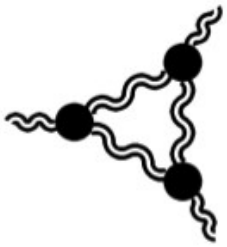
3 Loop 3PI for 3D QCD

Mark A. York, Guy Moore, Marcus Tassler

McGill University, Montreal

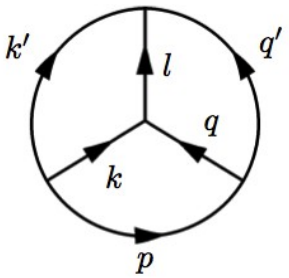
arXiv:1202.4756





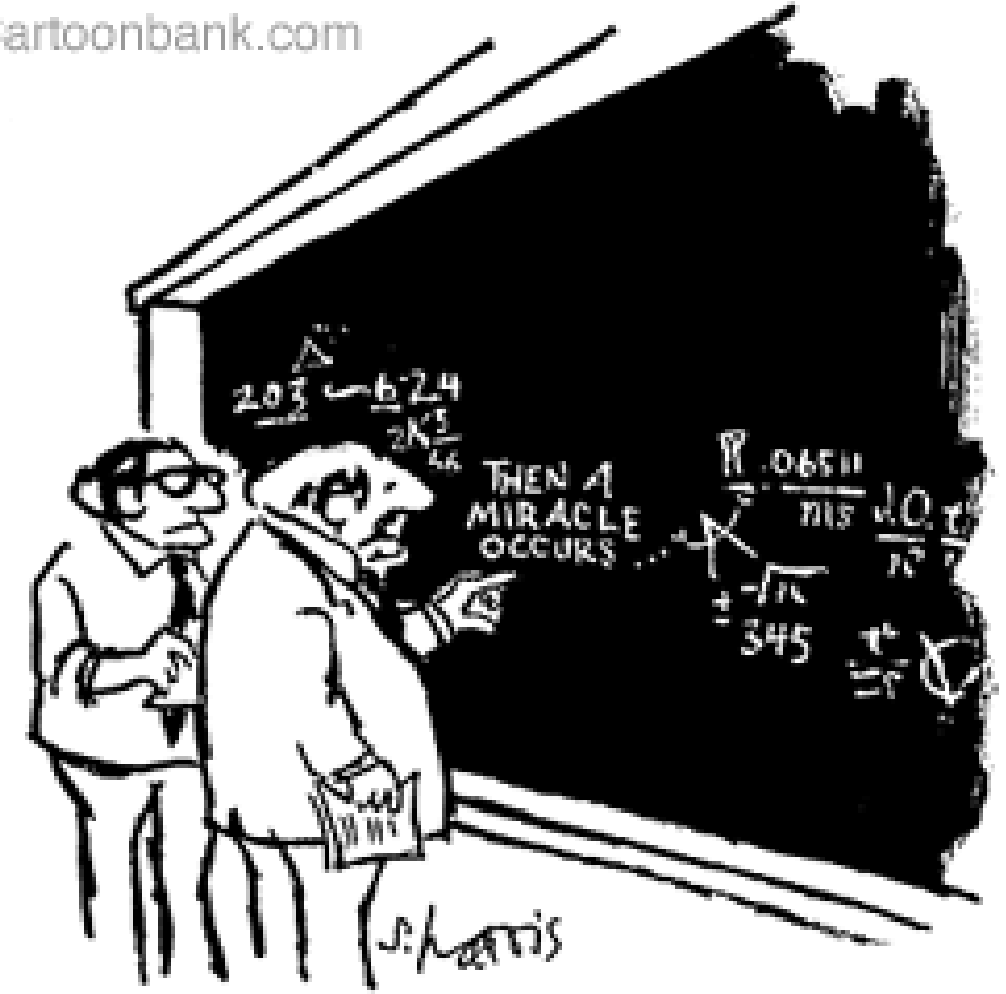
SEWM 2012

Swansea University

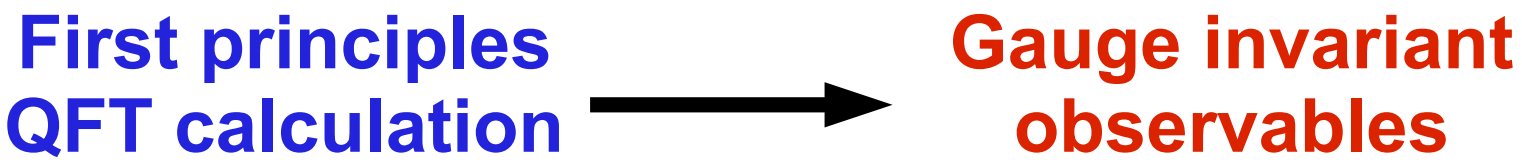




© Cartoonbank.com



"I think you should be more explicit here in step two."


