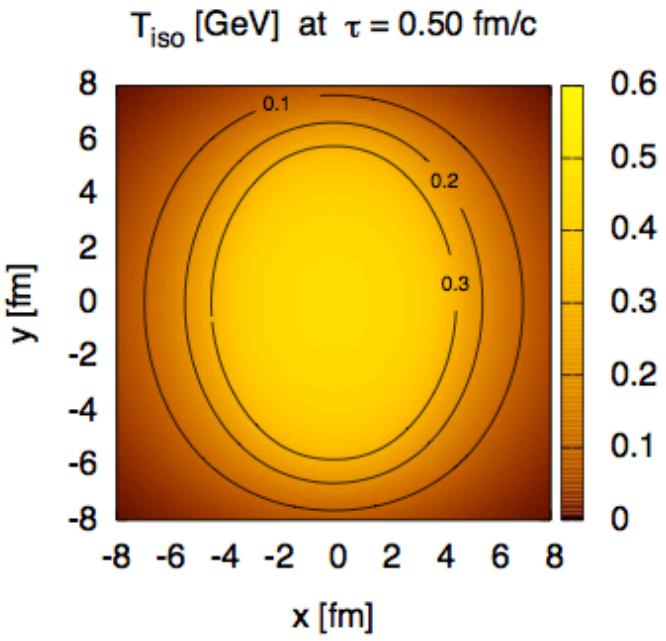


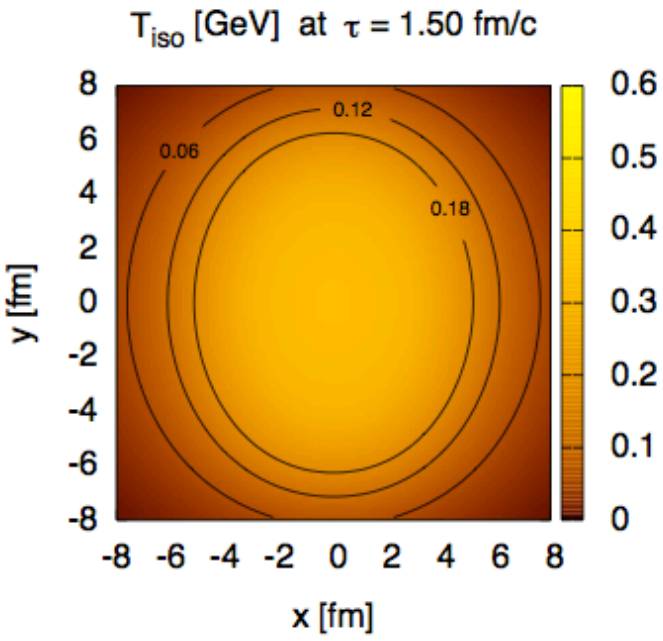
M. Martinez, R. Ryblewski, and MS, arXiv:1204.1473.

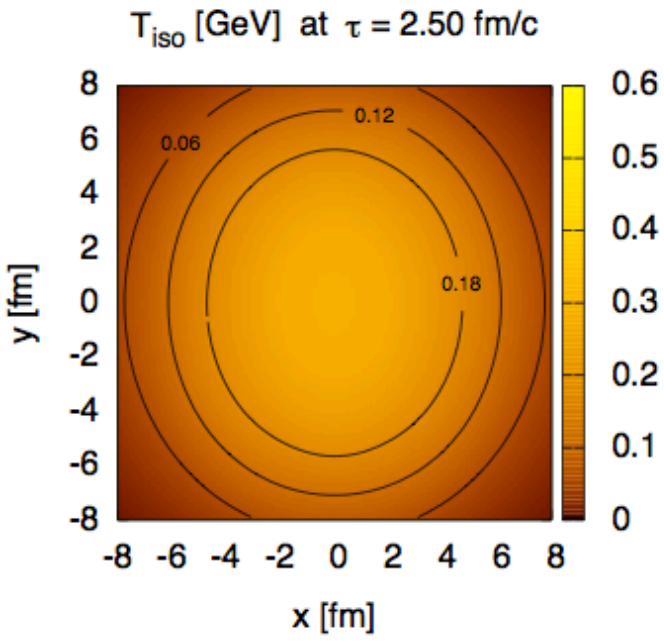
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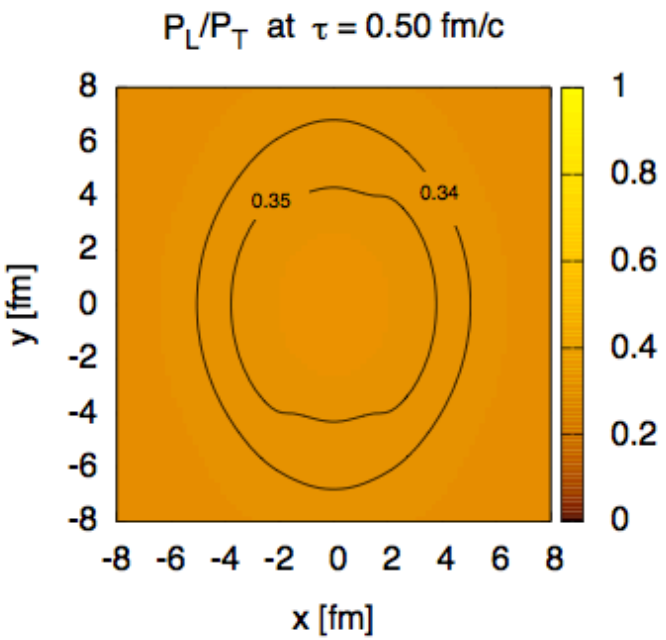
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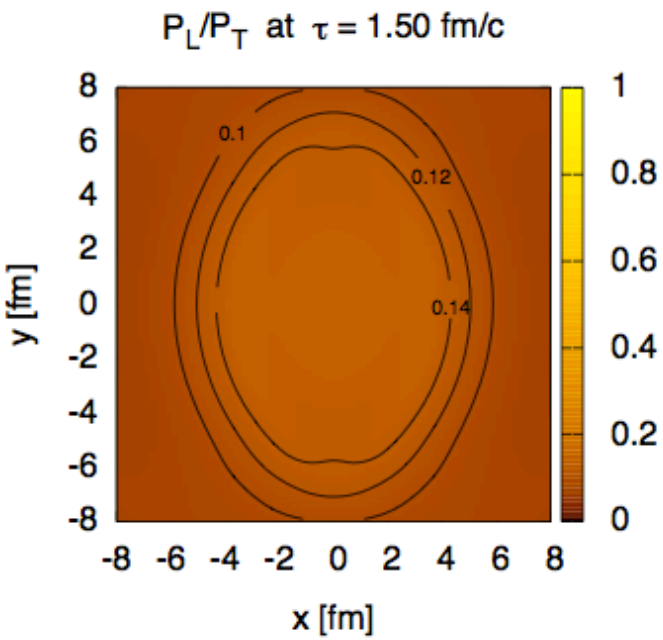
$$\frac{\eta}{s} = \frac{10}{4\pi}$$

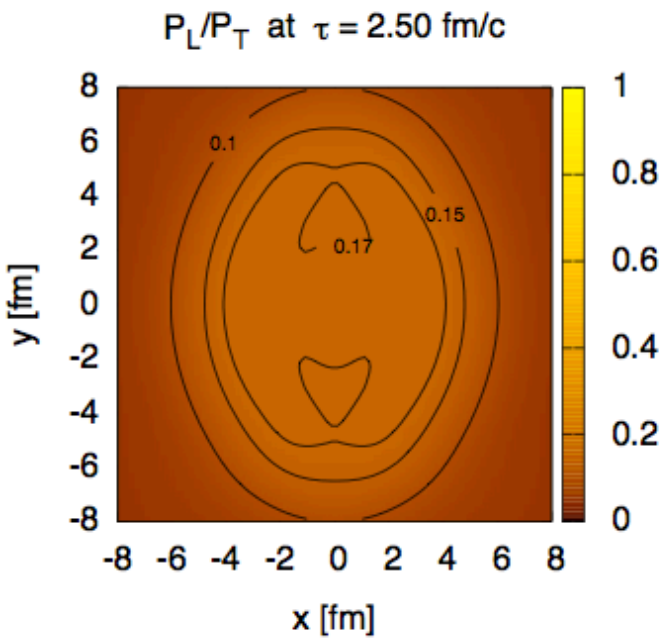










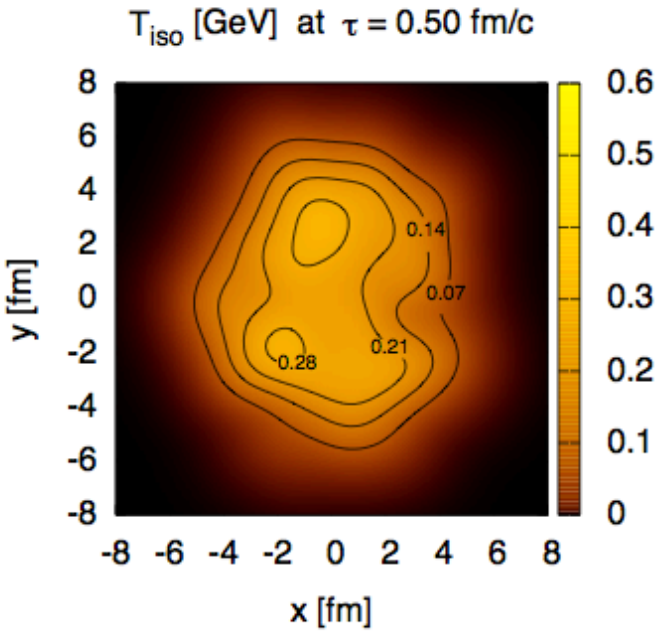


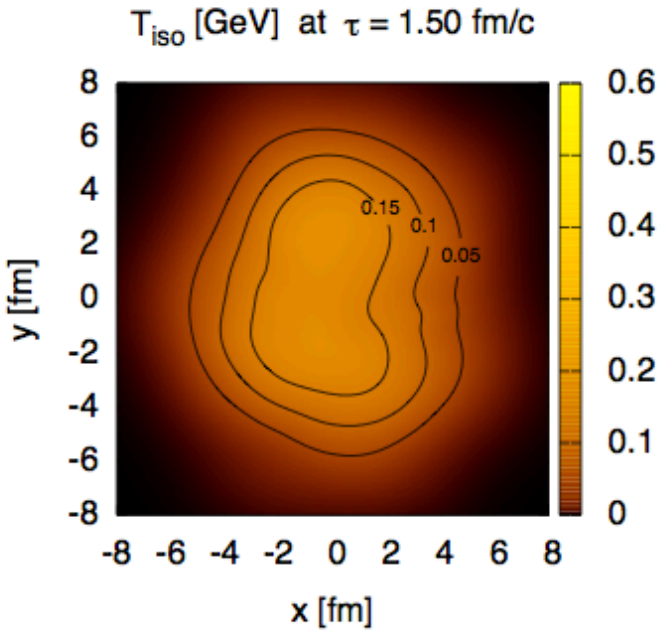
M. Martinez, R. Ryblewski, and MS, arXiv:1204.1473.

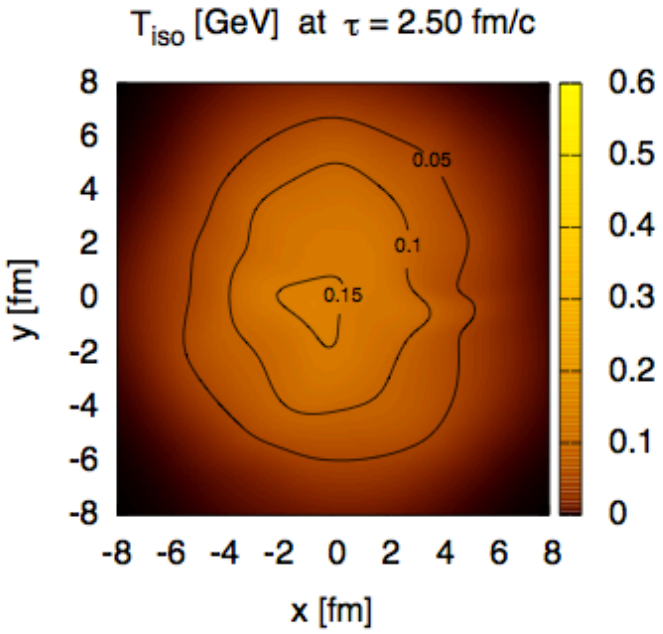
Transverse Dynamics

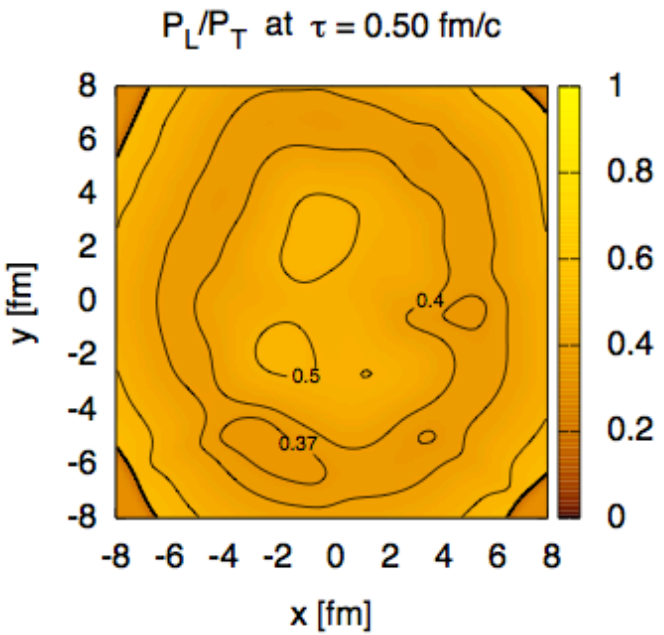
Pb-Pb @ 2.76 TeV
T_0 = 600 MeV
τ_0 = 0.25 fm/c
b = 7 fm

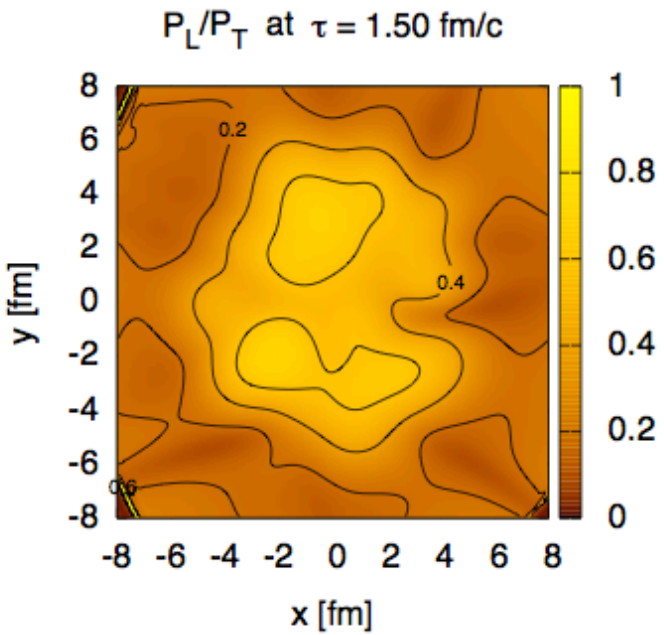
$$\frac{\eta}{\mathcal{S}} = \frac{1}{4\pi}$$