First principles
QFT calculation
Gauge invariant
observables

First principles QFT calculation

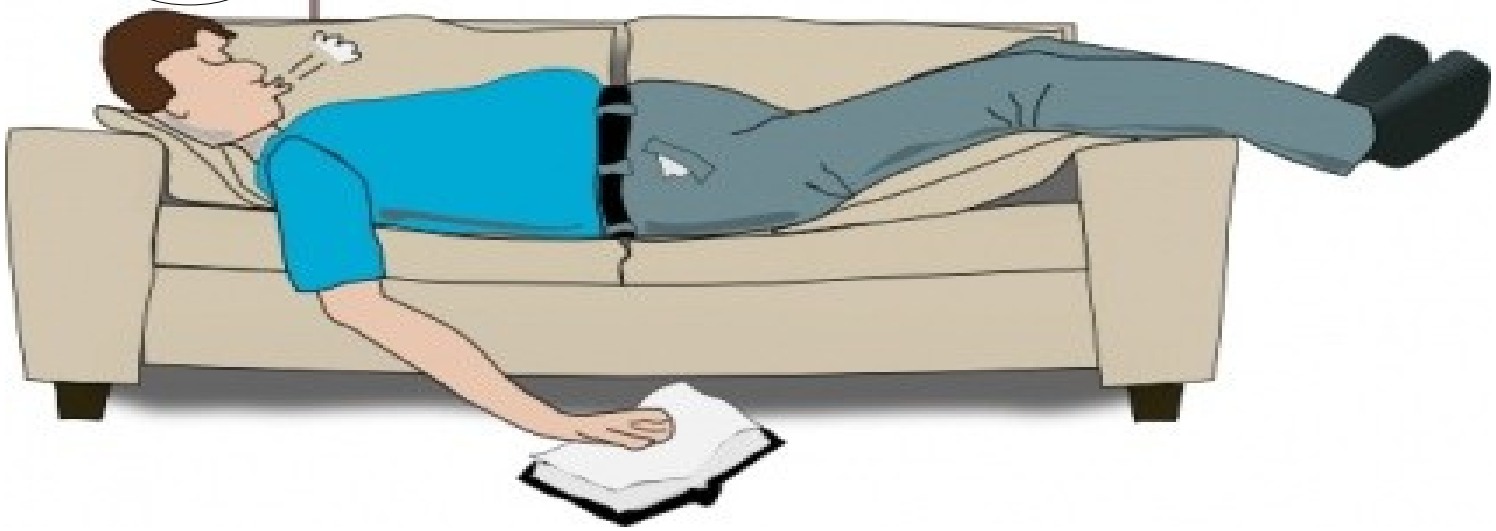
~~**Gauge invariant observables**~~

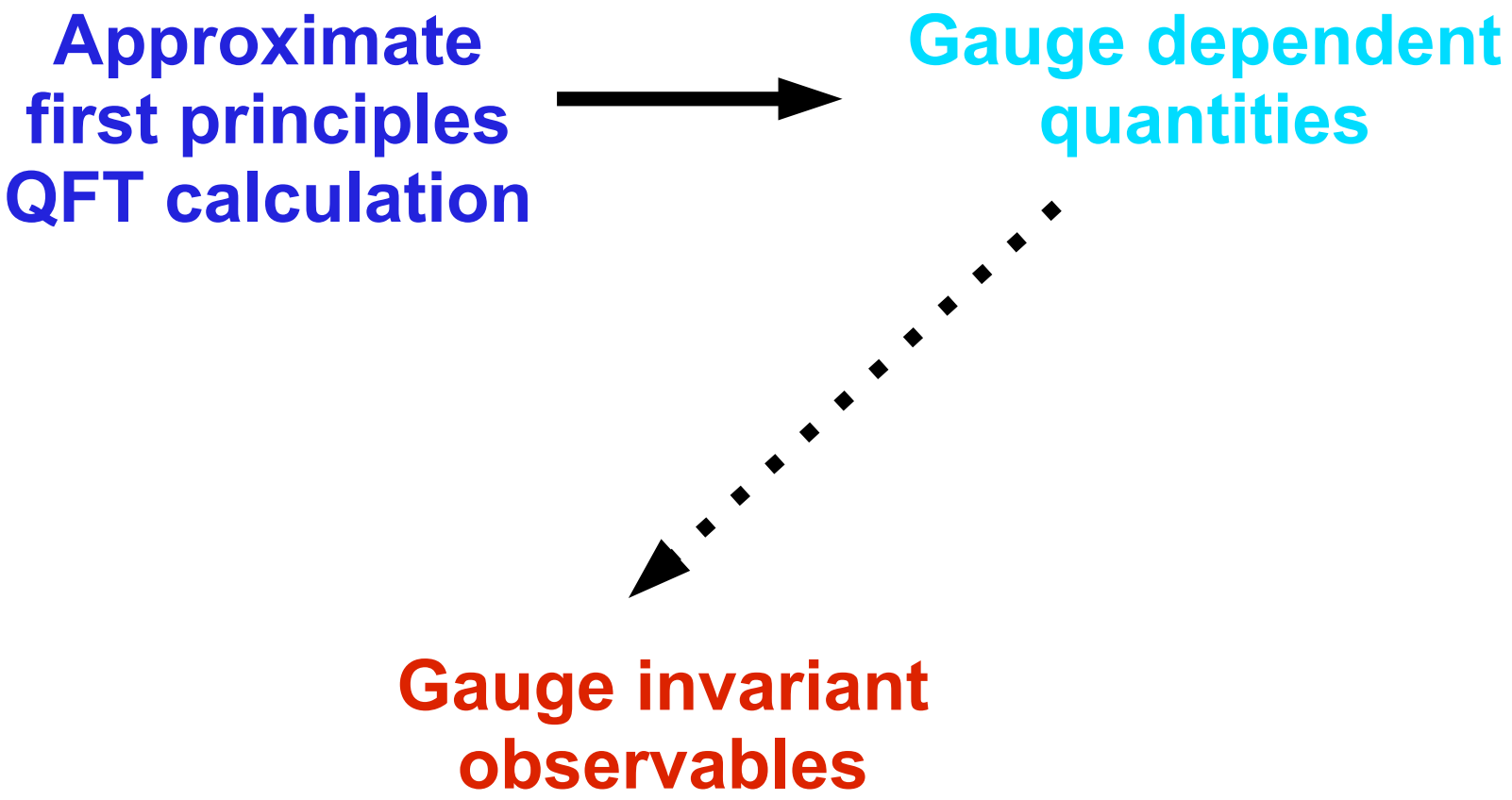
$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\ F_{\mu\nu}F^{\mu\nu}$$

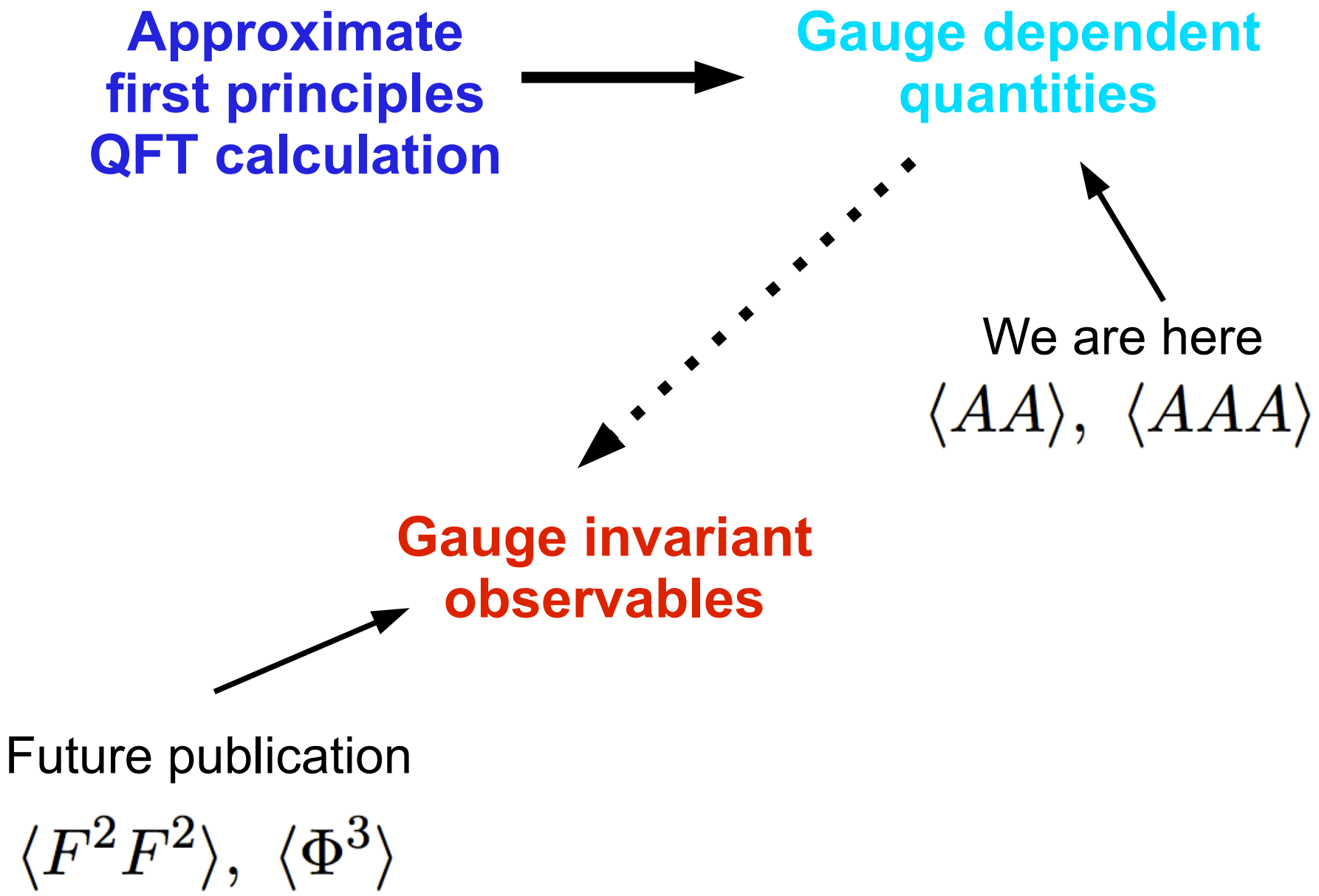
mathematically intractable,
Yang Mills not exactly solvable

First principles
QFT calculation

Gauge invariant
observables




Approximate
first principles
QFT calculation
Gauge dependent
quantities
Gauge invariant
observables


Approximate
first principles
QFT calculation
Gauge dependent
quantities
We are here
$\langle AA \rangle,\ \langle AAA \rangle$
Gauge invariant
observables
Future publication
$\langle F^2 F^2 \rangle,\ \langle \Phi^3 \rangle$

3 Loop 3PI for 3D Yang Mills

Remainder of talk:

- What is 3 loop 3PI?
- Why 3D Yang Mills?
- How do you solve it?
- Results and comparison

Main objects of 3PI with Yang Mills

Gluon propagator $G^{ab}_{\mu\nu} =$ 

Ghost propagator $\Delta^{ab} =$ 

Ghost-gluon vertex $\mathbb{V}^{abc}_{\mu} =$ 

Gluon 3-vertex $V^{abc}_{\mu\nu\rho} =$ 

Gluon 4-vertex $V^{(0)abcd}_{\mu\nu\rho\tau} =$ 

Main objects of 3PI with Yang Mills

Gluon propagator $G^{ab}_{\mu\nu} =$ 

Ghost propagator $\Delta^{ab} =$ 

Ghost-gluon vertex $\mathbb{V}^{abc}_{\mu} =$ 

Gluon 3-vertex $V^{abc}_{\mu\nu\rho} =$ 

Included in 3PI formalism

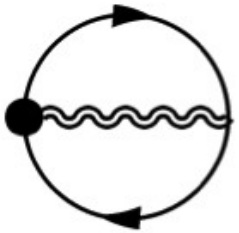
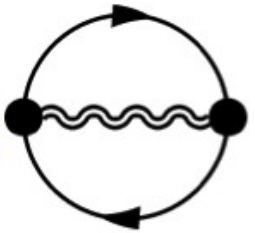
Gluon 4-vertex $V^{(0)abcd}_{\mu\nu\rho\tau} =$ 