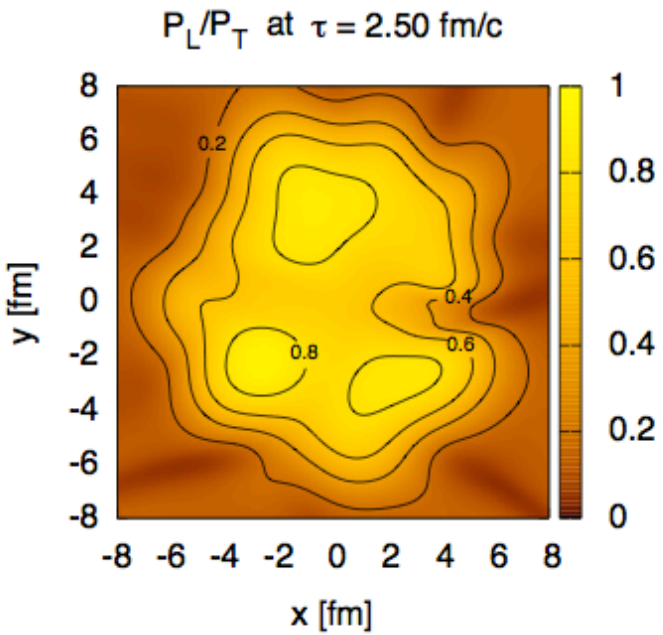






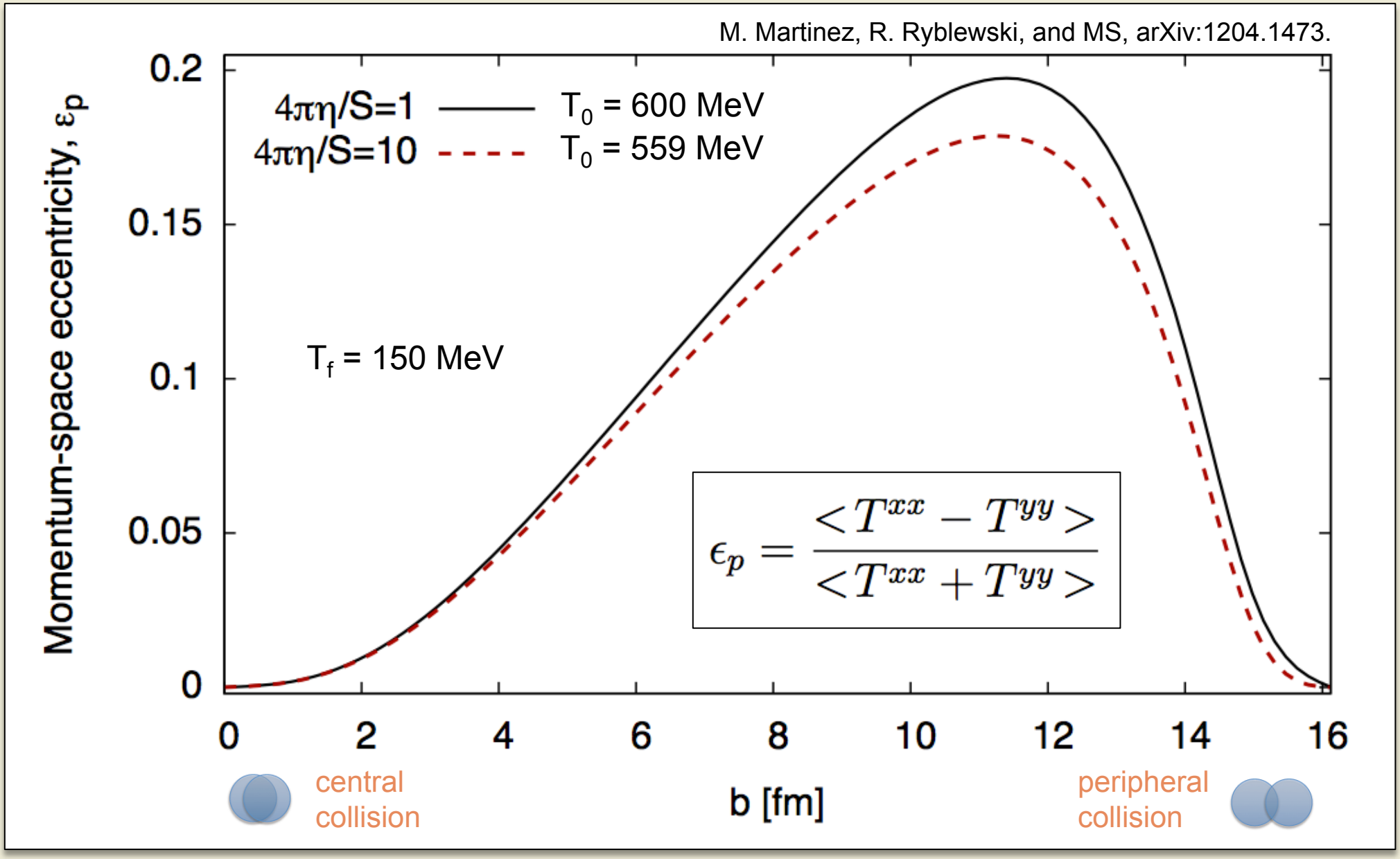






M. Martinez, R. Ryblewski, and MS, arXiv:1204.1473.

Collective Flow



Highly-anisotropic hydrodynamics in 3+1 space-time dimensions *

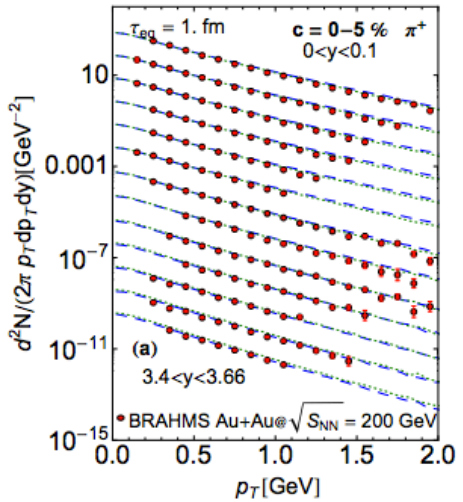
Radoslaw Ryblewski[1,†] and Wojciech Florkowski[2,1,‡]

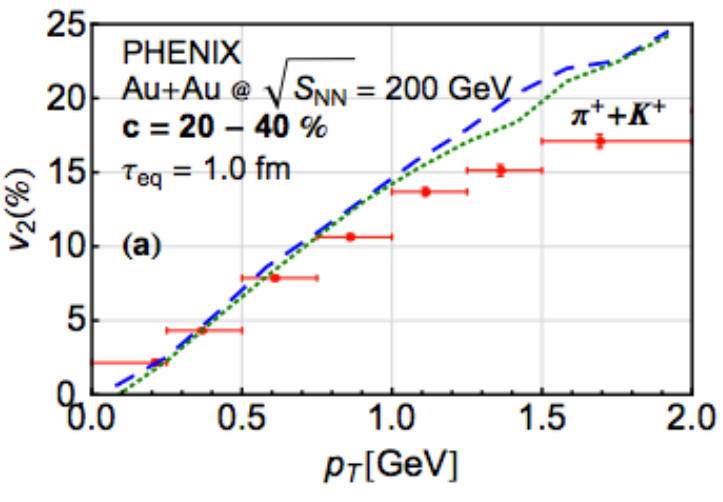
[1] *The H. Niewodniczański Institute of Nuclear Physics, Polish Academy of Sciences, PL-31342 Kraków, Poland*

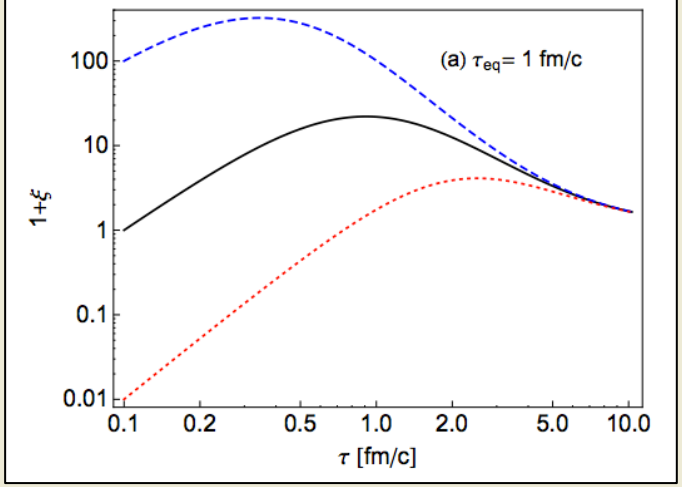
[2] *Institute of Physics, Jan Kochanowski University, PL-25406 Kielce, Poland*

(Dated: April 10, 2012)

arXiv:1204.2624v1



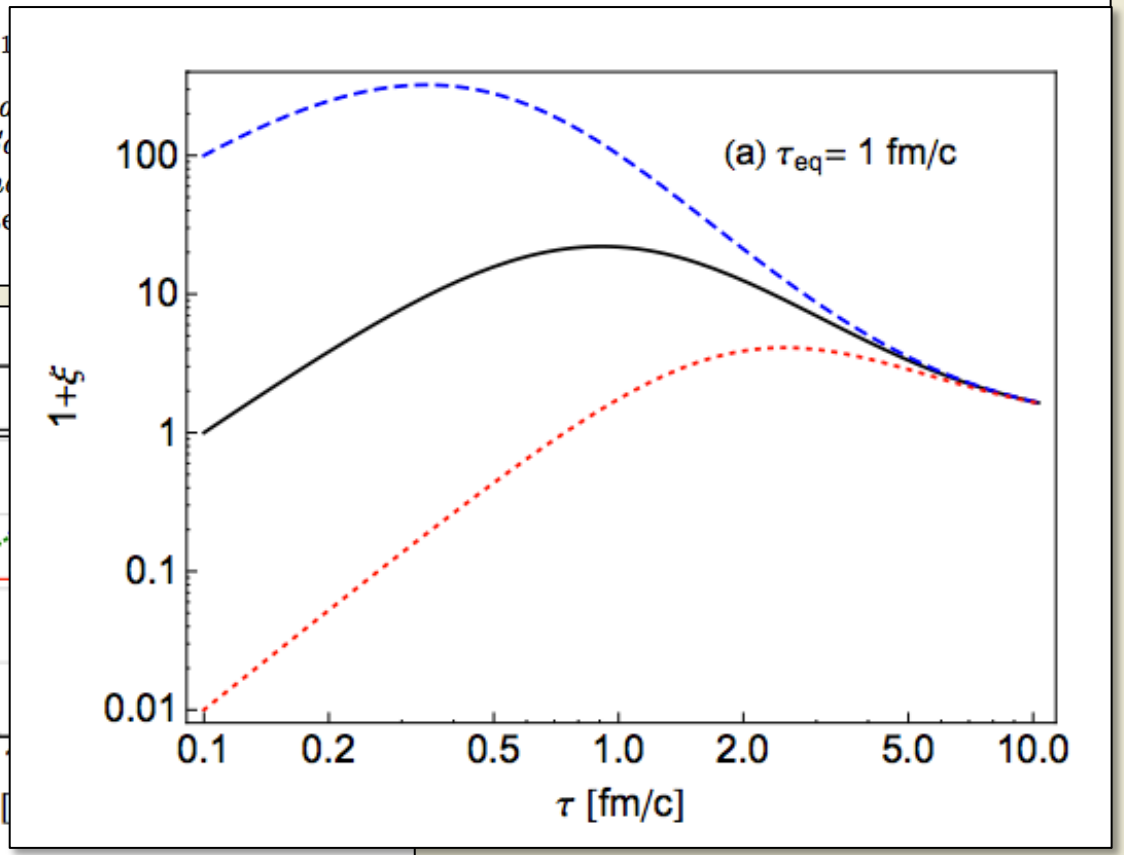


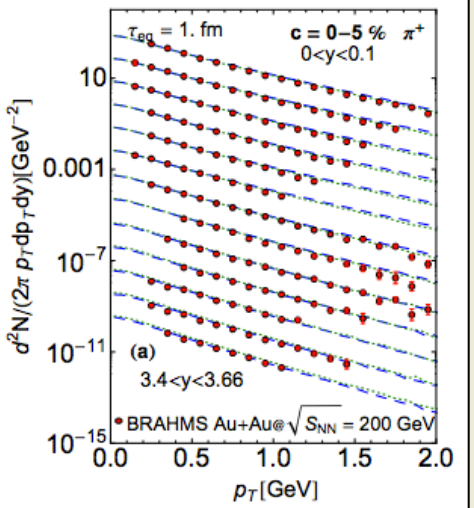


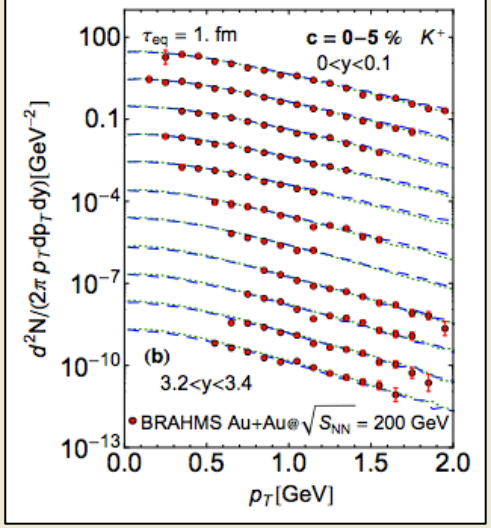
Results are insensitive to even large persistent momentum-space anisotropies!

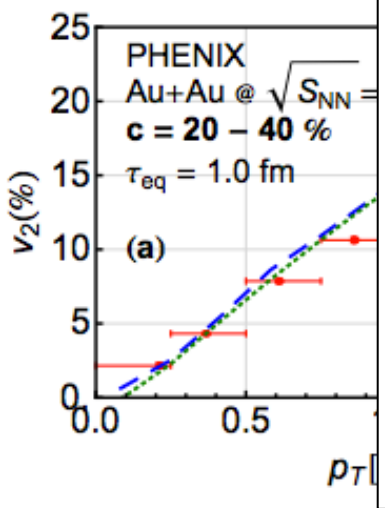
Highly-anisotropic hydrodynamics in 3+1 space-time dimensions *

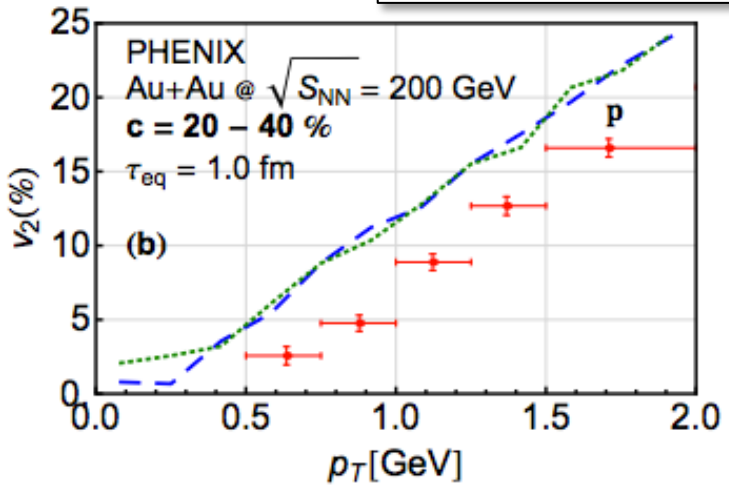
Radoslaw Ryblewski[1]
[1] The H. Niewodniczańs
Polish Academy of Sc
[2] Institute of Physics, Jan Koch
(Date











Results are insensitive to even large persistent momentum-space anisotropies!

Conclusions

- Plasma need not be isotropic in momentum space to describe the data
- Large momentum-space anisotropies cause trouble for traditional viscous hydrodynamical approaches
- Particularly worrisome near edges
- A practical way out is to change to an intrinsically anisotropic expansion point and consider fluctuations around that instead → “anisotropic hydrodynamics”