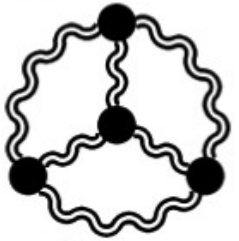
Need 4PI

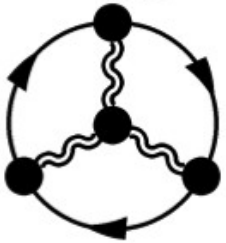
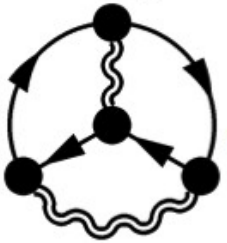
The 3PI effective action

$$\Gamma = S[A^\mu, \bar{c}, c] + \frac{1}{2}\mathrm{Tr}\log G - \frac{1}{2}\mathrm{Tr}[G^{(0)}]^{-1}G - \mathrm{Tr}\log\Delta + \mathrm{Tr}[\Delta^{(0)}]^{-1}\Delta$$

$+\frac{1}{6}$  $-\frac{1}{12}$  $+\frac{1}{8}$ 

$-$  $+\frac{1}{2}$ 

$+\frac{1}{48}$  $+\frac{1}{24}$  $+\frac{1}{8}$ 

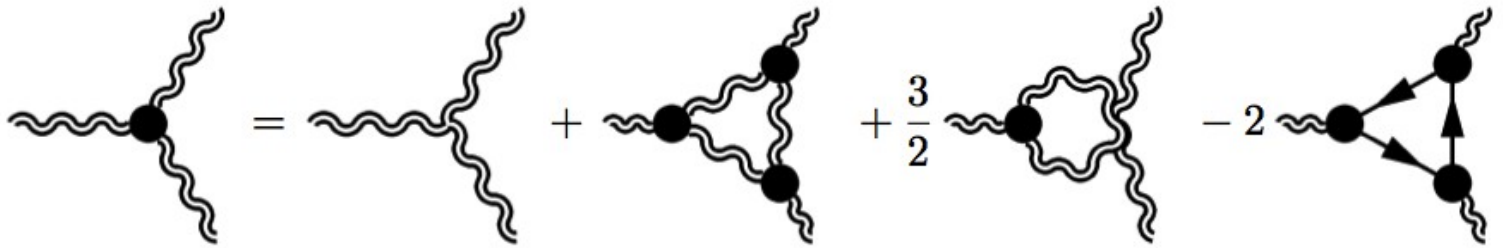
$-\frac{1}{3}$  $-\frac{1}{4}$ ,

$$\frac{\delta\Gamma}{\delta G} = \frac{\delta\Gamma}{\delta\Delta} = \frac{\delta\Gamma}{\delta V} = \frac{\delta\Gamma}{\delta\mathbb{V}} = 0$$

3PI resummation

$$\frac{\delta\Gamma}{\delta V} = 0$$

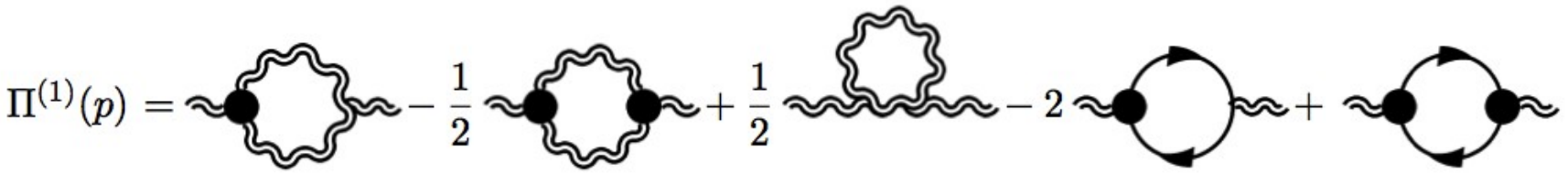
Vertex resummation



$$\frac{\delta\Gamma}{\delta G} = 0$$

Self-energy resummation

$$G^{-1}(p) = p^2 - \Pi^{(1)}(p) - \Pi^{(2)}(p)$$



Why 3D / Why resum?

One Loop self energy



$$\sim g^2 N_c \Gamma(-1/2-\epsilon)\sqrt{p^2}\left(\frac{p^2}{\mu^2_{\rm MS}}\right)^{\epsilon}$$

Two loops self energy



$$\sim g^4 N_c^2 \Gamma(\epsilon)\left(\frac{p^2}{\mu^2_{\rm MS}}\right)^{2\epsilon}$$



$$\sim \frac{1}{\epsilon_{\rm IR}}$$

Why 3D / Why resum?

One loop self energy



$$\sim g^2 N_c \Gamma(-1/2-\epsilon)\sqrt{p^2}\left(\frac{p^2}{\mu^2_{\rm MS}}\right)^{\epsilon}$$

One loop is ok

Two loop self energy



$$\sim g^4 N_c^2 \Gamma(\epsilon)\left(\frac{p^2}{\mu^2_{\rm MS}}\right)^{2\epsilon}$$

UV divergent, but theory is superrenormalizable



$$\sim \frac{1}{\epsilon_{\rm IR}}$$

Infrared divergence: BAD!

Perturbation theory fails at 2 loops, so...
What if we consider an infinite number of diagrams?

Why 3D / Why resum?

Why 3D?

- UV behaviour of Yang Mills much better in 3D than in 4D

Why resum?

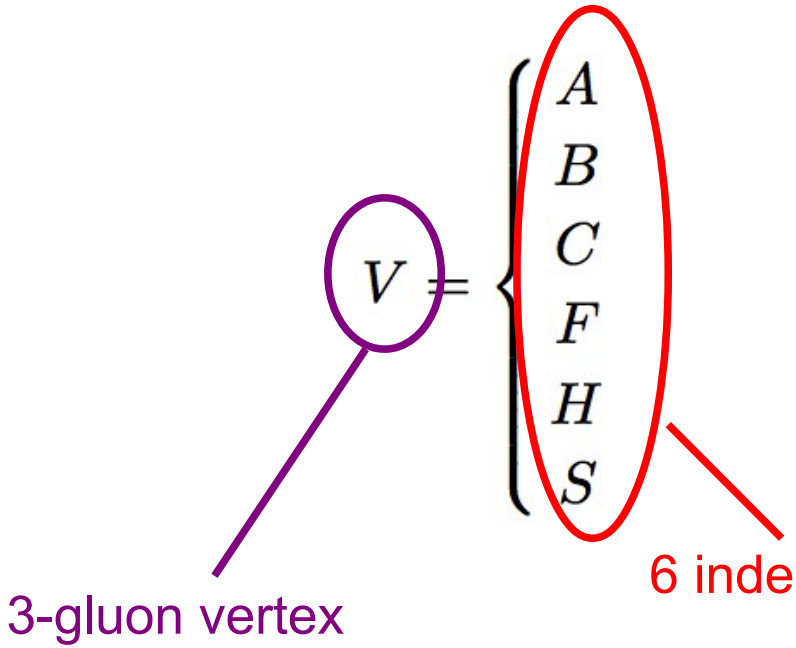
- IR divergences cured by generation of a mass; i.e. resummed gluon propagator is finite at p = 0.

Solving 3PI Self-Consistently

We want to solve

$$\frac{\delta\Gamma}{\delta G} = \frac{\delta\Gamma}{\delta\Delta} = \frac{\delta\Gamma}{\delta V} = \frac{\delta\Gamma}{\delta\mathbb{V}} = 0$$

And to do so, we must actually solve for 11 functions



$$\mathbb{V} = \begin{cases} \mathbb{A} \\ \mathbb{B} \end{cases}$$

$$G = \begin{cases} G_T \\ G_L \end{cases}$$

Δ

Why So Many Functions?