--- Backup Slides ---

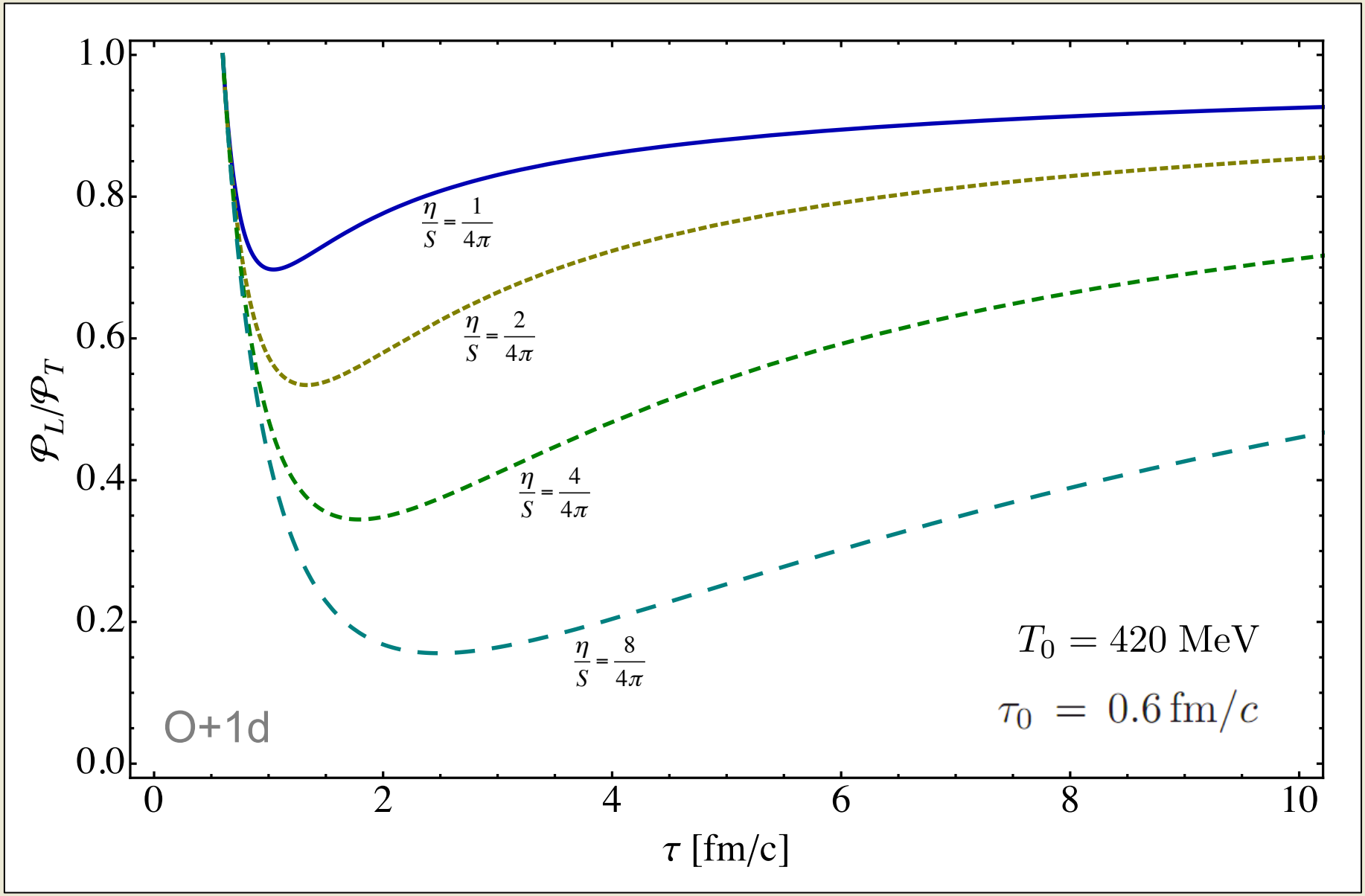
Observables sensitive to momentum-space anisotropy

- Jet collisional and radiative energy loss
  Romatschke & MS : hep-ph/0408275
  Dumitru, MS, et al: 0710.1223
  Schenke, MS, et al : 0810.1314
- Photons
  Schenke & MS: hep-ph/0611332
- Dileptons
  Martinez & MS: 0805.4552
  Martinez & MS: 0808.3969
- Quarkonium suppression
  MS: arXiv:1106.2571, arXiv:1112.2761
  Dumitru, Guo, & MS: 0711.4722
  Dumitru, Guo, & MS: 0903.4703

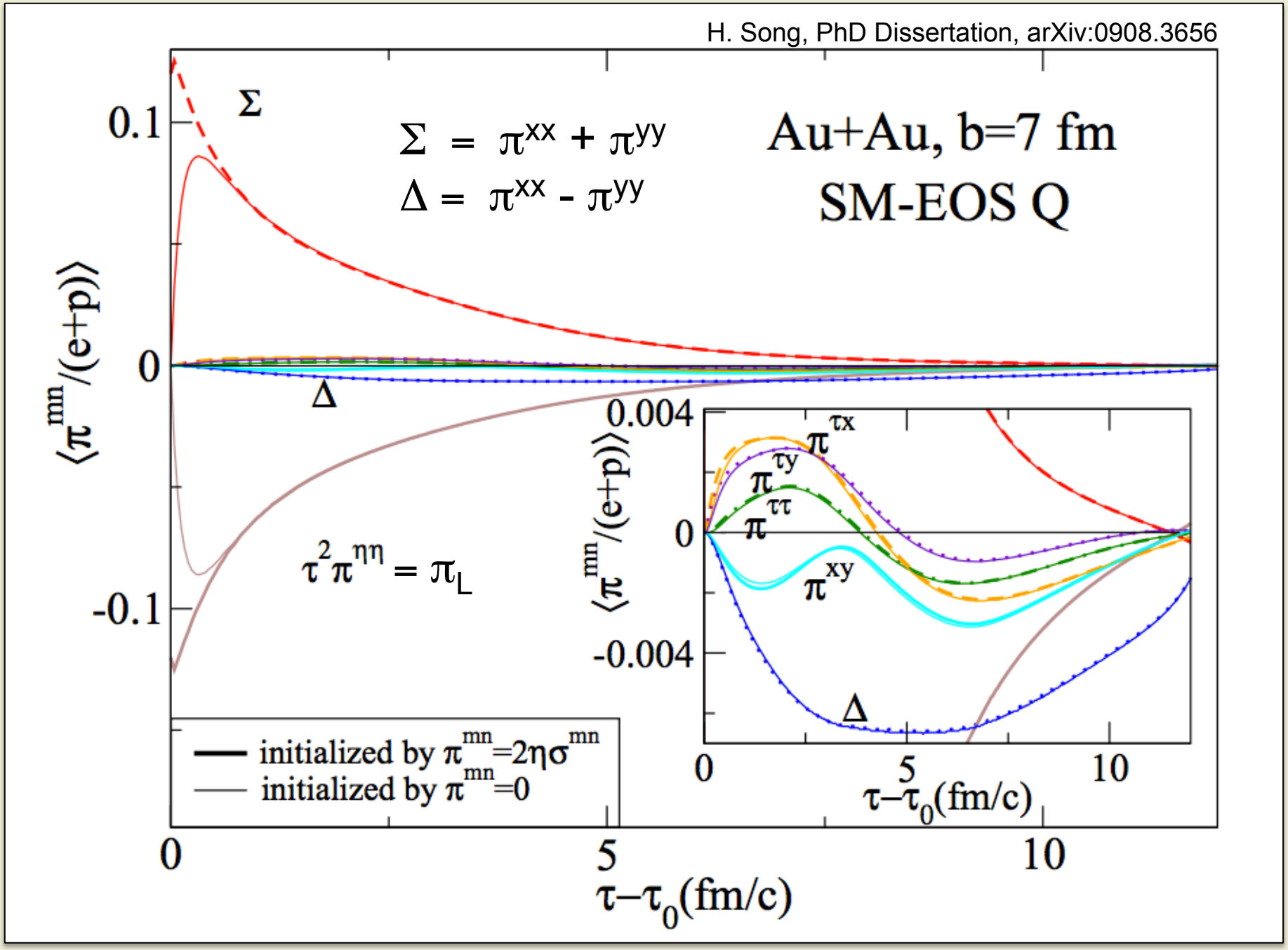
Work using the AdS/CFT to study anisotropic systems

Janik et al, 0806.2141, 1103.3452, 1203.0755
Mateos & Trancanelli, 1105.3472, 1106.1637
Chernicoff et al, 1202.3696, 1203.0561
Rebhan & Steineder, 1106.3539, 1110.6825
Giataganas, 1202.4436
Heller et al, 1202.0981

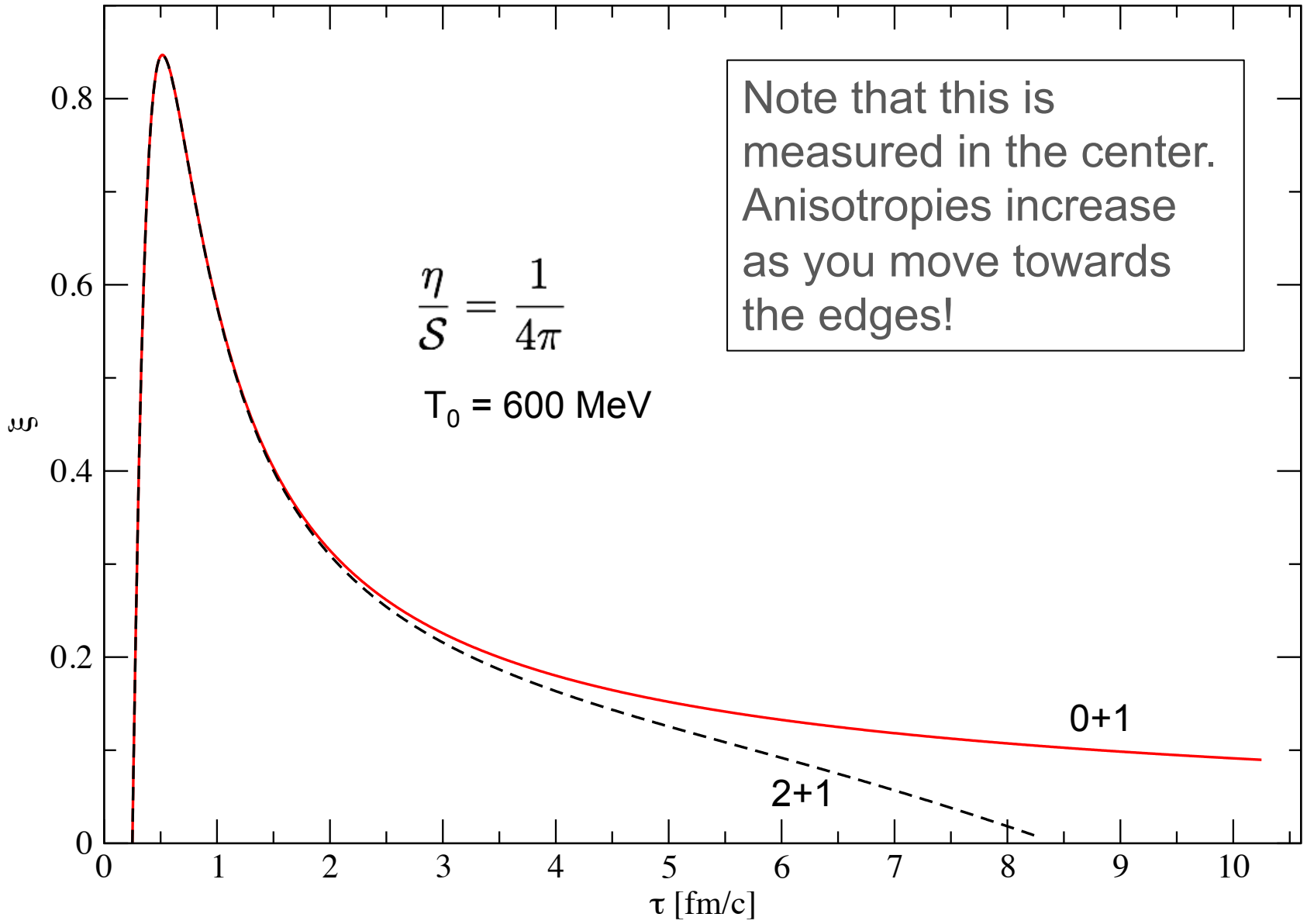
Viscous Hydro Momentum Anisotropy



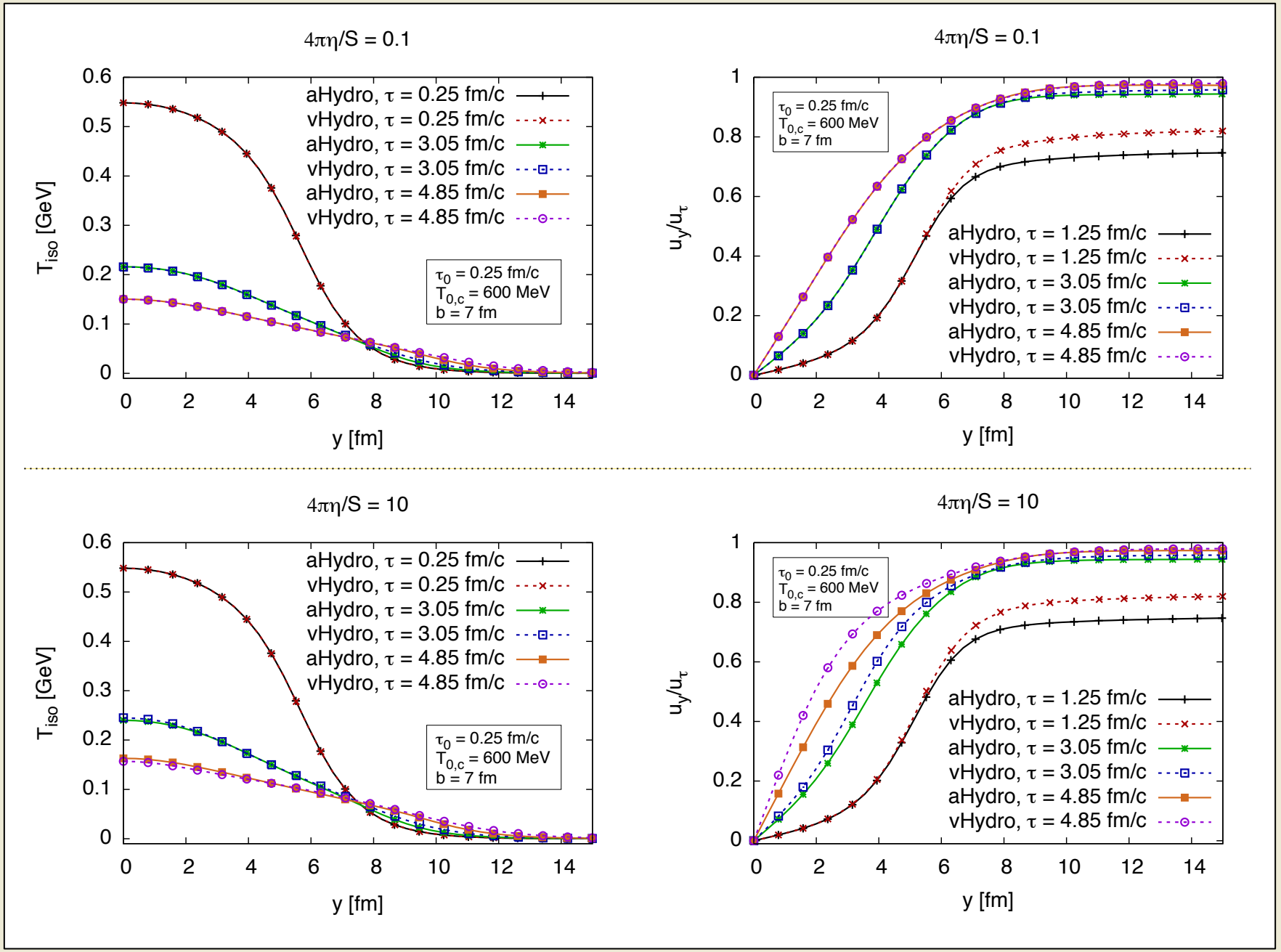
Including Transverse Dynamics



Effect of Transverse Expansion on Isotropization



Check against Viscous Hydro



Relativistic Hydro from Transport

• Describe evolution of the system using the Boltzmann equation

$$p^\alpha \partial_\alpha f = -C[f]$$

C[f] = Collisional Kernel

• Can extract hydro equations from the Boltzmann equation by taking "moments" of the equation using an integral operator

$$\hat{I}^{\mu\nu\cdots\sigma} \equiv \int \frac{d^3p}{2E}\, p^\mu p^\nu \cdots p^\sigma$$

eg.

$$\hat{I} \equiv \int \frac{d^3p}{2E}$$

0th moment operator

$$\hat{I}^\mu \equiv \int \frac{d^3p}{2E}\, p^\mu$$

1st moment operator

0th Moment

N^α : Particle Number and Current

$$\partial_\alpha N^\alpha = -\int \frac{d^3p}{E} C[f]$$

if number conserving

collisional kernel

$$\partial_\alpha N^\alpha = 0$$

Number conservation

If particle number changing processes in kernel, eg 2 → 3, RHS is nonzero

1st Moment

$T^{\alpha\beta}$: Energy-Momentum Tensor

$$\partial_\alpha T^{\alpha\beta} = -\int \frac{d^3p}{E} p^\beta C[f]$$

if energy conserving

collisional kernel

$$\partial_\alpha T^{\alpha\beta} = 0$$

Energy-momentum conservation!