Vertex basis tensors (Ball, Chiu):

$$\mathbf{A}_{\mu_1\mu_2\mu_3} = g_{\mu_1\mu_2}(p_1 - p_2)_{\mu_3}$$
$$\mathbf{B}_{\mu_1\mu_2\mu_3} = g_{\mu_1\mu_2}(p_1 + p_2)_{\mu_3}$$
$$\mathbf{C}_{\mu_1\mu_2\mu_3} = \left[p_1 \cdot p_2 g_{\mu_1\mu_2} - p_{1\mu_2}p_{2\mu_1}\right](p_1 - p_2)_{\mu_3}$$
$$\mathbf{F}_{\mu_1\mu_2\mu_3} = \left[p_1 \cdot p_2 g_{\mu_1\mu_2} - p_{1\mu_2}p_{2\mu_1}\right]\left[(p_2 \cdot p_3)p_{1\mu_3} - (p_1 \cdot p_3)p_{2\mu_3}\right]$$
$$\mathbf{H}_{\mu_1\mu_2\mu_3} = g_{\mu_1\mu_2}\left[(p_1 \cdot p_3)p_{2\mu_3} - (p_2 \cdot p_3)p_{1\mu_3}\right] + \frac{1}{3}(p_{1\mu_3}p_{2\mu_1}p_{3\mu_2} - p_{1\mu_2}p_{2\mu_3}p_{3\mu_1})$$
$$\mathbf{S}_{\mu_1\mu_2\mu_3} = p_{1\mu_3}p_{2\mu_1}p_{3\mu_2} + p_{1\mu_2}p_{2\mu_3}p_{3\mu_1}.$$

Propagator basis tensors:

$$\mathbf{L}^{\mu\nu} \equiv \frac{p^\mu p^\nu}{p^2}, \qquad \mathbf{T}^{\mu\nu} \equiv g^{\mu\nu} - \mathbf{L}^{\mu\nu}$$

ALL INDEPENDENT!

Variational Ansaetze

Transverse gluon propagator:

$$G_T(p) = \frac{1}{p^2 - \Pi_T(p)}$$

$$\Pi_T(p) = \frac{\xi^2 + 2\xi + 11}{64} p + \frac{\sum_{i=0}^{N_{\max}} a_i^{\{G_T\}} p^i}{\sum_{i=1}^{N_{\max}} b_i^{\{G_T\}} p^i + 1}$$

Variational Ansaetze

Transverse gluon propagator:

$$G_T(p) = \frac{1}{p^2 - \Pi_T(p)}$$

$$\Pi_T(p) = \frac{\xi^2 + 2\xi + 11}{64} p + \frac{\sum_{i=0}^{N_{\max}} a_i^{\{G_T\}} p^i}{\sum_{i=1}^{N_{\max}} b_i^{\{G_T\}} p^i + 1}$$

One loop result inserted by hand

Pade approximant, O(1) in the UV

Variational Ansaetze

3-gluon vertex A-function:

$$A(p_1, p_2; p_3) = 1 + A^{(1)}(p_1, p_2; p_3) + A^{\mathrm{NP}}(p_1, p_2; p_3)$$

Variational Ansaetze

3-gluon vertex A-function:

$$A(p_1,p_2;p_3) = 1 + A^{(1)}(p_1,p_2;p_3) + A^{\rm NP}(p_1,p_2;p_3)$$

Tree level

One loop result from pert. theory

Pade approximant

$$A^{\rm NP}(p_1,p_2;p_3) = \frac{1}{p_1^2+p_2^2+p_3^2+\omega^2} \frac{\sum_{i\geq j} a_{ijk}^{\{A\}} (p_1^i p_2^j + p_1^j p_2^i) p_3^k}{\sum_{i\geq j} b_{ijk}^{\{A\}} (p_1^i p_2^j + p_1^j p_2^i) p_3^k}$$

Similar for all of the other functions

Solving 3PI

$$\frac{\delta\Gamma}{\delta G} = \frac{\delta\Gamma}{\delta\Delta} = \frac{\delta\Gamma}{\delta V} = \frac{\delta\Gamma}{\delta\mathbb{V}} = 0 \quad \longrightarrow \quad \frac{\partial\Gamma}{\partial c_i} = 0$$

Root finding (numerically):

- Conjugate gradient descent
- Newton-Raphson

Caveat:

- Numerical integrals must converge

3PI Regularization

$$\frac{\partial}{\partial c_i}$$  $$\sim \underbrace{\int d^3p \, \frac{\partial G}{\partial c_i}}_{p-integral} \overbrace{\int d^3q \, G^2 V^2}^{subdiagram}$$

3PI Regularization



$$\sim \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2 + g^2 p + m^2} \sim \Lambda$$

Linearly divergent
(numerically)

3PI Regularization



$$\sim \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2 + g^2 p + m^2} \sim \Lambda$$

Linearly divergent (numerically)

Add “0”



$$= 0$$

3PI Regularization



$$\sim \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2 + g^2 p + m^2} \sim \Lambda$$

Linearly divergent (numerically)

Add “0”



$$= 0$$



$$+ \quad 0 =$$

3PI Regularization



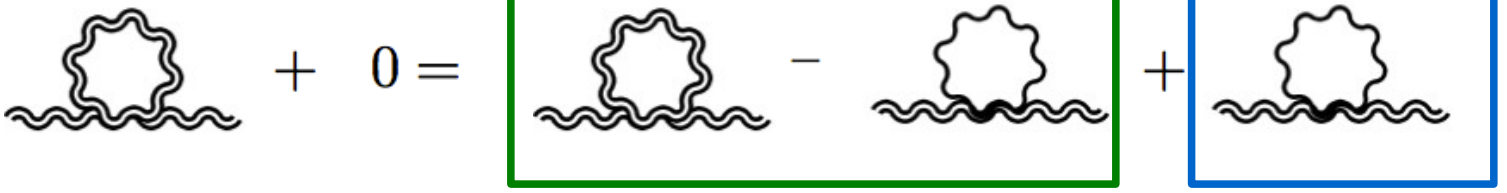
$$\sim \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2 + g^2 p + m^2} \sim \Lambda$$

Linearly divergent (numerically)

Add “0”



$$= 0$$



$$+ \quad 0 =$$

$$-$$

$$+$$

~ log numerically

finite, computable in dim. reg.

Other divergences...

In short, suppose

$$G_T(p) = \frac{c_0}{p+\omega} + \frac{c_1}{p^2+\omega^2} + \text{Padé}$$

i.e.

$$G_T(p) \sim \frac{c_0}{p} \nsim \left.\frac{1}{p^2}\right|_{p\gg 1}$$

Other divergences...

In short, suppose

$$G_T(p) = \frac{c_0}{p+\omega} + \frac{c_1}{p^2+\omega^2} + \text{Padé}$$

i.e.

$$G_T(p) \sim \frac{c_0}{p} \nsim \frac{1}{p^2}\bigg|_{p\gg 1} \longrightarrow \frac{\partial\Gamma}{\partial c_0} = \infty$$

Simple solution:

$$G_T(p) \sim \text{bare} + \text{one loop} + \text{Ansatz at } \mathcal{O}(1/p^4)\bigg|_{p\gg 1}$$

Enforce the correct asymptotic UV behaviour in G, V etc

When all the dust settles...

$$G^{-1}(p) = G_0^{-1}(p) - \Pi^{(1)}(p) - \Pi^{(2)}(p)$$

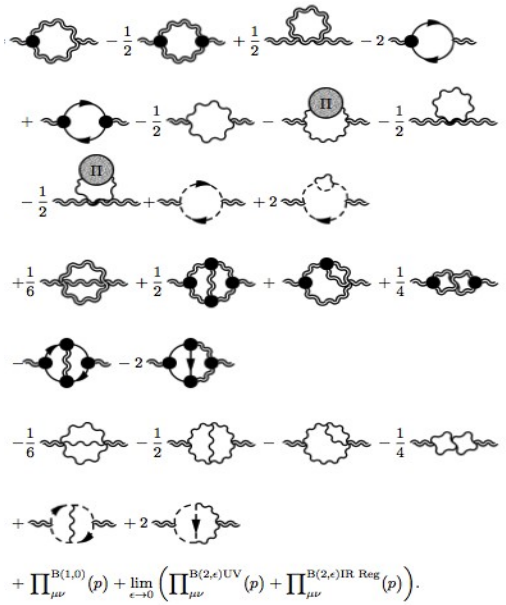
$$\frac{\partial \Gamma}{\partial c_i} \sim \int \frac{d^3p}{(2\pi)^3} \frac{\partial G_T}{\partial c_i} \left(p^2 - \Pi_T^{\text{Ansatz}} - p^2 + \Pi_T^{(1)} + \Pi_T^{(2)} \right)$$

When all the dust settles...

$$G^{-1}(p) = G_0^{-1}(p) - \Pi^{(1)}(p) - \Pi^{(2)}(p)$$

$$\frac{\partial \Gamma}{\partial c_i} \sim \int \frac{d^3p}{(2\pi)^3} \frac{\partial G_T}{\partial c_i} \left(p^2 - \Pi_T^{\text{Ansatz}} - p^2 + \Pi_T^{(1)} + \Pi_T^{(2)} \right)$$

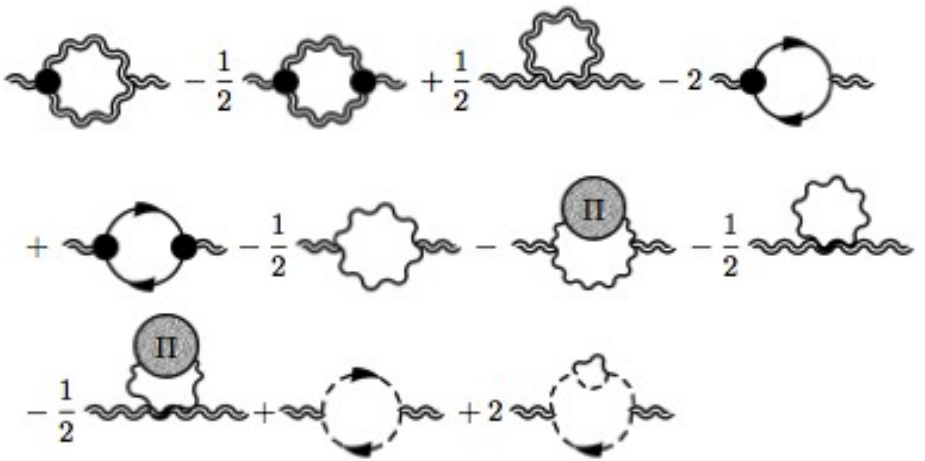
$$\Pi_{\mu\nu}^{(1)} + \Pi_{\mu\nu}^{(2)} =$$

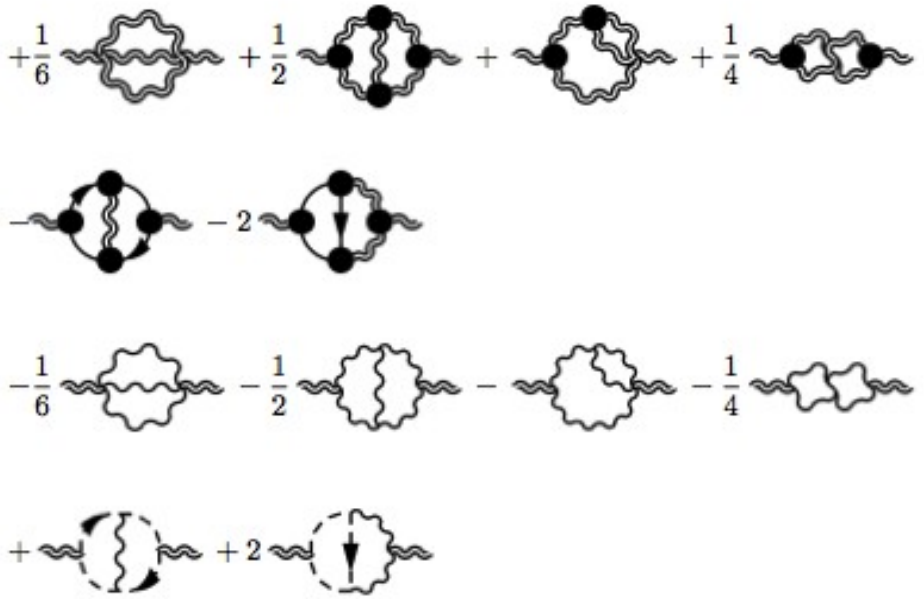


$$+ \Pi_{\mu\nu}^{\mathrm{B}(1,0)}(p) + \lim_{\epsilon \to 0} \left(\Pi_{\mu\nu}^{\mathrm{B}(2,\epsilon)\mathrm{UV}}(p) + \Pi_{\mu\nu}^{\mathrm{B}(2,\epsilon)\mathrm{IR\ Reg}}(p) \right).$$

Derivative vanishes
when SD eqn. is solved!

$$\Pi^{(1)}_{\mu\nu} + \Pi^{(2)}_{\mu\nu} =$$





$$+ \prod^{\mathrm{B}(1,0)}_{\mu\nu}(p) + \lim_{\epsilon\to 0}\left(\prod^{\mathrm{B}(2,\epsilon)\mathrm{UV}}_{\mu\nu}(p) + \prod^{\mathrm{B}(2,\epsilon)\mathrm{IR\ Reg}}_{\mu\nu}(p)\right).$$

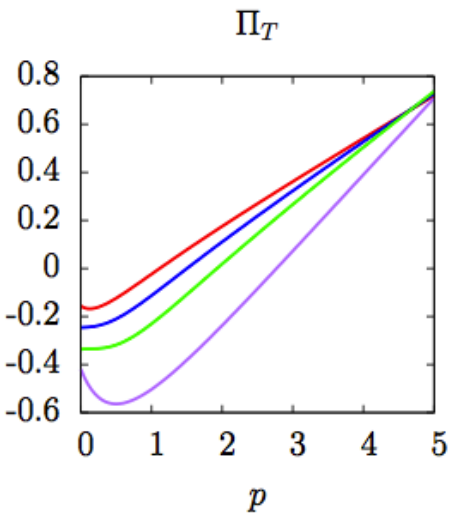
$$\Pi^{(1)}_{\mu\nu} + \Pi^{(2)}_{\mu\nu} =$$