Collisional Kernel

- Relaxation time approximation

$$p^\alpha \partial_\alpha f = -C[f]$$

$$C[f(\tau,\mathbf{x},\mathbf{p})] = p_\mu u^\mu \, \Gamma \left[f(\tau,\mathbf{x},\mathbf{p}) - f_{\rm eq}(|\mathbf{p}|, T(\tau,\mathbf{x})) \right]$$

- Where Γ is the relaxation rate
- Γ will be fixed by matching to 2nd order viscous hydro in the weak anisotropy limit
- $\mathrm{T}(\tau,\boldsymbol{x})$ is the self-consistent isotropic temperature which can be fixed by requiring energy conservation at all proper times [Baym '84]

Numerical Checks 1

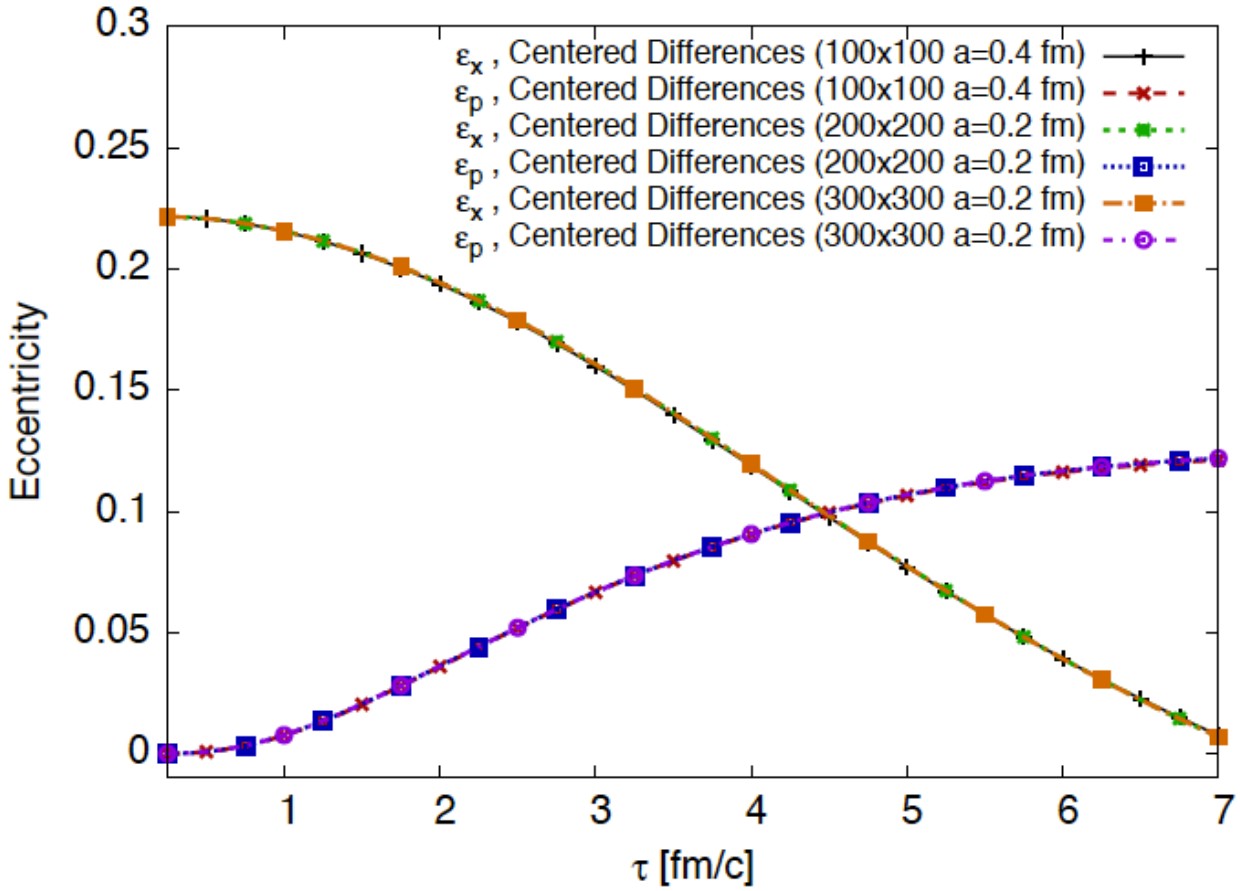


FIG. 1: Spatial and momentum eccentricities as a function of proper time for a smooth Glauber wounded-nucleon transverse profile with $b = 7$ fm, $\Lambda_0 = T_0 = 0.6$ GeV, $\xi_0 = 0$, and $u_{\perp,0} = 0$ at $\tau_0 = 0.25$ fm/c assuming $4\pi\eta/\mathcal{S} = 1$. In all three cases we used a RK4 temporal step size of $\epsilon = 0.01$ fm/c.

Numerical Checks 2

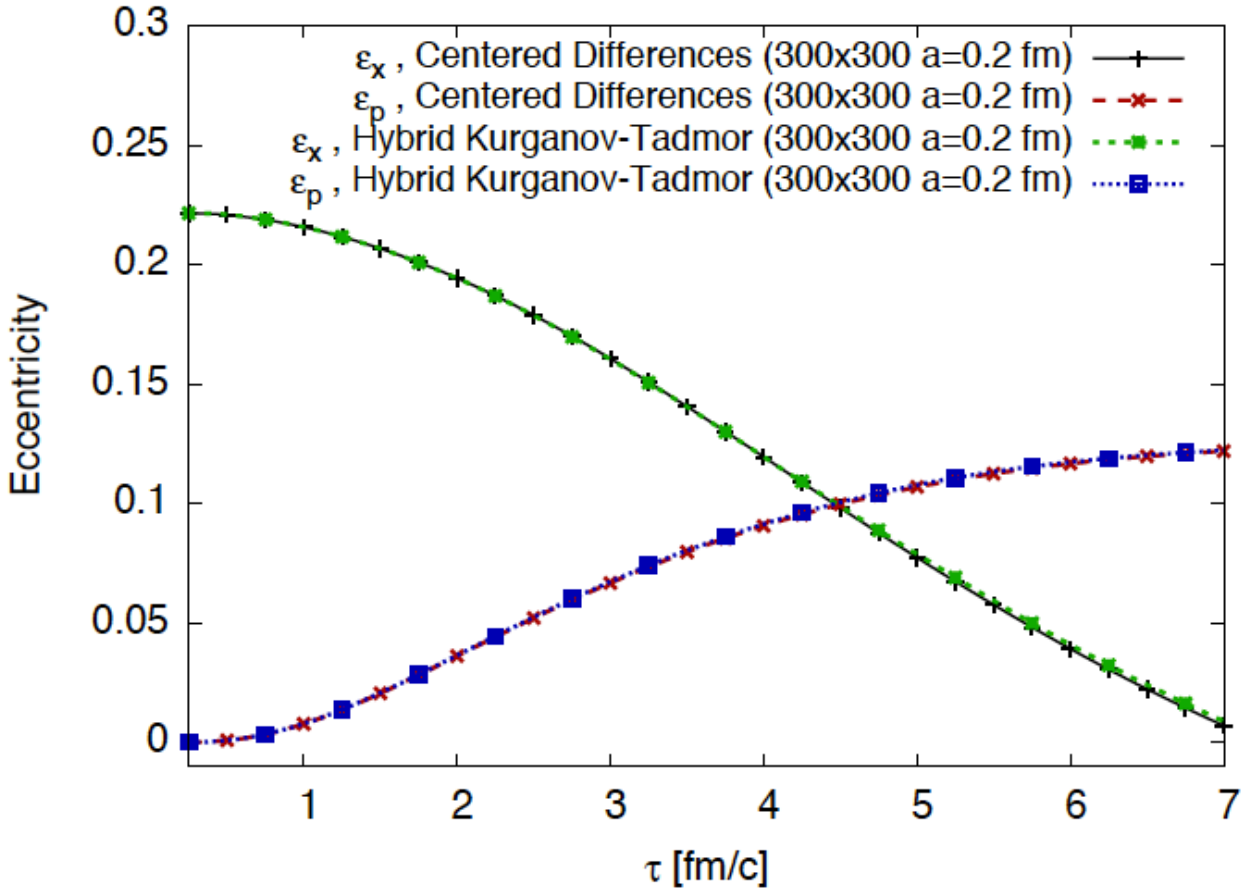


FIG. 2: Spatial and momentum eccentricities as a function of proper time for a smooth Glauber wounded-nucleon transverse profile with $b = 7$ fm, $\Lambda_0 = T_0 = 0.6$ GeV, $\xi_0 = 0$, and $u_{\perp,0} = 0$ at $\tau_0 = 0.25$ fm/c assuming $4\pi\eta/\mathcal{S} = 1$. Here we compare the centered differences and Hybrid Kurganov-Tadmor algorithms.