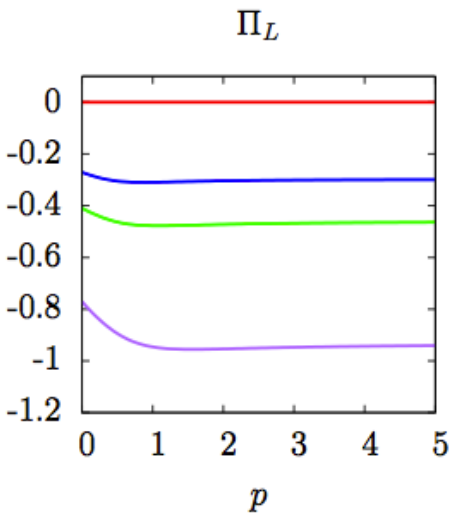
```
pq*qz*xz*gll[4]*glt[5]*zf010[5]*zf200[11]*zf212[6] - pq*qx*z2*gll[4]*glt[5]*zf010[5]*zf200[11]*zf212[6] +
pq*qz*xz*gll[4]*gll[5]*zf020[5]*zf200[11]*zf212[6] - pq*qz*xz*gll[4]*glt[5]*zf020[5]*zf200[11]*zf212[6] +
pq*qx*z2*gll[4]*glt[5]*zf020[5]*zf200[11]*zf212[6] - pq*qz*xz*z2*gll[4]*gll[5]*zf111[5]*zf200[11]*zf212[6] +
pq*qz*xz*z2*gll[4]*gll[5]*zf121[5]*zf200[11]*zf212[6] - kz*pq*qz*yz*gll[4]*gll[5]*zf020[11]*zf211[5]*zf212[6] +
kz*pq*qz*yz*gll[4]*glt[5]*zf020[11]*zf211[5]*zf212[6] - ky*pq*qz*z2*gll[4]*glt[5]*zf020[11]*zf211[5]*zf212[6] +
kz*pq*qz*z2*gll[4]*gll[5]*zf100[11]*zf211[5]*zf212[6] - kz*pq*qz*yz*z2*gll[4]*gll[5]*zf121[11]*zf211[5]*zf212[6] -
kz*pq*qz*xy*z2*gll[4]*gll[5]*zf122[11]*zf211[5]*zf212[6] + kz*pq*qz*xz*gll[4]*gll[5]*zf200[11]*zf211[5]*zf212[6] -
kz*pq*qz*xz*gll[4]*glt[5]*zf200[11]*zf211[5]*zf212[6] + kx*pq*qz*z2*gll[4]*glt[5]*zf200[11]*zf211[5]*zf212[6] +
kz*pq*pz*qz*gll[5]*glt[4]*zf020[11]*zf111[6]*zf221[5] - kz*pq*pz*qz*glt[4]*glt[5]*zf020[11]*zf111[6]*zf221[5] +
kp*pq*qz*z2*glt[4]*glt[5]*zf020[11]*zf111[6]*zf221[5] - kz*pq*qz*z2*gll[5]*glt[4]*zf100[11]*zf111[6]*zf221[5] -
kz*pq*qz*yz*gll[4]*gll[5]*zf020[11]*zf112[6]*zf221[5] + kz*pq*qz*yz*gll[4]*glt[5]*zf020[11]*zf112[6]*zf221[5] -
ky*pq*qz*z2*gll[4]*glt[5]*zf020[11]*zf112[6]*zf221[5] + kz*pq*qz*z2*gll[4]*gll[5]*zf100[11]*zf112[6]*zf221[5] -
kz*pz*qy*qz*gll[5]*glt[4]*zf020[11]*zf121[6]*zf221[5] + kz*pz*qy*qz*glt[4]*glt[5]*zf020[11]*zf121[6]*zf221[5] -
kp*qy*qz*z2*glt[4]*glt[5]*zf020[11]*zf121[6]*zf221[5] + kz*qy*qz*z2*gll[5]*glt[4]*zf100[11]*zf121[6]*zf221[5] +
kz*pq*pz*qz*z2*gll[5]*glt[4]*zf111[6]*zf121[11]*zf221[5] - kz*pq*qz*yz*z2*gll[4]*gll[5]*zf112[6]*zf121[11]*zf221[5] -
kz*pz*qy*qz*z2*gll[5]*glt[4]*zf121[6]*zf121[11]*zf221[5] + kz*qy*qz*yz*gll[4]*gll[5]*zf020[11]*zf122[6]*zf221[5] -
kz*qy*qz*yz*gll[4]*glt[5]*zf020[11]*zf122[6]*zf221[5] + ky*qy*qz*z2*gll[4]*glt[5]*zf020[11]*zf122[6]*zf221[5] -
kz*qy*qz*z2*gll[4]*gll[5]*zf100[11]*zf122[6]*zf221[5] + kz*qy*qz*yz*z2*gll[4]*gll[5]*zf121[11]*zf122[6]*zf221[5] -
kz*pq*qz*z2*gll[5]*glt[4]*zf010[6]*zf122[11]*zf221[5] + kz*qy*qz*z2*gll[5]*glt[4]*zf020[6]*zf122[11]*zf221[5] +
kz*qx*qz*z2*gll[5]*glt[4]*zf100[6]*zf122[11]*zf221[5] + kz*pq*px*qz*z2*gll[5]*glt[4]*zf111[6]*zf122[11]*zf221[5] -
kz*pq*qz*xy*z2*gll[4]*gll[5]*zf112[6]*zf122[11]*zf221[5] - kz*px*qy*qz*z2*gll[5]*glt[4]*zf121[6]*zf122[11]*zf221[5] +
kz*qy*qz*xy*z2*gll[4]*gll[5]*zf122[6]*zf122[11]*zf221[5] - kz*qx*qz*z2*gll[5]*glt[4]*zf122[11]*zf200[6]*zf221[5] -
kz*pq*qz*xz*gll[5]*glt[4]*zf111[6]*zf200[11]*zf221[5] + kz*pq*qz*xz*glt[4]*glt[5]*zf111[6]*zf200[11]*zf221[5] -
kx*pq*qz*z2*glt[4]*glt[5]*zf111[6]*zf200[11]*zf221[5] + kz*pq*qz*xz*gll[4]*gll[5]*zf112[6]*zf200[11]*zf221[5] -
kz*pq*qz*xz*gll[4]*glt[5]*zf112[6]*zf200[11]*zf221[5] + kx*pq*qz*z2*gll[4]*glt[5]*zf112[6]*zf200[11]*zf221[5] +
kz*qy*qz*xz*gll[5]*glt[4]*zf121[6]*zf200[11]*zf221[5] - kz*qy*qz*xz*glt[4]*glt[5]*zf121[6]*zf200[11]*zf221[5] +
kx*qy*qz*z2*glt[4]*glt[5]*zf121[6]*zf200[11]*zf221[5] - kz*qy*qz*xz*gll[4]*gll[5]*zf122[6]*zf200[11]*zf221[5] +
kz*qy*qz*xz*gll[4]*glt[5]*zf122[6]*zf200[11]*zf221[5] - kx*qy*qz*z2*gll[4]*glt[5]*zf122[6]*zf200[11]*zf221[5] -
kz*pq*pz*qz*gll[5]*glt[4]*zf020[11]*zf211[6]*zf221[5] + kz*pq*pz*qz*glt[4]*glt[5]*zf020[11]*zf211[6]*zf221[5] -
kp*pq*qz*z2*glt[4]*glt[5]*zf020[11]*zf211[6]*zf221[5] + kz*pq*qz*z2*gll[5]*glt[4]*zf100[11]*zf211[6]*zf221[5] -
kz*pq*pz*qz*z2*gll[5]*glt[4]*zf121[11]*zf211[6]*zf221[5] - kz*pq*px*qz*z2*gll[5]*glt[4]*zf122[11]*zf211[6]*zf221[5] +
kz*pq*qz*xz*gll[5]*glt[4]*zf200[11]*zf211[6]*zf221[5] - kz*pq*qz*xz*glt[4]*glt[5]*zf200[11]*zf211[6]*zf221[5] +
kx*pq*qz*z2*glt[4]*glt[5]*zf200[11]*zf211[6]*zf221[5] + kz*pq*qz*yz*gll[4]*gll[5]*zf020[11]*zf212[6]*zf221[5] -
kz*pq*qz*yz*gll[4]*glt[5]*zf020[11]*zf212[6]*zf221[5] + ky*pq*qz*z2*gll[4]*glt[5]*zf020[11]*zf212[6]*zf221[5] -
kz*pq*qz*z2*gll[4]*gll[5]*zf100[11]*zf212[6]*zf221[5] + kz*pq*qz*yz*z2*gll[4]*gll[5]*zf121[11]*zf212[6]*zf221[5] +
kz*pq*qz*xy*z2*gll[4]*gll[5]*zf122[11]*zf212[6]*zf221[5] - kz*pq*qz*xz*gll[4]*gll[5]*zf200[11]*zf212[6]*zf221[5] +
kz*pq*qz*xz*gll[4]*glt[5]*zf200[11]*zf212[6]*zf221[5] - kx*pq*qz*z2*gll[4]*glt[5]*zf200[11]*zf212[6]*zf221[5] +
pz*qy*qz*gll[5]*glt[4]*zf001[5]*zf020[11]*zf221[6] - pz*qy*qz*glt[4]*glt[5]*zf001[5]*zf020[11]*zf221[6] +
pz*qy*qz*gll[5]*glt[4]*zf010[5]*zf020[11]*zf221[6] - pz*qy*qz*glt[4]*glt[5]*zf010[5]*zf020[11]*zf221[6] +
pq*qy*z2*glt[4]*glt[5]*zf010[5]*zf020[11]*zf221[6] - pz*qy*qz*gll[5]*glt[4]*zf020[5]*zf020[11]*zf221[6] +
pz*qy*qz*glt[4]*glt[5]*zf020[5]*zf020[11]*zf221[6] - pq*qy*z2*glt[4]*glt[5]*zf020[5]*zf020[11]*zf221[6] -
qy*qz*z2*gll[5]*glt[4]*zf001[5]*zf100[11]*zf221[6] - qy*qz*z2*gll[5]*glt[4]*zf010[5]*zf100[11]*zf221[6] +
qy*qz*z2*gll[5]*glt[4]*zf020[5]*zf100[11]*zf221[6] + pz*qy*qz*z2*gll[5]*glt[4]*zf020[11]*zf111[5]*zf221[6] -
qy*qz*z4*gll[5]*glt[4]*zf100[11]*zf111[5]*zf221[6] - pz*qy*qz*z2*gll[5]*glt[4]*zf020[11]*zf121[5]*zf221[6] +
qy*qz*z4*gll[5]*glt[4]*zf100[11]*zf121[5]*zf221[6] + pz*qy*qz*z2*gll[5]*glt[4]*zf001[5]*zf121[11]*zf221[6] +
```

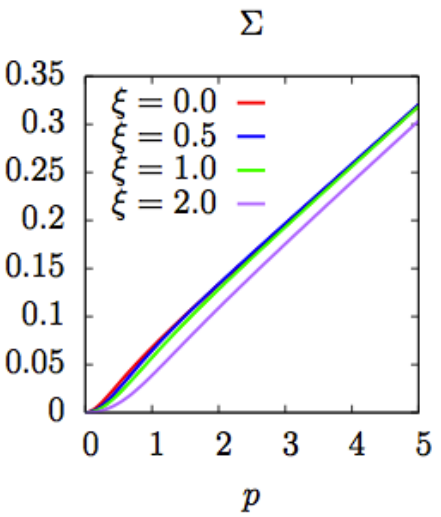
Solution

$a_0^{\{G_T\}}$	-1.56e-01	$a_1^{\{G_T\}}$	-1.03e+00	$a_2^{\{G_T\}}$	-9.14e-01	$a_3^{\{G_T\}}$	-4.47e-01	$b_1^{\{G_T\}}$	4.11e+00	$b_2^{\{G_T\}}$	4.10e+00
$b_3^{\{G_T\}}$	3.82e+00	$a_2^{\{\Delta\}}$	1.79e-01	$a_3^{\{\Delta\}}$	1.13e-01	$b_1^{\{\Delta\}}$	1.73e+00	$b_2^{\{\Delta\}}$	3.49e+00	$b_3^{\{\Delta\}}$	1.75e+00
$a_{000}^{\{H\}}$	1.02e-01	$a_{100}^{\{H\}}$	-4.98e-02	$a_{110}^{\{H\}}$	-2.73e-01	$a_{111}^{\{H\}}$	-5.15e-01	$a_{200}^{\{H\}}$	-8.11e-01	$a_{210}^{\{H\}}$	1.40e-01
$a_{300}^{\{H\}}$	5.49e-01	$b_{100}^{\{H\}}$	1.19e+00	$b_{110}^{\{H\}}$	1.29e-01	$b_{111}^{\{H\}}$	2.38e-01	$b_{200}^{\{H\}}$	1.04e-01	$b_{210}^{\{H\}}$	-1.77e-01
$b_{300}^{\{H\}}$	3.20e-01	$a_{000}^{\{A\}}$	-2.77e-01	$a_{001}^{\{A\}}$	-7.66e-01	$a_{002}^{\{A\}}$	3.70e-01	$a_{003}^{\{A\}}$	8.59e-02	$a_{100}^{\{A\}}$	-9.45e-01
$a_{101}^{\{A\}}$	4.90e-01	$a_{102}^{\{A\}}$	-5.26e-02	$a_{110}^{\{A\}}$	7.11e-01	$a_{111}^{\{A\}}$	-8.68e-01	$a_{200}^{\{A\}}$	7.03e-02	$a_{201}^{\{A\}}$	-3.46e-01
$a_{210}^{\{A\}}$	-2.06e+00	$a_{300}^{\{A\}}$	-7.44e-02	$b_{001}^{\{A\}}$	1.54e+00	$b_{002}^{\{A\}}$	5.90e-01	$b_{003}^{\{A\}}$	6.73e-01	$b_{100}^{\{A\}}$	6.22e+00
$b_{101}^{\{A\}}$	8.78e-01	$b_{102}^{\{A\}}$	7.12e-01	$b_{110}^{\{A\}}$	1.85e+00	$b_{111}^{\{A\}}$	-6.23e-01	$b_{200}^{\{A\}}$	1.93e+00	$b_{201}^{\{A\}}$	-2.77e-01
$b_{210}^{\{A\}}$	-1.69e-01	$b_{300}^{\{A\}}$	3.42e-01	$a_{000}^{\{C\}}$	-1.53e+00	$a_{001}^{\{C\}}$	-1.74e+00	$a_{002}^{\{C\}}$	-6.63e-01	$a_{003}^{\{C\}}$	3.15e-02
$a_{100}^{\{C\}}$	5.94e-02	$a_{101}^{\{C\}}$	-2.47e-01	$a_{102}^{\{C\}}$	-2.09e-02	$a_{110}^{\{C\}}$	3.44e-01	$a_{111}^{\{C\}}$	-4.98e-01	$a_{200}^{\{C\}}$	-2.97e+00
$a_{201}^{\{C\}}$	6.72e-01	$a_{210}^{\{C\}}$	-4.27e-02	$a_{300}^{\{C\}}$	5.05e-01	$b_{001}^{\{C\}}$	4.11e-01	$b_{002}^{\{C\}}$	6.14e-01	$b_{003}^{\{C\}}$	7.04e-01
$b_{100}^{\{C\}}$	2.19e-01	$b_{101}^{\{C\}}$	5.53e-01	$b_{102}^{\{C\}}$	6.94e-01	$b_{110}^{\{C\}}$	4.13e-01	$b_{111}^{\{C\}}$	6.53e-01	$b_{200}^{\{C\}}$	3.68e-01
$b_{201}^{\{C\}}$	6.20e-01	$b_{210}^{\{C\}}$	5.21e-01	$b_{300}^{\{C\}}$	4.46e-01	$a_{000}^{\{F\}}$	-5.67e-02	$a_{001}^{\{F\}}$	-3.53e-03	$a_{002}^{\{F\}}$	9.77e-03
$a_{003}^{\{F\}}$	4.70e-01	$a_{100}^{\{F\}}$	-4.18e-02	$a_{101}^{\{F\}}$	-1.96e-02	$a_{102}^{\{F\}}$	3.20e-01	$a_{110}^{\{F\}}$	-5.84e-02	$a_{111}^{\{F\}}$	2.00e-01
$a_{200}^{\{F\}}$	-5.23e-02	$a_{201}^{\{F\}}$	2.57e-01	$a_{210}^{\{F\}}$	1.37e-01	$a_{300}^{\{F\}}$	2.28e-01	$b_{001}^{\{F\}}$	9.96e-01	$b_{002}^{\{F\}}$	9.96e-01
$b_{003}^{\{F\}}$	9.97e-01	$b_{100}^{\{F\}}$	9.96e-01	$b_{101}^{\{F\}}$	9.97e-01	$b_{102}^{\{F\}}$	9.98e-01	$b_{110}^{\{F\}}$	9.97e-01	$b_{111}^{\{F\}}$	9.98e-01
$b_{200}^{\{F\}}$	9.97e-01	$b_{201}^{\{F\}}$	9.98e-01	$b_{210}^{\{F\}}$	9.99e-01	$b_{300}^{\{F\}}$	9.99e-01	$a_{000}^{\{A\}}$	-8.12e-01	$a_{001}^{\{A\}}$	-1.02e-01
$a_{002}^{\{A\}}$	2.02e-01	$a_{003}^{\{A\}}$	2.12e-01	$a_{010}^{\{A\}}$	3.59e-01	$a_{011}^{\{A\}}$	-6.42e-01	$a_{012}^{\{A\}}$	-3.44e-01	$a_{020}^{\{A\}}$	-3.01e-01
$a_{021}^{\{A\}}$	-1.23e-01	$a_{030}^{\{A\}}$	2.68e-01	$a_{100}^{\{A\}}$	-1.23e-01	$a_{101}^{\{A\}}$	-9.41e-02	$a_{102}^{\{A\}}$	-1.16e-01	$a_{110}^{\{A\}}$	-4.18e-01
$a_{111}^{\{A\}}$	4.75e-01	$a_{120}^{\{A\}}$	2.53e-02	$a_{200}^{\{A\}}$	3.50e-02	$a_{201}^{\{A\}}$	4.01e-01	$a_{210}^{\{A\}}$	-2.83e-01	$a_{300}^{\{A\}}$	-1.65e-01
$b_{001}^{\{A\}}$	6.26e+00	$b_{002}^{\{A\}}$	2.89e+00	$b_{003}^{\{A\}}$	1.82e+00	$b_{010}^{\{A\}}$	6.31e+00	$b_{011}^{\{A\}}$	2.81e+00	$b_{012}^{\{A\}}$	1.71e+00
$b_{020}^{\{A\}}$	2.96e+00	$b_{021}^{\{A\}}$	1.64e+00	$b_{030}^{\{A\}}$	1.57e+00	$b_{100}^{\{A\}}$	5.13e+00	$b_{101}^{\{A\}}$	2.43e+00	$b_{102}^{\{A\}}$	1.55e+00
$b_{110}^{\{A\}}$	2.36e+00	$b_{111}^{\{A\}}$	1.44e+00	$b_{120}^{\{A\}}$	1.36e+00	$b_{200}^{\{A\}}$	2.30e+00	$b_{201}^{\{A\}}$	1.43e+00	$b_{210}^{\{A\}}$	1.31e+00
$b_{300}^{\{A\}}$	1.38e+00	$a_{000}^{\{B\}}$	1.43e-01	$a_{001}^{\{B\}}$	-3.00e-02	$a_{002}^{\{B\}}$	1.92e-01	$a_{010}^{\{B\}}$	1.06e-01	$a_{011}^{\{B\}}$	-1.90e-01
$a_{020}^{\{B\}}$	2.20e-01	$a_{100}^{\{B\}}$	2.53e-01	$a_{101}^{\{B\}}$	3.20e-01	$a_{110}^{\{B\}}$	-3.44e-02	$a_{200}^{\{B\}}$	-4.49e-01	$a_{00}^{\{B\}}$	2.52e-01
$a_{01}^{\{B\}}$	-5.05e-01	$a_{02}^{\{B\}}$	8.21e-02	$a_{10}^{\{B\}}$	-4.71e-02	$a_{11}^{\{B\}}$	2.61e-01	$a_{20}^{\{B\}}$	2.67e-01	$b_{001}^{\{B\}}$	1.00e+00
$b_{002}^{\{B\}}$	1.00e+00	$b_{003}^{\{B\}}$	1.00e+00	$b_{010}^{\{B\}}$	9.99e-01	$b_{011}^{\{B\}}$	1.00e+00	$b_{012}^{\{B\}}$	1.00e+00	$b_{020}^{\{B\}}$	1.00e+00
$b_{021}^{\{B\}}$	1.00e+00	$b_{030}^{\{B\}}$	1.00e+00	$b_{100}^{\{B\}}$	1.00e+00	$b_{101}^{\{B\}}$	1.00e+00	$b_{102}^{\{B\}}$	1.00e+00	$b_{110}^{\{B\}}$	1.00e+00
$b_{111}^{\{B\}}$	1.00e+00	$b_{120}^{\{B\}}$	1.00e+00	$b_{200}^{\{B\}}$	1.00e+00	$b_{201}^{\{B\}}$	1.00e+00	$b_{210}^{\{B\}}$	1.00e+00	$b_{300}^{\{B\}}$	1.01e+00