Numerical Checks 2

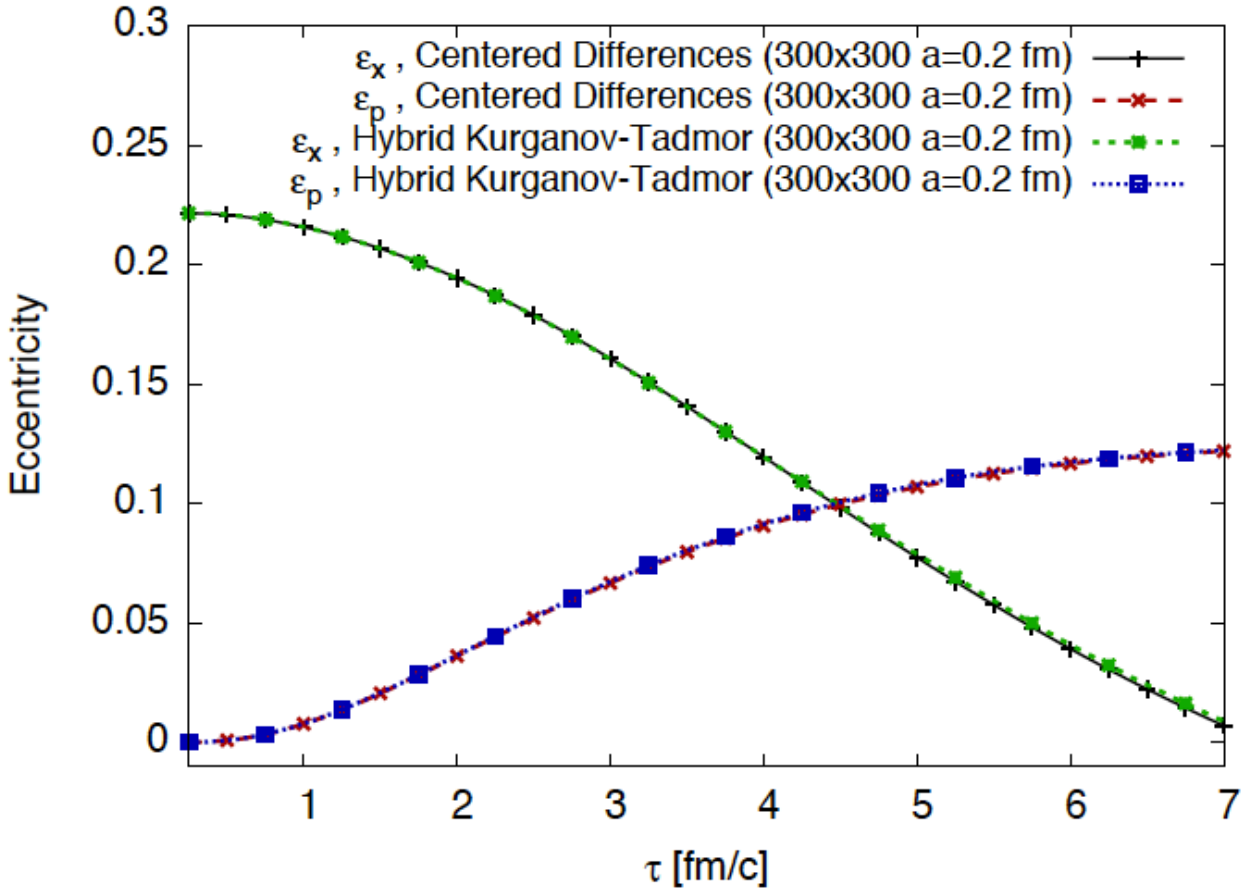


FIG. 2: Spatial and momentum eccentricities as a function of proper time for a smooth Glauber wounded-nucleon transverse profile with $b = 7$ fm, $\Lambda_0 = T_0 = 0.6$ GeV, $\xi_0 = 0$, and $u_{\perp,0} = 0$ at $\tau_0 = 0.25$ fm/c assuming $4\pi\eta/\mathcal{S} = 1$. Here we compare the centered differences and Hybrid Kurganov-Tadmor algorithms.

Numerical Checks 3

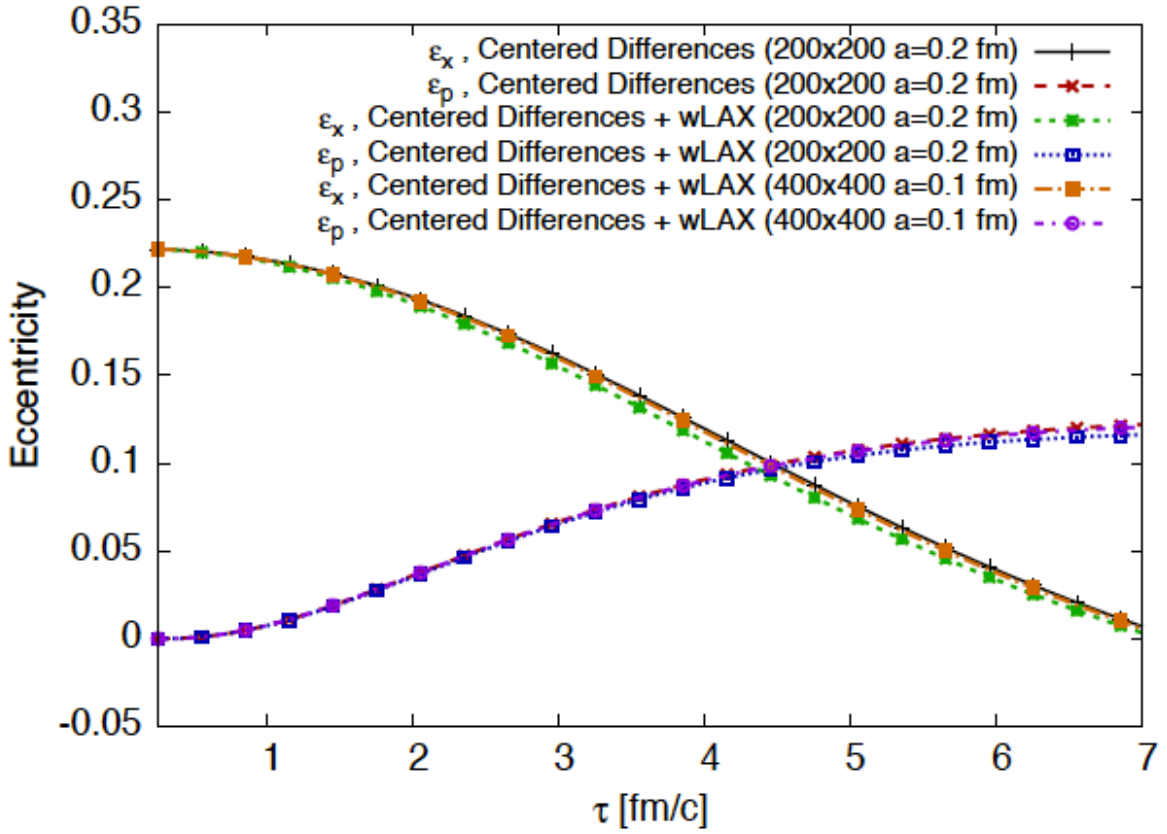


FIG. 10: Spatial and momentum eccentricities as a function of proper time for a smooth Glauber wounded-nucleon transverse profile with $b = 7$ fm, $\Lambda_0 = T_0 = 0.6$ GeV, $\xi_0 = 0$, and $u_{\perp,0} = 0$ at $\tau_0 = 0.25$ fm/c assuming $4\pi\eta/\mathcal{S} = 1$. Here we demonstrate the convergence of the wLAX algorithm with $\lambda = 0.05$ to the result obtained without any spatial averaging as one decreases the lattice spacing. In all cases RK4 with a temporal step size $\epsilon = 0.01$ fm/c was used.