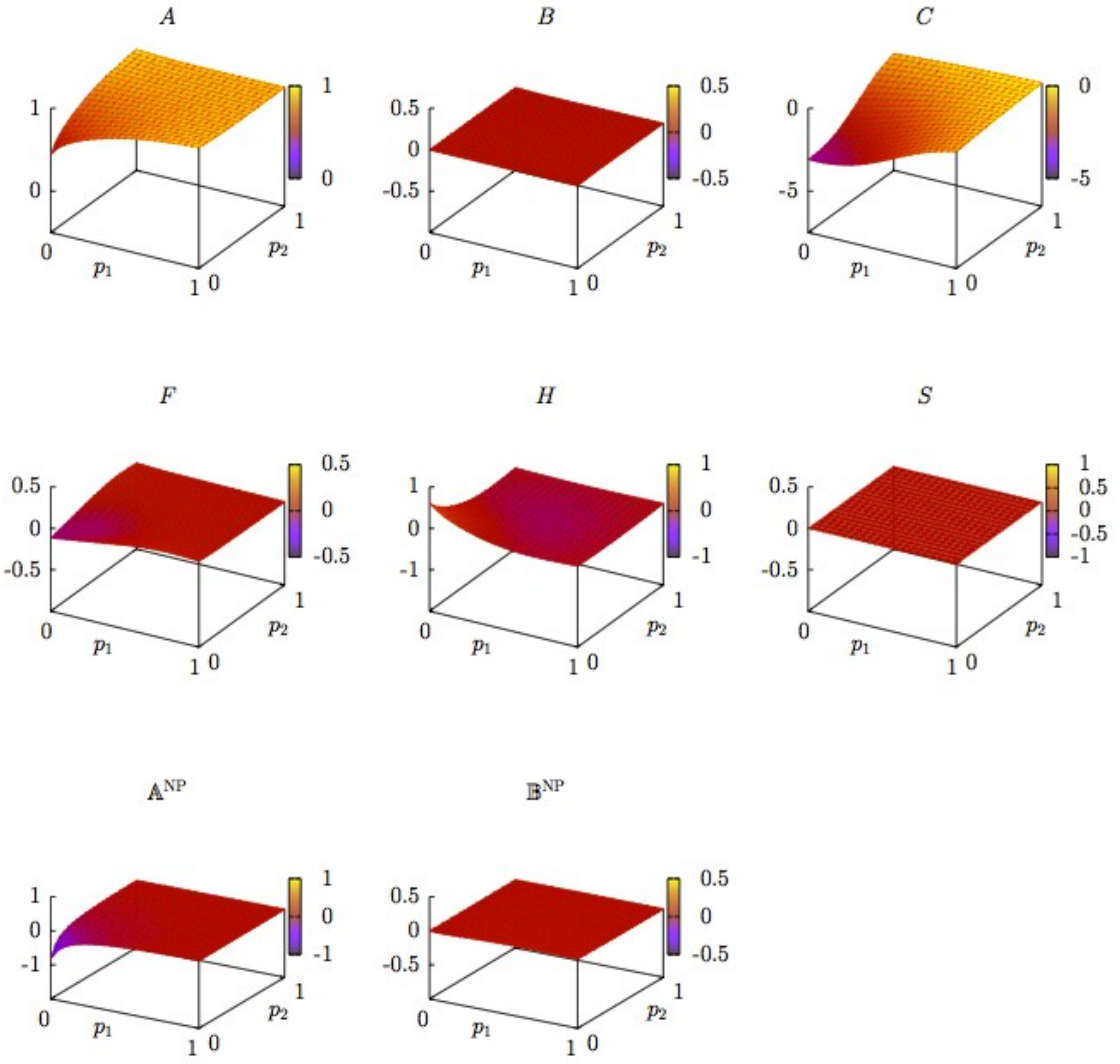
Results


Π_T
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0 1 2 3 4 5
p

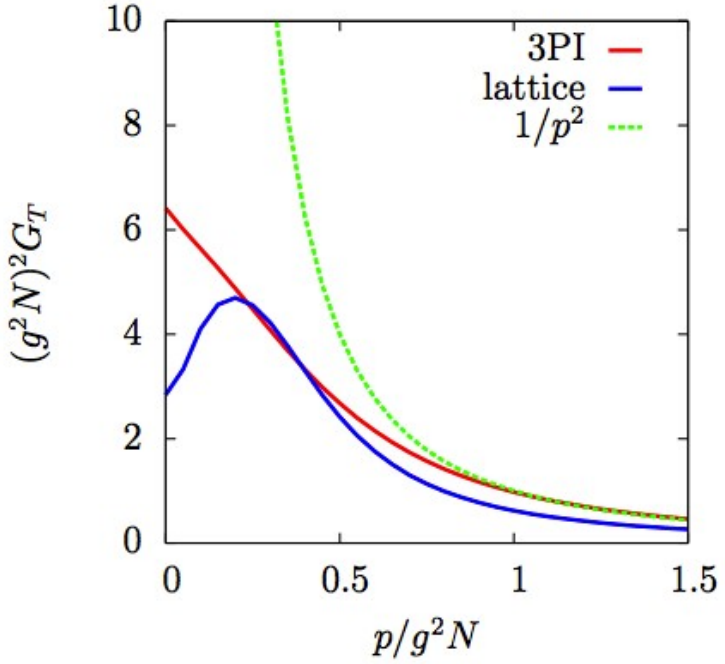

Π_L
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0 1 2 3 4 5
p


Σ
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5
p
$\xi = 0.0$
$\xi = 0.5$
$\xi = 1.0$
$\xi = 2.0$

Results



Results



Lattice data: A. Cucchieri and T. Mendes (~ 2009)

Summary:

- The 3PI variational problem **is** well posed
- In 3D divergences can be regulated
- Integrals and “extremum-finding” can be performed numerically
- Resummed 2 and 3 point functions were obtained

Outlook:

- Method should be tested with 3D Yang Mills + Higgs
- Extension to 4PI, 4-loops, i.e. resummed 4-point functions
- ***Implementation in 4 dimensions...***